CONTROLS

NOTES

GATE 2009

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DATE : May. 20, 2007.

EE

7.00 - 1.00 ⇒ Control Systems → Hall 7

2.30 - 5.30 ⇒ Digital electronics → Hall 7.

21-05-07

CONTROL SYSTEMS → 15 Marks.

1. Nagrath & Gopal.
2. B.C. KUO
3. IES / IAS papers G.K. publishers.
4. A.K. jairath

→ T/f, Block diagram, signal flow — 2 M

→ Time Domain Analysis → 4 M {f/b changes the location of poles.}

→ stability [RH/ RL / BP/NP] → 4 to 6 M} → for closed loop

→ Compensators (PID controllers) → 2 M

→ state space ~~steady state~~ (Ogota) Analysis → 2 to 4 M → Multi i/p, Multi o/p.

Transfer functions

→ order of the system → no. of (or) one time constant storage elements

T/f is a mathematical equivalent Model for a system.

T/f valid for → Linear time invarient (LTI)

TDA (time domain Analysis) → to know about the performance {Time domain specifications} of the system. w.r.t. time.

→ for unbounded signals (ramp) we donot find the stability

State space Analysis $\xrightarrow{\text{for}}$ Dynamic Systems [linear / Non-linear / time varient / invarient]

→ -ve f/b → poles shifted to left

+ve f/b → poles shifted to right

→ In closed loop systems if order of the system is very high, it is difficult to find roots of T/F. so we use

* RH → char. eq to find CL. stability ↲ RH, RL/BP/NP

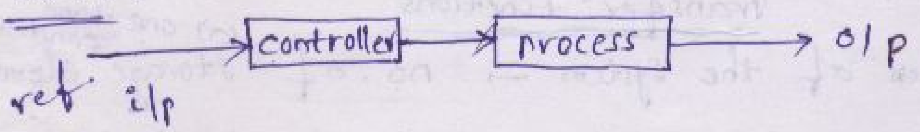
* RL / BP / NP → O/L

* order → NP, RL, BP, RH.

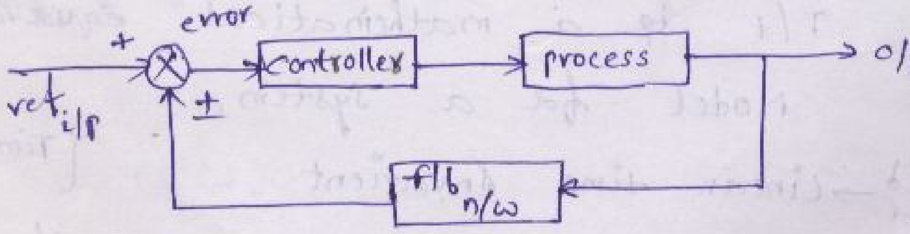
⇒ Control system: It is an arrangement of group of phy. components in such a way that it gives the desired o/p by means of controller. either direct method or indirect.

→ Based on the controller action, control systems ⊏ O/L system / C/L system.

OLCS :-



CLCS :-



OLCS :- A system in which the controller action is inde. of o/p. Eg :- fans, heater, normal, iron box, traffic lights

Eg :- Any system which not sense the o/p.

CL CS :- The controller action is totally depends on o/p. Eg :- Any m/c with Automatic {Refrigerator, Iron box automatic} which sense the o/p.

⇒ **F/b n/w :-** It is nothing but a transducer which converts energy from one form to the another form.

* It consists passive elements R, L, C. The max. value of f/b n/w ratio is one.

⇒ f/b is the property of the CL system which brings the o/p to the actual i/p ~~is~~ ~~used~~ to compare with ref i/p and generates error signal, then the controller is adjusted such that error becomes zero.

⇒ **T/F :-** It is a mathematical equivalent model for the system.

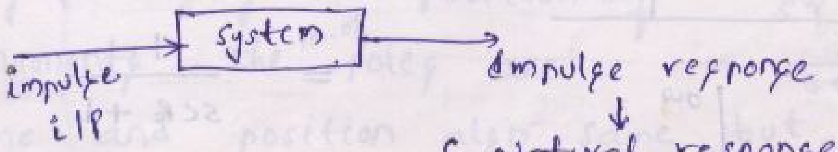
DEF: A T/F of a Linear time invariant (LTI) is defined as ratio of L.T o/p to L.T i/p. with all initial condi.s are zero.

(low pass → Integrator)

Linear systems → Transfer function

Non- linear → Decribing function

DEF 2: A T/F of a LTI, is also defined as L.T. of impulse response with all initial condi.s are zero.

* 

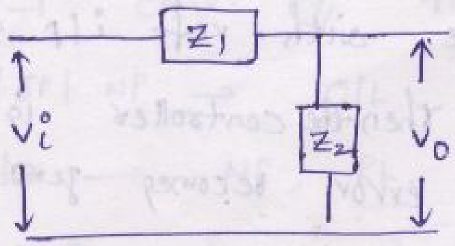
↓

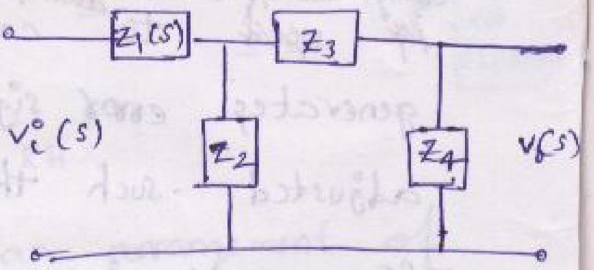
{ Natural response or actual system response or free }
forced response.

* for, ramp, step → forced response.

⇒ T/f → Electrical n/ω
→ Differential eq.
→ Signal response

↪ Electrical n/ω :-



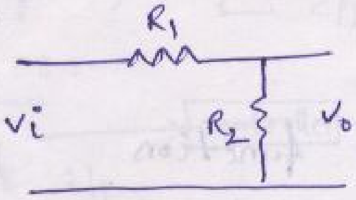


$$\frac{V_o(s)}{V_i(s)} = \frac{Z_2(s)}{Z_1(s)+Z_2(s)}$$

$$\frac{V_o(s)}{V_i(s)} = \frac{Z_2(s)\cdot Z_4(s)}{Z_1(s)[Z_2(s)+Z_3+Z_4] + Z_2[Z_3+Z_4]}$$

Q. find the T/f for the following :- and represent poles and zeros in s-plane.

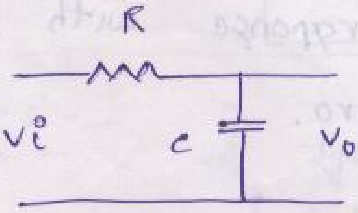
(i).



$$\frac{V_o}{V_i} = \underset{\uparrow}{\frac{R_2}{R_1+R_2}}$$

* attenuation factor
[No poles & zeros]
because no storage elements.

(ii).



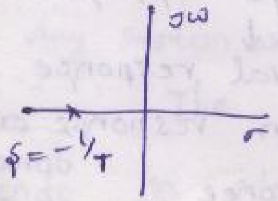
$$\frac{V_o}{V_i} = \frac{1/cs}{R + 1/cs}$$

$$= \frac{1}{scR+1}$$

take $\tau = RC$

$$= \frac{1}{s\tau+1}$$

(first order) standard form



* A pole is nothing but −ve of inverse of system time constant at which the magnitude of T/F is ~~zero~~ infinity.

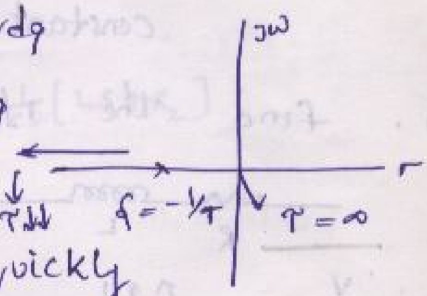
*→ Behaviour of the system is given by τ.

* If τ ↑, (large) system response is slow.

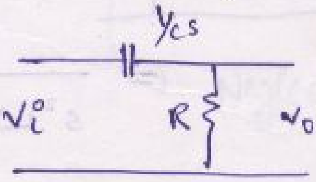
* τ at origin is infinity.

→ τ is nothing but −ve of inverse of dominent pole location τ = −1/pole.

* As the pole moves towards to the left, the τ is decreased and system reaches steady state quickly and becomes more stable.



(iii).



$$T/F : \frac{v_o}{v_i} = \frac{R}{R + 1/cs}$$

$$= \frac{csR}{scR + 1}$$

Let τ = RC

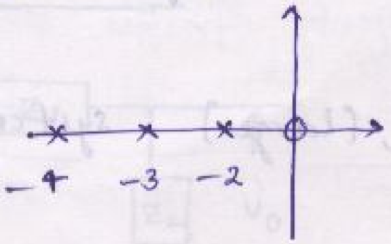
$$= \frac{s\tau}{s\tau + 1}$$

$$= \frac{s}{s + 1/\tau}$$

* By changing the position of components the no. of poles are same and position also same but the no. of zeros changes and ~~gives~~ position changes.

⟹ A zero is −ve of inverse of system time constant at which magnitude of T/f is zero.

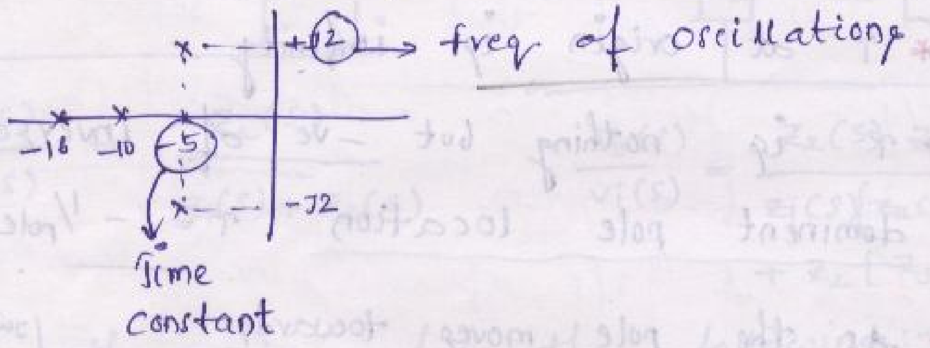
(iii). findout time constant,



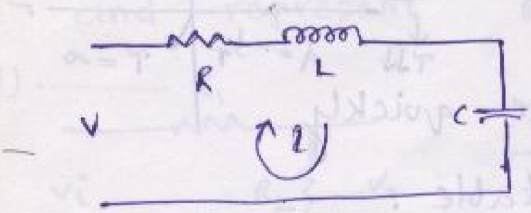
$\tau = -\frac{1}{(-2)}$

$= 0.5$

(iv).



(v). find the T/f.



2 storage elements → 2 order.

$V(s) = I(s)\left[R + sL + \frac{1}{sC}\right]$

$T/f = \frac{I}{V} = \frac{1}{R + sL + \frac{1}{sC}}$

$= \frac{sC}{s^2LC + sCR + 1}$

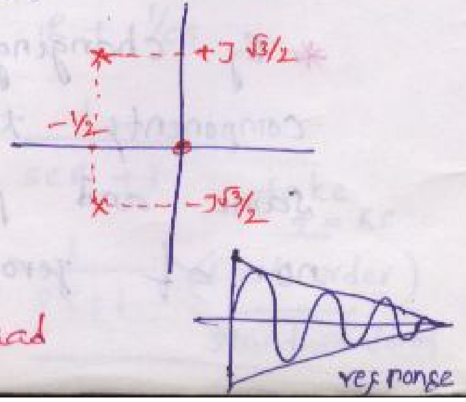
Let $L = 1H$
$C = 1F$
$R = 1\Omega$

Then locate poles & zeros. and explain what type of response.

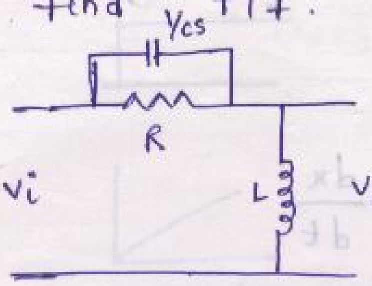
$\frac{I}{V} = \frac{s}{s^2 + s + 1}$

Time constant $= 2$

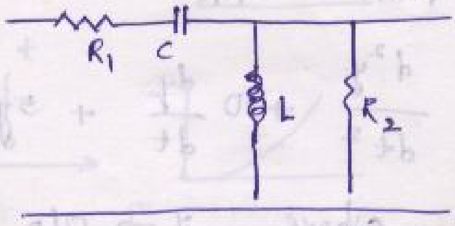
freq of oscillation $= \frac{\sqrt{3}}{2}$ rad



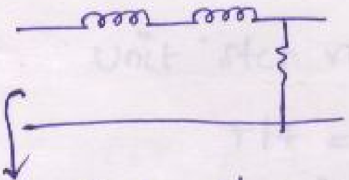
(vi). find T/F.



(vii).

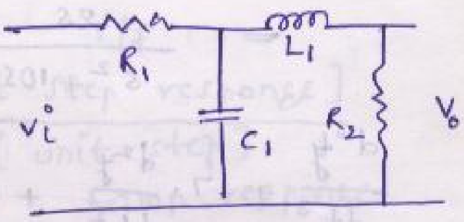


→ for electrical n/ω, Modern control system by A.K. Jairath.



Only first order

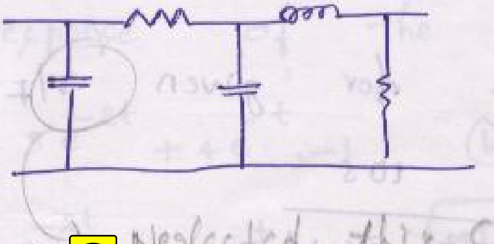
(viii).



Ans. (viii).

$$\frac{V_o}{V_i} = \frac{\frac{1}{cs}\cdot R_2}{R_1\left[\frac{1}{cs}+Ls+R_2\right]+\frac{1}{cs}\left[Ls+R_2\right]}$$

Eg :-



Neglected this capacitance.

→ Differential equations :- [D.E.]

1. find T/f.

$$\frac{d^2y}{dt^2} + 10\frac{dy}{dt} + 5y = 2\frac{dx}{dt}$$

where $y \rightarrow$ o/p & $x \rightarrow$ i/p

$$\frac{Y(S)}{X(S)} = \frac{\text{i/p related terms}}{\text{o/p related terms}}$$

$$= \frac{2S}{S^2 + 10S + 5}$$

2. $$\frac{d^3y}{dt^3} + 7 \cdot \frac{d^2y}{dt^2} + 10 = 5 \cdot \frac{d^2x}{dt^2}$$

$$T/F = \frac{5}{S+7}$$

* Here 10 is a initial condi. So in T/f evaluation initial condi-s are zero.

3. Obtain D.E for given T/f.

$$\frac{Y(S)}{X(S)} = \frac{10S}{S^2 + 7S + 6}$$

$$\frac{d^2y}{dt^2} + 7 \cdot \frac{dy}{dt} + 6y = 10 \cdot \frac{dx}{dt}$$

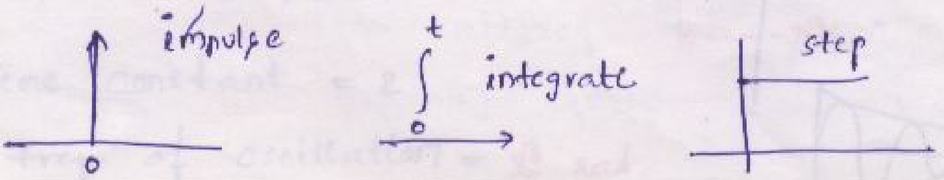
$\downarrow$

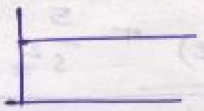
+k

→ Signals Response :-

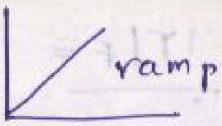
TF = L[impulse response]

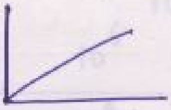
(1).



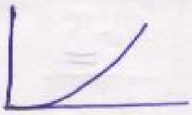
(2).


$$\int_0^t \longrightarrow$$



(3).


$$\int_0^t \longrightarrow$$



⇒ Types of questions :-

Given	find
(1). Step response	T/f

$$T/f = L[\text{Impulse response}] = 0$$

(2). Unit step response	T/f

$$T/f = \frac{L[\text{unit step response}]}{L[\text{unit step}]}$$

(3). Impulse response	Ramp response

$$\int_0^t \int_0^t - \, dt$$

1. The unit impulse response of a system is $c(t) = -4.e^{-t} + 6.e^{-2t}$, $(t \geq 0)$. The step response of the system is ?

ⓐ. $-3e^{-2t} + 4e^{-t} - 1$ ⓑ. $-3e^{-2t} - 4e^{-t} - 1$

ⓒ. $3e^{-2t} + 4e^{-t} - 1$

(4). Ramp Step

$$T/f = \frac{L[U.R.R]}{L[U.R.]}$$

just do integrate

$$\int_0^t c(t) =$$

2. The unit step response is $ø(t) = \frac{5}{2} - \frac{5}{2}e^{-2t} + 5t$

The T/f is —?

$$T/F = \frac{L[\text{unit step response}]}{L[\text{unit step}]}$$

$$T/f = \frac{\frac{5}{2s} - \frac{5}{2(s+2)} + \frac{5}{s^2}}{1/s}$$

$$=$$

3. A system described by,

$\frac{d^2y}{dt^2} + 3 \cdot \frac{dy}{dt} + 2y = x(t)$ is initially at rest, for the i/p $x(t) = 2\,u(t)$. the o/p $y(t)$ is — ?

for response / o/p :-

ⓐ. first find T/f.

ⓑ. substitute i/p

ⓒ. partial fractions.

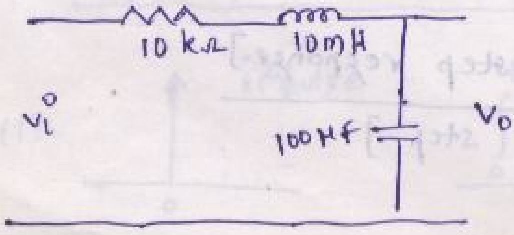
ⓓ. Apply L.T.

$$T/f = \frac{Y(s)}{X(s)} = \frac{1}{s^2+3s+2}, \quad X(s) = \frac{2}{s}$$

$$\rightarrow Y(s) = \frac{2}{s(s^2+3s+2)}$$

Ans. $2(1-2e^{-t}+e^{-2t})\,u(t)$

4. for the ckt shown in fig. initial condi.s are zero. its T/f is — ?



① $\frac{1}{s^2+10^6 s+10^6}$

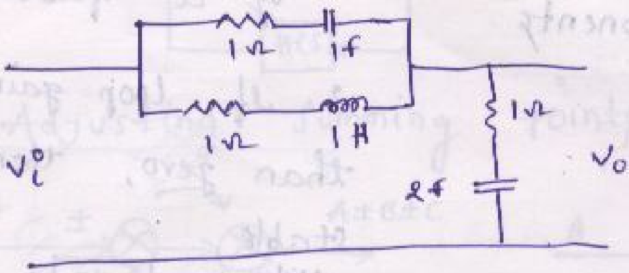
② $\frac{10^6}{s^2+10^3 s+10^6}$

③ $\frac{10^3}{s^2+10^3 s+10^6}$

$$\frac{V_o}{V_i} = \frac{\dfrac{1}{100 \times 10^{-6} s}}{\dfrac{1}{10^{-4} s} + 10^{4} + 10^{-2} s}$$

Ans. $\dfrac{10^6}{s^2 + 10^6 s + 10^6}$

5.



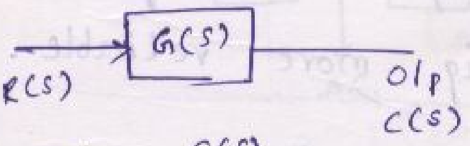
$$\frac{\left(1+\frac{1}{s}\right)\left(1+s\right)}{2+s+1/s} = 1 , \quad \frac{V_o}{V_i} = \frac{1+\frac{1}{2s}}{1+1+\frac{1}{2s}}$$

$$= \frac{2s+1}{4s+1}$$

⇒ **Block diagram :-**

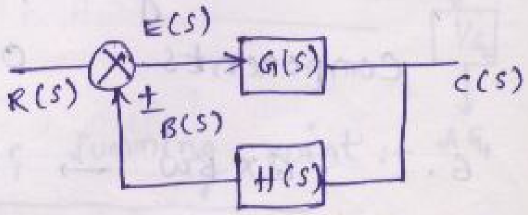
It is a short hand pictorial representation of system b/w i/p & o/p.

OLCS



$T/F = \dfrac{C(s)}{R(s)} = G(s)$

CL CS



G(s) – forward path gain $= \dfrac{C(s)}{E(s)}$.

H(s) – f/b path gain $= \dfrac{B(s)}{C(s)}$

G(s). H(s) — open loop T/F

This represents actual system $= \dfrac{B(s)}{E(s)}$

CL, $T/F = \dfrac{G(s)}{1 \mp G(s).H(s)}$

* Oscillators → +ve f/b → unstable

* Multivibrators → +ve f/b → stable

Comparision :-

OLCS	CLCS
1. No f/b	1. gain will be reduced by a factor (1+ GH).
2. Less components	2. If loop gain more than zero, then more stable, stability so depends on loop gain
3.	3. Accuracy is depends on the f/b n/w.
	4. Less sensitive with f/b the sensitivity improved, the sensitivity factor is less.

The better is less sensitive.

5. Reliability depends on no. of components. OLCS is more reliable.

6. G × BW → constant.

operating area is a bandwidth.

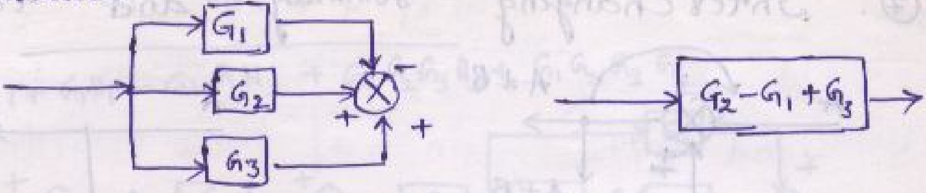
so for CLCS bandwidth is more

→ BLOCK DIAGRAM REDUCTION RULES :-

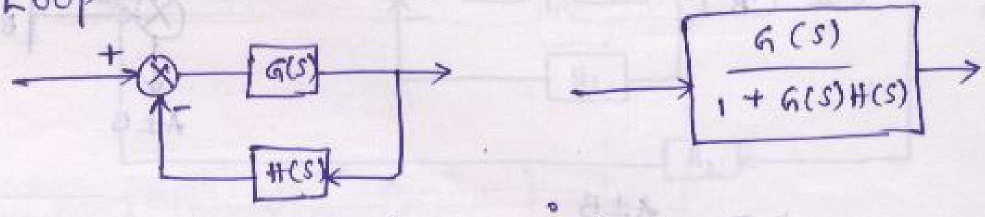
① . Cascade / series

valid for signal flow graph also

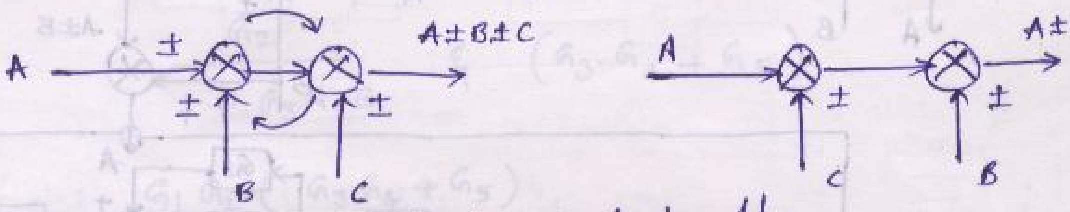
②. Parallel



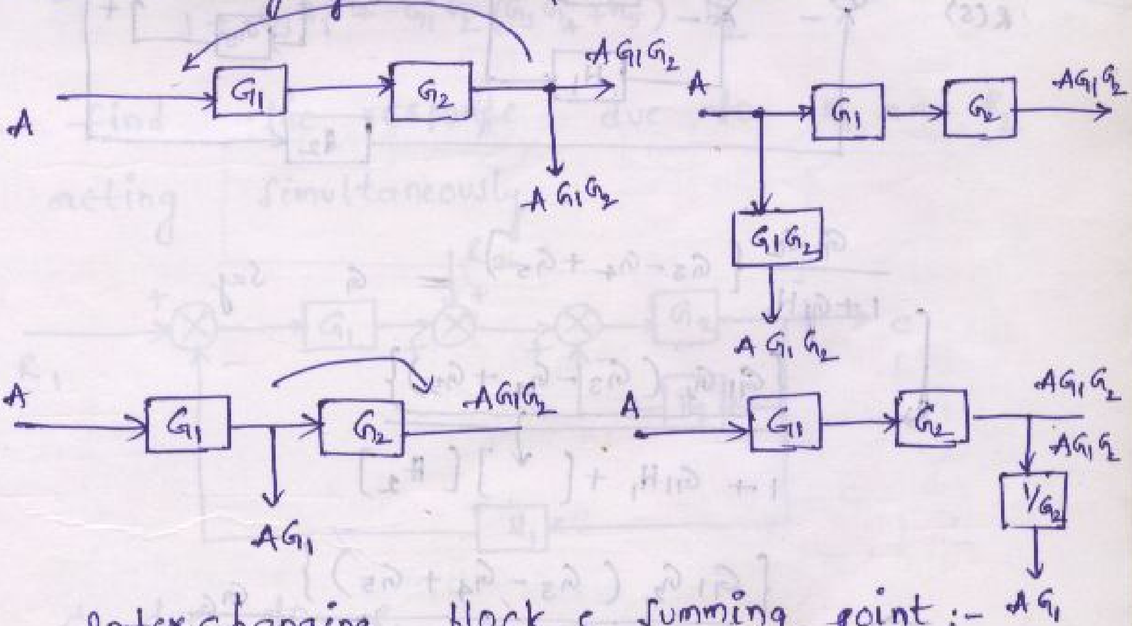
③. Loop



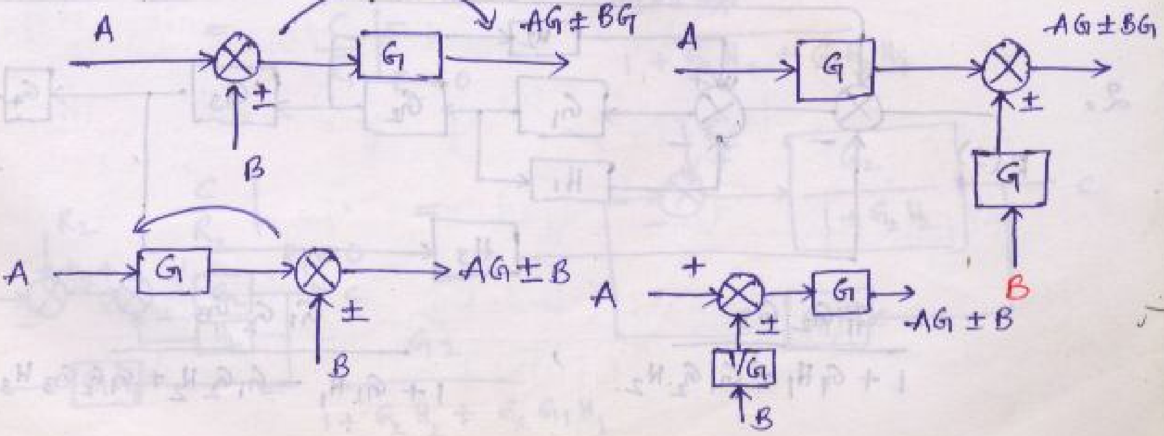
④. Adjusting summing points



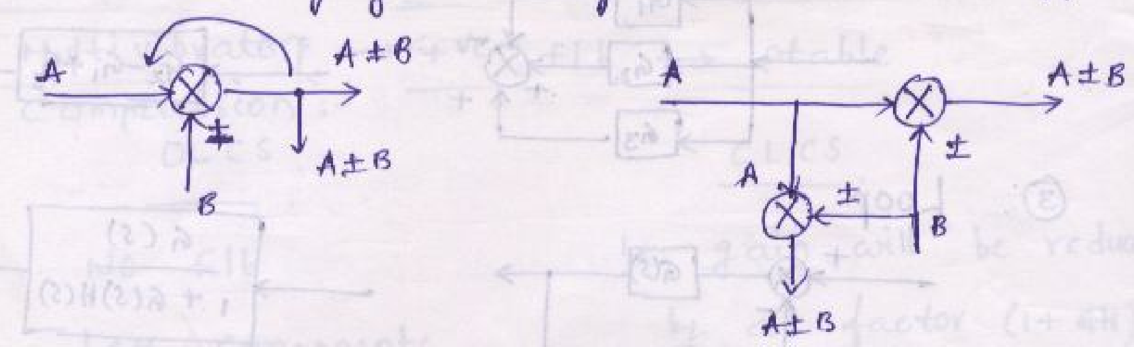
⑤. Interchanging block & take off :-

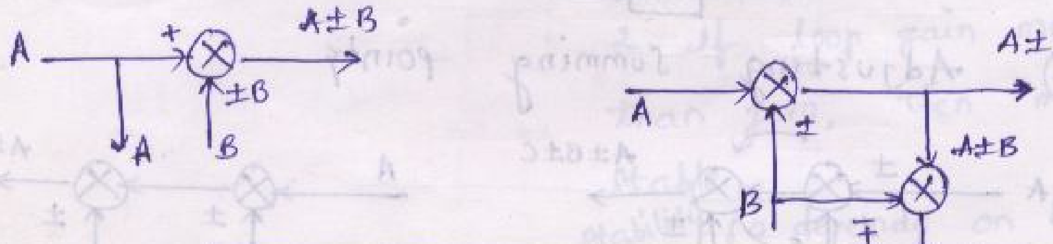


⑥. Interchanging block & summing point :-

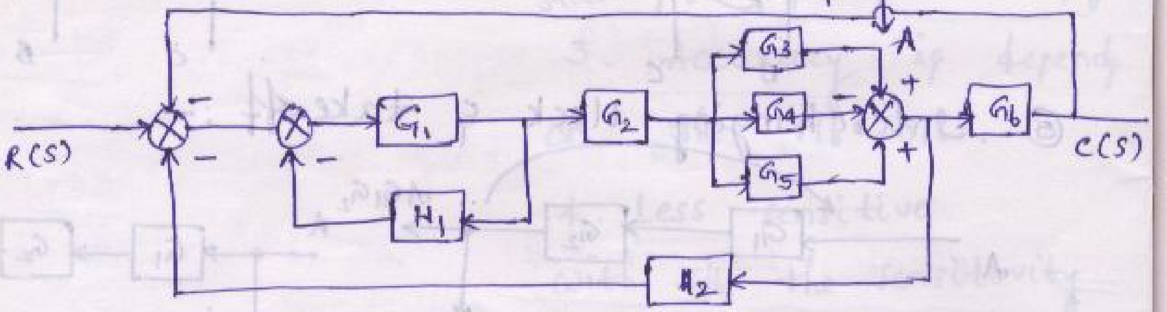


⑦. Interchanging summing and take-off :-





1.

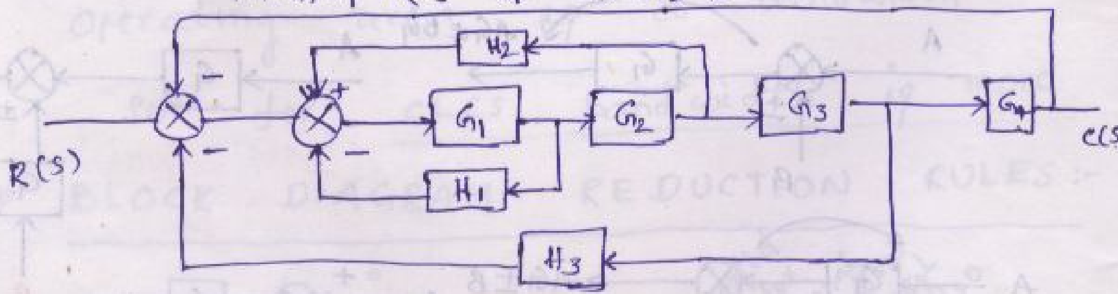


$$\frac{G_1 G_2}{1+G_1 H_1}\{G_3 - G_4 + G_5\} = G \quad \text{Say}$$

$$\frac{\{G_1 G_2 (G_3 - G_4 + G_5)\}}{1 + G_1 H_1 + [\quad][H_2]}$$

$$\Rightarrow \frac{\{G_1 G_2 (G_3 - G_4 + G_5)\}\, G_6}{1 + G_1 H_1 + (G_3 - G_4 + G_5) G_1 G_2 H_2 + [\quad]1.}$$

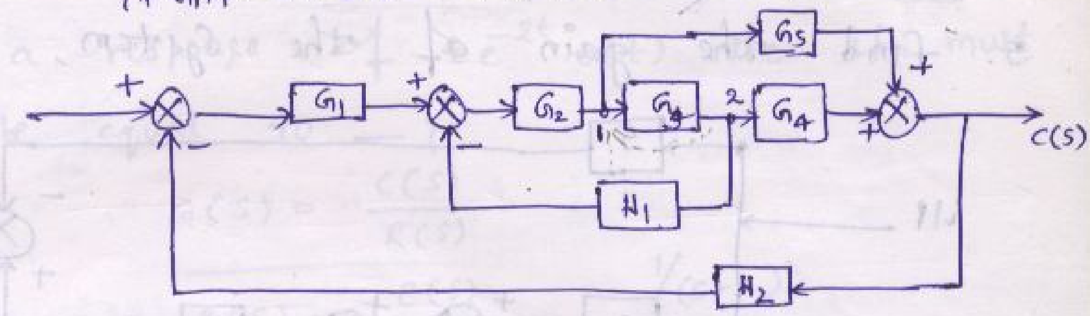
2.



$$\frac{G_1 G_2 G_3}{1 + G_1 H_1 - G_1 G_2 H_2}, \qquad \frac{G_1 G_2 G_3}{1 + G_1 H_1 - G_1 G_2 H_2 + G_1 G_2 G_3 H_3}$$

$$= \frac{G_1 G_2 G_5 G_4}{1 + G_1 H_1 - G_1 G_2 H_2 + G_1 G_2 G_5 H_3 + G_1 G_2 G_3 G_4}$$

3.

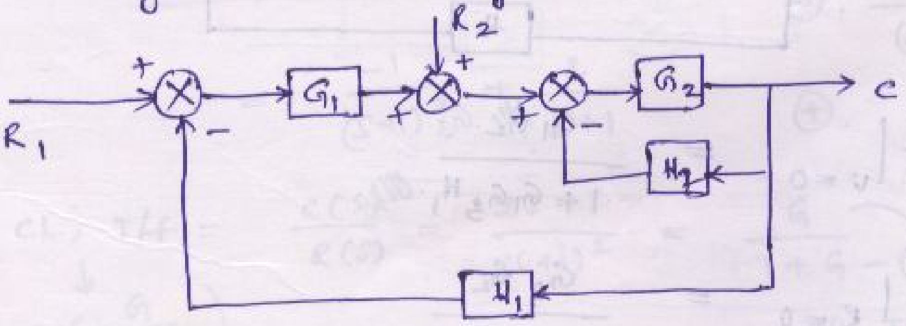


2 to 1 :-

$$\frac{G_2}{1 + G_3 H_1 \cdot G_2} \quad \& \quad (G_3 \cdot G_4 + G_5)$$

$$= \frac{G_1 G_2 (G_3 G_4 + G_5)}{1 + G_3 H_1 \cdot G_2 + G_1 G_2 (G_3 G_4 + G_5) \cdot H_2}$$

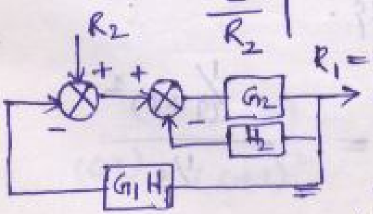
4. find the response due to R_1 and R_2 acting simultaneously.



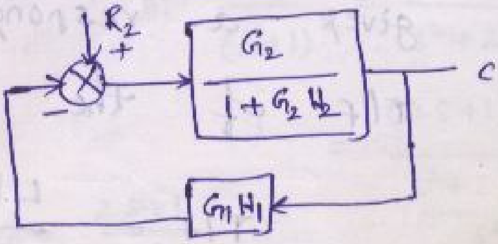
C due to R_1,

$$\left.\frac{C}{R_1}\right|_{R_2=0} = \frac{G_1 G_2}{1 + G_2 H_2 + G_1 G_2 H_1}$$

$$\left.\frac{C}{R_2}\right|_{R_1=0}$$



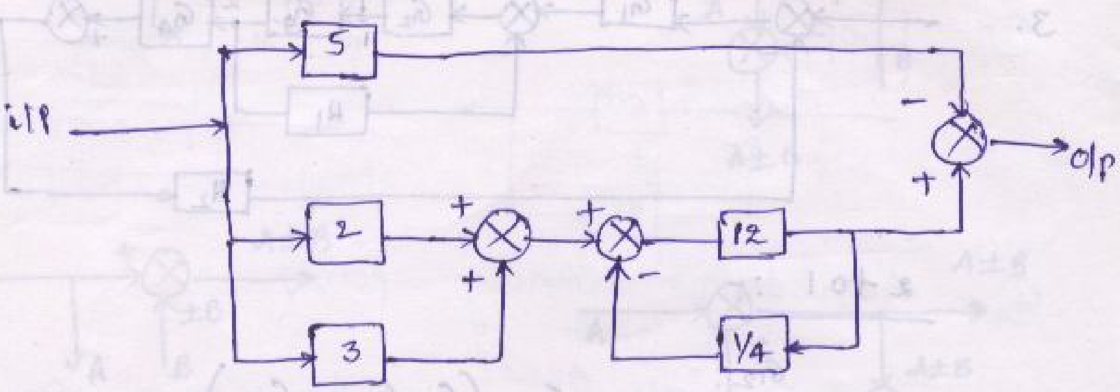
$=$



$$= \frac{G_2}{1 + G_2 H_2 + G_2 G_1 H_1}$$

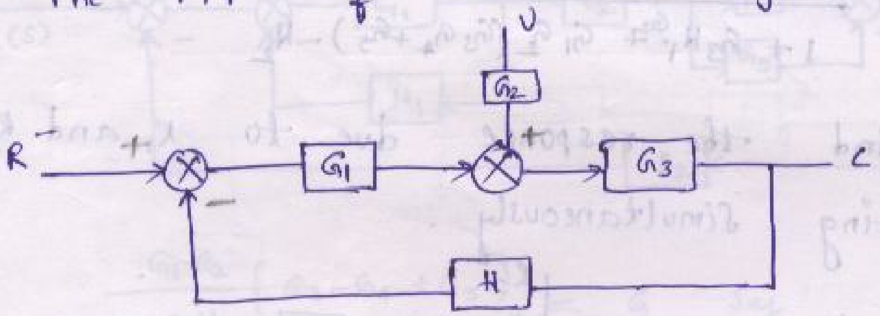
$$\therefore \quad C = \frac{G_1 G_2 R_1 + G_2 R_2}{1 + G_2 H_2 + G_1 G_2 H_1}$$

5. find the gain of the system,



Ans: 10.

6. The T/f of the block diagram below is



$$\frac{C}{R}\Big|_{U=0} = \frac{G_1 \cancel{G_2} G_3}{1 + G_1 G_3 H_1 \cdot \cancel{G_2}}$$

$$\frac{C}{U}\Big|_{R=0} = \frac{G_3 G_2}{1 + G_1 G_3 H_1 \cancel{G_2}}$$

7. A linear time invariant system initially at rest, when subjected to unit step, gives a response of $y = te^{-t}$. The T/f of the system is.

$$T/f = \frac{L[U.S.R]}{L[U.S]} = \frac{1/(s+1)^2}{1/s}$$

✓8. The impulse response of an initially relaxed linear system is $e^{-2t}u(t)$ to produce a response of $te^{-2t}u(t)$ the i/p must be equal to — ?

$$G(s) = \frac{C(s)}{R(s)}$$

$$\Rightarrow R(s) = \frac{C(s)}{G(s)} = \frac{1/(s+2)^2}{1/(s+2)}$$

$$= \frac{1}{s+2} = e^{-2t}.u(t).$$

✓9. The unit impulse response of an unity (H = 1) f/b control system is $c(t) = (-te^{-t} + 2e^{-t}).u(t)\ t \geq 0$

The open ~~cl~~ loop T/f (G) = ?

① . $\frac{2s+1}{(s+1)^2}$

② . $\frac{s+1}{s^2}$

③ . $\frac{s+1}{(s+2)^2}$

④ . $\frac{2s+1}{s^2}$,

cl . T/f = L [impulse response] with initial condi = 0

$$= \frac{-1}{(s+1)^2} + \frac{2}{s+1}$$

$$\text{CL, T/f} = \frac{C(s)}{R(s)} = \frac{2s+1}{(s+1)^2} = \frac{G}{1+G-(\;)}$$

↓ $\left(\frac{G}{1+GH}\right)$

At H = 1, $\frac{G}{1+G}$

O/L T/f = G

$$= \frac{2s+1}{(s+1)^2 - (\;)}$$

$$\text{OL T/f (G)} = \frac{2s+1}{(s+1)^2 - 2s - 1}$$

$$= \frac{2s+1}{s^2}$$

$$\frac{2s+1}{(s+1)^2} = \frac{2s+1}{s^2+2s+1}$$

$$= \frac{(2s+1)/s^2}{\frac{s^2+2s+1}{s^2}}$$

(or) $\frac{2s+1}{(s+1)^2} = \frac{G}{1+G} \Rightarrow G = \frac{2s+1}{s^2}$

10. find OL DC gain of a unity f/b system of CL T/f is $\frac{s+4}{s^2+7s+13}$

① 4/13
② 4/9
③ 4
④ 14

DC gain means f = 0, ie sub. s = 0, $s = j\omega = j2\pi f$

$\rightarrow \frac{4}{13}$

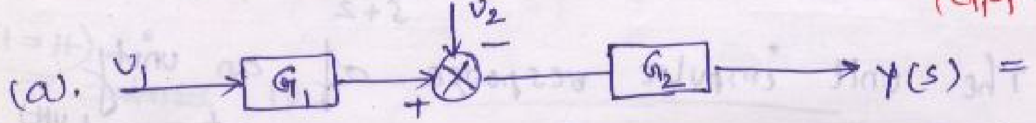
$\sqrt{G}$ OL gain

$\frac{G}{1+G-G}$

11. In block diagram shown the o/p Y(s) = ?

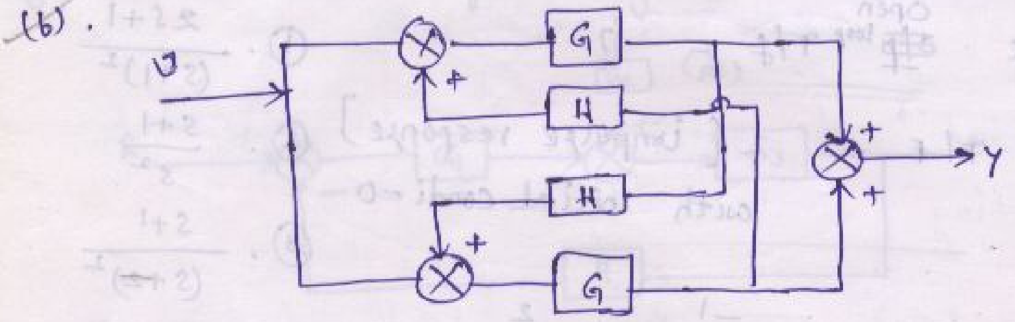
(a).

$(G_1U_1 - U_2)G_2$



$G_1G_2U_1 - G_2U_2$

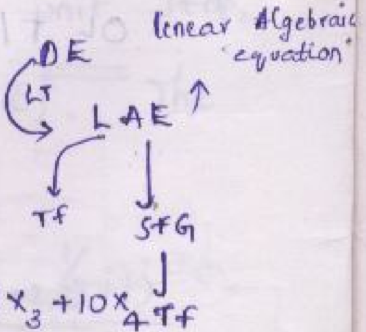
(b).



$$Y(s) = \frac{G}{1-GH} + \frac{G}{1-GH} = \frac{2G}{1-GH}$$

⇒ Signal flow Graph :-

It is a graphical representation of the system b/w the set of linear algebraic eq.s.



Q. construct signal flow graph for the following LAE s.

a). $x_2 = Ax_1$ b) $x_5 = x_1 - 2x_2 + 3x_3 + 10x_4$

c). $x_2 = 7x_1$, $x_3 = 5x_1$, $x_4 = 3x_1$

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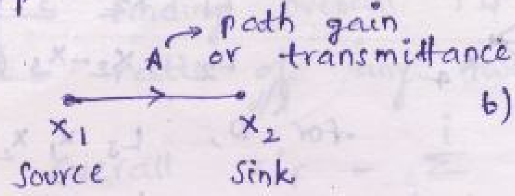
d). $y = mx + c$

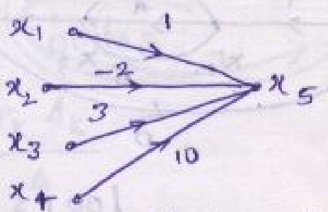
o/p ← y, i/p ← x

Ans:-

a).

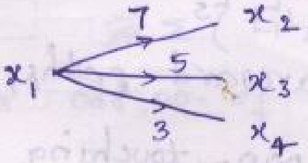


b).



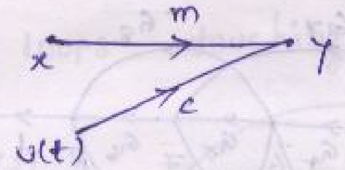
If all the signals are added at a particular node called as additional rule.

c).



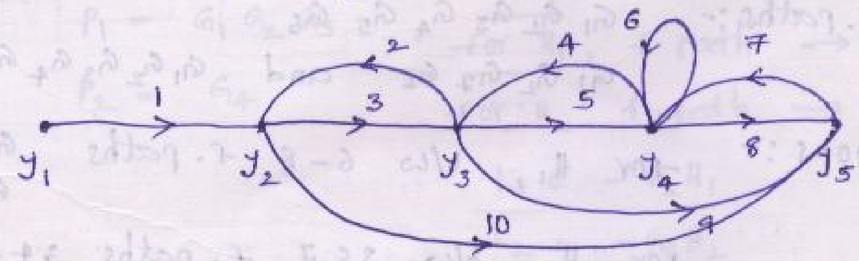
If the signal is transmitted from single node to many called transmission rule.

d).



Q. $y_2 = y_1 + 2y_3$, $y_3 = 3y_2 + 4y_4$, $y_4 = 5y_3 + 6y_4 + 7y_5$

$y_5 = 8y_4 + 9y_3 + 10y_2$



→ A node should be touched only once while selecting forward path/Loop.

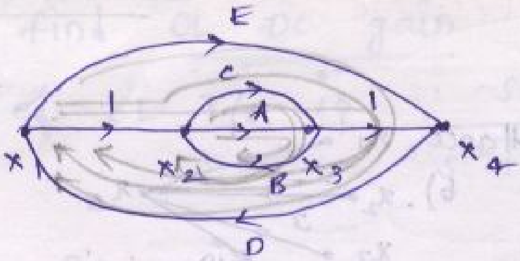
Loop:-

It is a path which terminates on the same node where it is started.

Non-touching loop:-

If there is no common node b/w 2 or more loop then it is called as non-touching loop.

Q.



f/w path B, $x_2 - x_3$

L_1 $x_2 - x_3$ Ⓐ

L_2 $x_2 - x_3$ Ⓒ

for f/b D, L_3 $x_1 x_2 x_3 x_4$

L_4 $x_1 x_2 x_3 x_4$

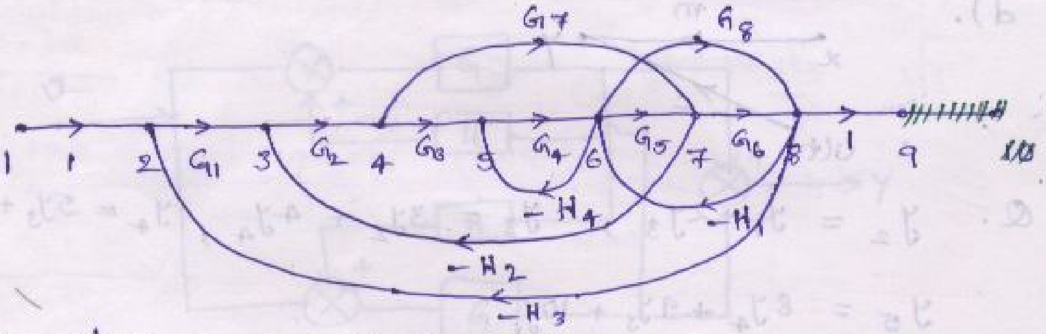
L_5 $x_1 x_4$

loops = 5

for non-touching, L_1 L_3, L_4, L_5 | L_2 L_3, L_4, L_5

= $L_1 L_5$ & $L_2 L_5$

Q. find the no. of farward paths, loops, 2 non-touching and 3 non-touching loops.



f. paths :- $G_1 G_2 G_3 G_4 G_5 G_6$

$G_1 G_2 G_7 G_6$ and $G_1 G_2 G_3 G_4 G_8$

loops :- for H_1, b/w 6-8 f. paths $G_5 G_6$, G_8 } 2

for H_2, b/w 3 & 7, f. paths 34567, 347 } 2

for H_3, b/w 2 & 8, f. paths 2345678, 23478, 234568 } 3

for H_4, b/w 5 & 6, f. paths 56 } 1

Total no. of loops: 8

Two-non touching loops → 3

Three-non touching loops → 0

[To findout Three-non touching loop, first select two-non touching loops and then check with other].

Mason's Gain formula :-

→ finding overall T/f
→ ratio of any two nodes

$$\text{Overall } T/F = \sum_{k=1}^{i} \frac{P_K \Delta_K}{\Delta}$$

P_K – kth farward path gain

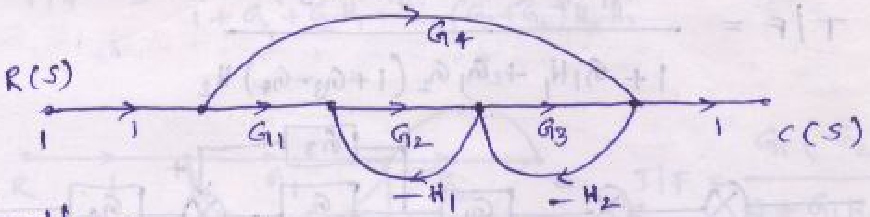
$$\Delta = 1 - \Sigma(L_1 + L_2 + L_3 + \cdots) + \overset{\text{two-non touching}}{\Sigma(L_1L_2 + L_1L_3 + \cdots)}$$

$$- \overset{\text{Three-non touching}}{\Sigma(L_1L_2L_3 + \cdots)} + \Sigma(L_1L_2L_3L_4 + \cdots)$$

for odd no. of non-touching loops, take opposite sign for loop gain & for same sign for even.

Δ_K - is obtained from Δ, by removing the loops touching the k th farward path.

Q.



f. paths :

$P_1 = G_1 G_2 G_3$

$P_2 = G_4$

Loops:-

for H_1, f. path → 1
for H_2, f. path → 1 } ② loops

$L_1 = -G_2 H_1$

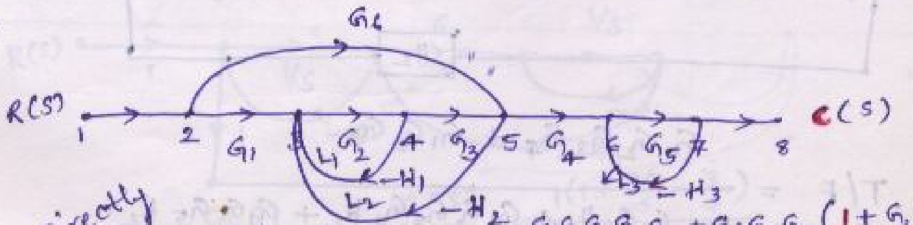
$L_2 = -G_3 H_2$

$$\Delta = 1 - (-G_2 H_1 - G_3 H_2)$$

$$\Delta_1 = 1 \quad ; \quad \Delta_2 = 1 - (-G_2 H_1)$$

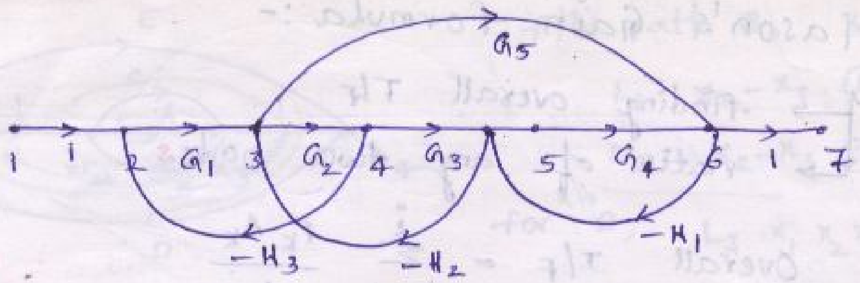
$$T/f = \frac{P_1 \Delta_1 + P_2 \Delta_2}{\Delta} = \frac{G_1 G_2 G_3 + G_4 (1 + G_2 H_1)}{1 + G_2 H_1 + G_3 H_2}$$

Q.



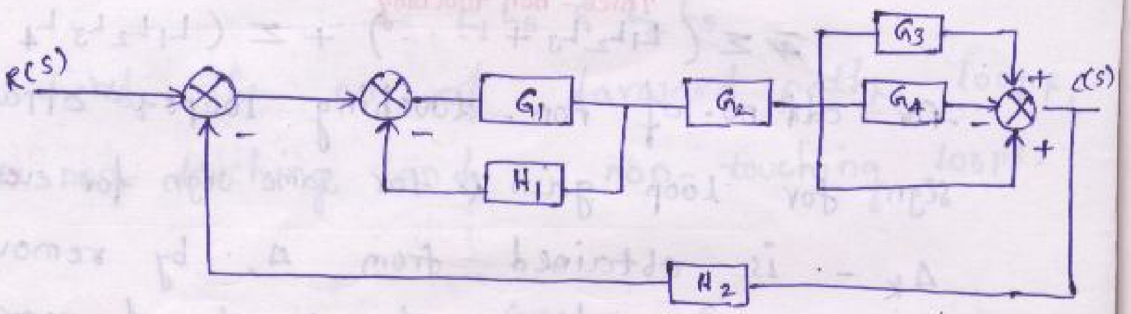
Directly write the Transfer function of → $T/F = \dfrac{G_1G_2G_3G_4G_5 + G_6G_4G_5(1 + G_2H_1)}{1 + G_2H_1 + G_2G_3H_2 + G_5H_3 + L_1L_3 + L_2L_3}$

Q.



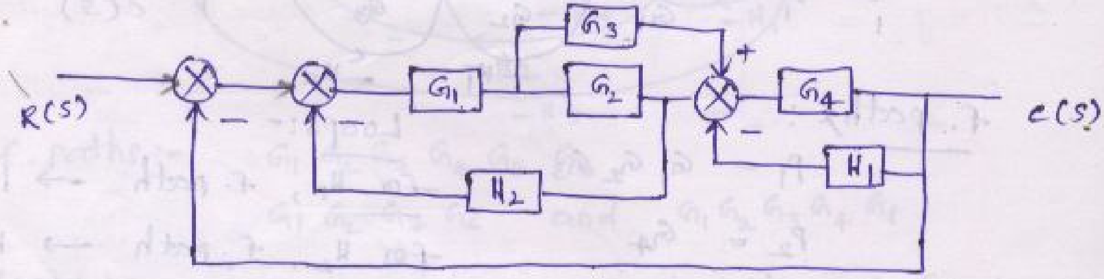
$$T/F = \frac{G_1G_2G_3G_4 + G_1G_5(1+0)}{1 + G_1G_2H_3 + G_2G_3H_2 + G_4H_1 + G_1G_2H_3G_4H_1 - G_5H_1H_2}$$

Q.



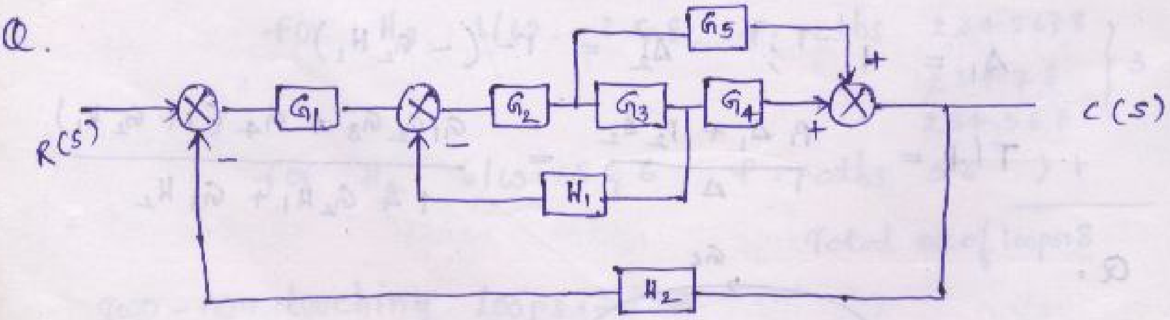
$$T/F = \frac{G_1G_2(1 - G_4 + G_3)}{1 + G_1H_1 + G_1G_2(1 + G_3 - G_4)H_2}$$

Q.



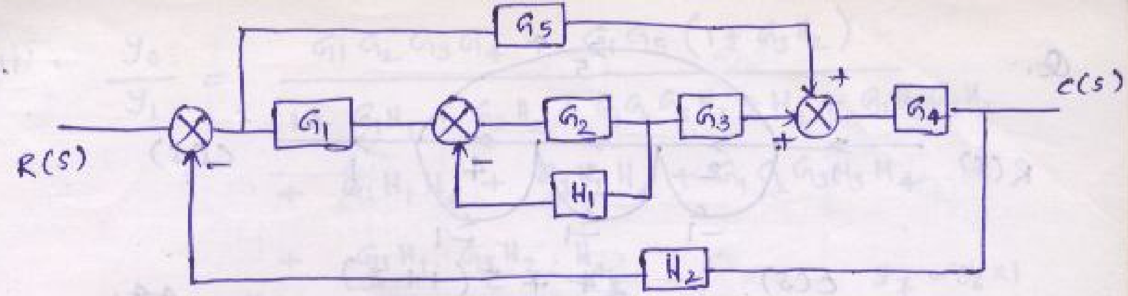
$$T/F = \frac{G_1G_2G_4 + G_1G_3G_4(1+0)}{1 + G_1G_2H_2 + G_4H_1 + G_1G_2G_4 + G_1G_3G_4 + G_1G_2H_2G_4H_1}$$

Q.



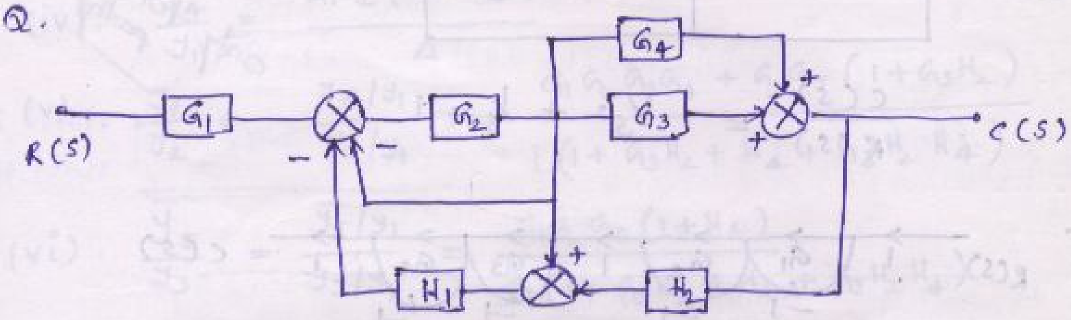
$$T/F = \frac{G_1G_2G_3G_4 + G_1G_2G_5}{1 + G_2G_3H_1 + G_1G_2G_3G_4H_2 + G_1G_2G_5H_2}$$

Q.



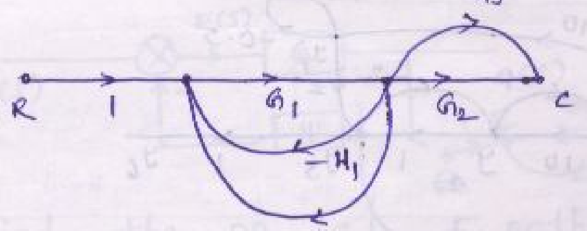
$$T/F = \frac{G_1G_2G_3G_4 + G_5G_4(1+G_2H_1)}{1+G_2H_1+G_1G_2G_3G_4H_2+G_5G_4H_2+G_2H_1G_5G_4H_2}$$

Q.



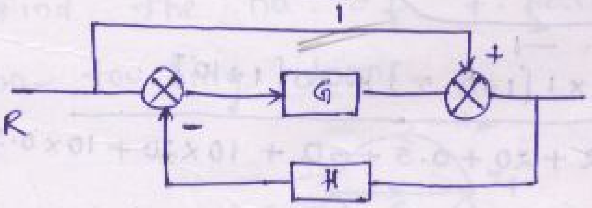
$$T/F = \frac{G_1G_2(G_3+G_4)}{1+G_2+G_2H_1+G_2(G_3+G_4)H_2H_1}$$

Q.



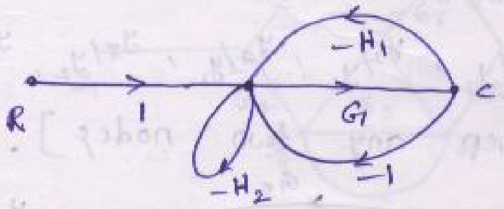
$$T/F = \frac{G_1(G_2+G_3)}{1+G_1H_1+(-G_1)}$$

Q.



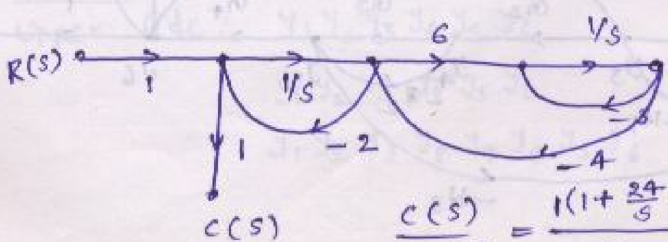
$$T/F = \frac{1+G}{1+GH}$$

Q.



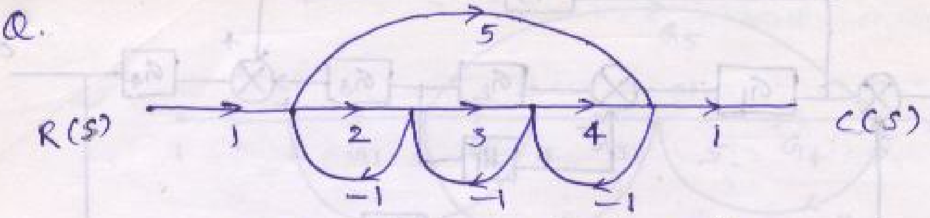
$$T/F = \frac{G}{1+H_2+G+GH_1}$$

Q.



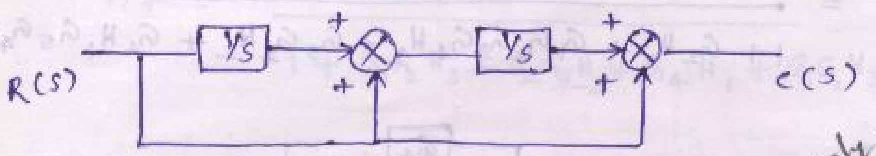
$$\frac{C(S)}{R(S)} = \frac{1(1+\frac{24}{s}+\frac{3}{s})}{1+2/s+\frac{24}{s}+\frac{3}{s}+\frac{6}{s^2}}$$

Q.



$$\frac{C(s)}{R(s)} = \frac{24 + 5(1+3)}{1+2+3+4+5+8} = \frac{44}{23}$$

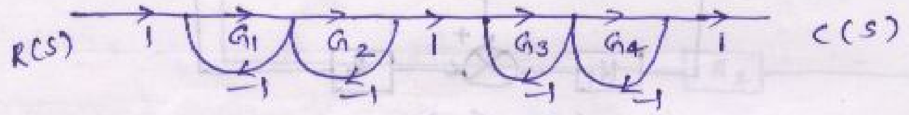
Q.



only forward paths

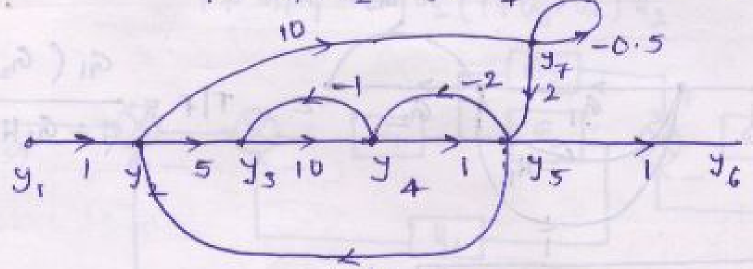
$$\frac{C(s)}{R(s)} = 1/s^2 + 1/s + 1$$

Q.



$$T/f = \frac{G_1 G_2 G_3 G_4}{1 + G_1 + G_2 + G_3 + G_4 + G_1 G_3 + G_1 G_4 + G_2 G_3 + G_2 G_4}$$

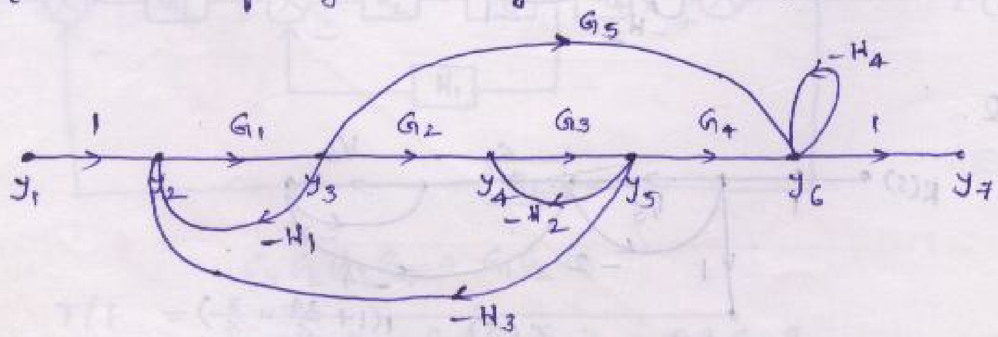
Q.



$$T/f = \frac{10\times5\times1[1+0.5] + 20[1+10]}{1+10+2+20+0.5+50+10\times20+10\times0.5 + 2\times0.5 + 50\times0.5} = 0.9$$

Q. Find y_6/y_1, y_7/y_1, y_2/y_1, y_4/y_1, y_7/y_2, y_5/y_3, y_4/y_3

[Ratio of given any two nodes].



(i). $$\frac{y_6}{y_1} = \frac{G_1G_2G_3G_4 + G_1G_5(1+G_3H_2)}{\begin{array}{l}1+G_1H_1+G_3H_2+G_1G_2G_3H_3+H_4+G_1H_1G_3H_2\\ +\ G_1H_1H_4+G_3H_2H_4+G_1G_2G_3H_3H_4\\ +\ G_1H_1.G_3H_2.H_4\end{array}}$$

$y_7 = y_6 \times 1$
$= y_6$

(ii). $$\frac{y_7}{y_1} = \frac{y_6}{y_1}$$ $(1-(L_1+L_2)+L_1L_2)$

(iii). $$\frac{y_2}{y_1} = \frac{1(1+G_3H_2+H_4+G_3H_2H_4)}{\Delta}$$

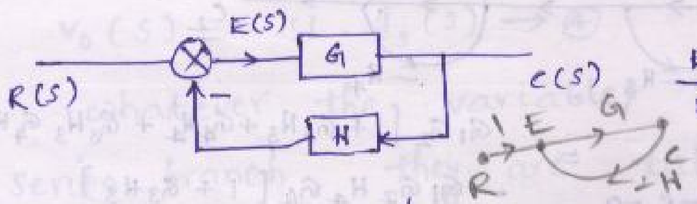
(iv). $$\frac{y_4}{y_1} = \frac{G_1G_2(1+H_4)}{\Delta}$$

(v). $$\frac{y_7}{y_2} = \frac{y_7/y_1}{y_2/y_1} = \frac{G_1G_2G_3G_4 + G_1G_5(1+G_3H_2)}{1(1+G_3H_2+H_4+G_3H_2H_4)}$$

(vi). $$\frac{y_5}{y_3} = \frac{y_5/y_1}{y_3/y_1} = \frac{G_1G_2G_3(1+H_4)}{G_1(1+G_3H_2+H_4+G_3H_2H_4)}$$

(vii). $$\frac{y_4}{y_3} = \frac{y_4/y_1}{y_3/y_1} = \frac{G_1G_2(1+H_4)}{G_1(1+G_3H_2+H_4+G_3H_2H_4)}$$

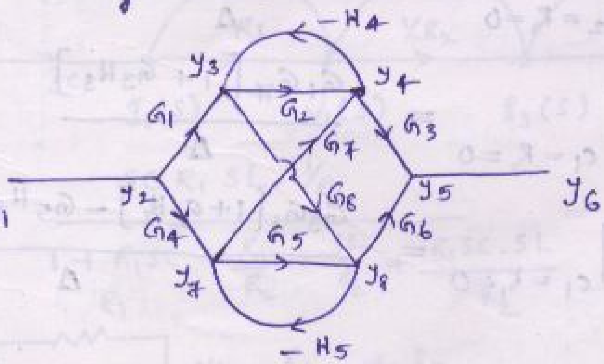
Q.



$$\frac{E(S)}{R(S)} = ?$$

Ans : $$\frac{1}{1+GH}$$

Q. find the no. of f. paths, loops and two non-touching loops.

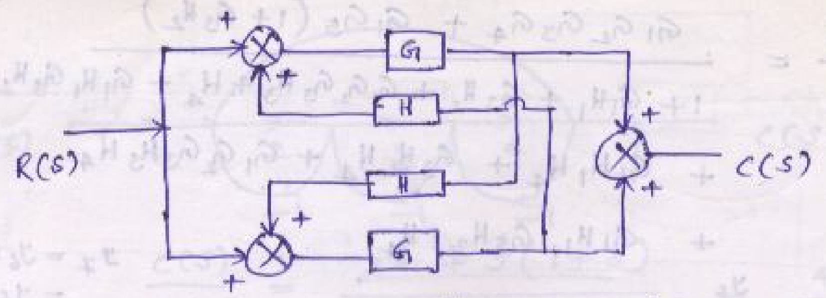


f. paths:-

on upper side : $y_1y_2y_3y_4y_5y_6$
$y_1y_2y_3y_8y_5y_6$
$y_1y_2y_3y_8y_7y_4y_5y_6$

} 3

Q.

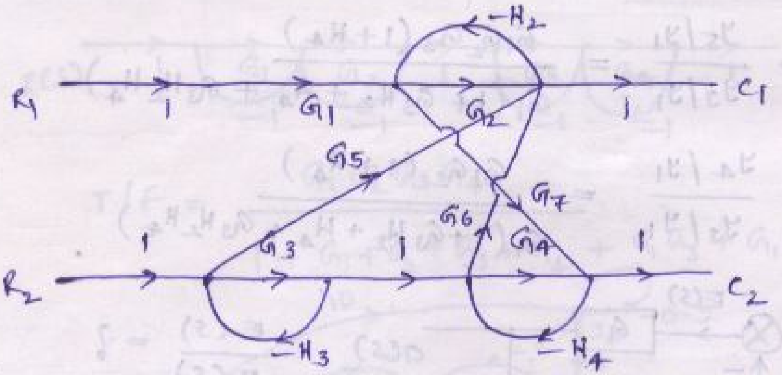


$$\frac{2G}{1-GH}$$

$$T/f = \frac{G + G^{2}H_{2} + G + G^{2}H}{1 - G^{2}H^{2}}$$

$$= \frac{2G\,[1+GH]}{(1+GH)\,(1-GH)} = \frac{2G}{1-GH}$$

Q. find C_1/R_1, C_1/R_2, C_2/R_1, C_2/R_2. [Multi i/p Multi o/p]



(i). $$\left.\frac{C_1}{R_1}\right|_{C_2=R_2=0} = \frac{G_1G_2\,[1+G_3H_3+G_4H_4+G_3H_3\,G_4H_4] - G_1G_7H_4G_6\,[1+G_3H_3]}{\Delta}$$

(ii). $$\left.\frac{C_1}{R_2}\right|_{C_2=R_1=0} = \frac{G_5\,[1+G_4H_4] + G_3G_6}{\Delta}$$

(iii). $$\left.\frac{C_2}{R_1}\right|_{C_1=R_2=0} = \frac{-G_1G_7\,[1+G_3H_3]}{\Delta}$$

(iv). $$\left.\frac{C_2}{R_2}\right|_{C_1=R_1=0} = \frac{G_3G_4\,[1+G_2H_2] - G_5H_2G_7 - G_3G_6H_2G_7}{\Delta}$$

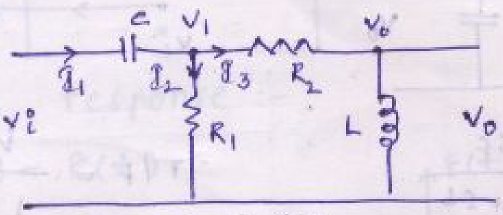
SFG's for Electrical N/ω :-

Ref:
1. Ogota
2. B.C. kuo

Steps :-

1. Select Branch 'current' or node voltage
2. Apply L.T. to all the var's & System components.
3. write the eq's of V/I
4. construct SFG.

Eg :-



$T/F = \frac{V_0}{V_i}$

$$= \frac{R_1 SL}{\frac{1}{SC}[R_1+R_2+SL]+R_1[R_2+SL]}$$

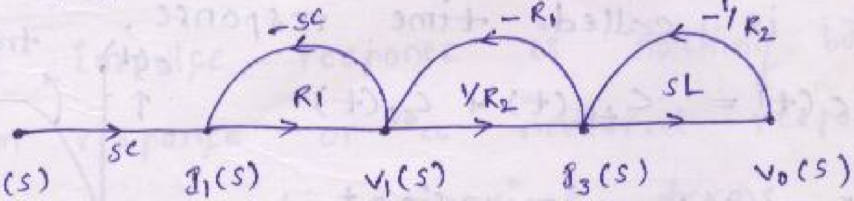
$$I_1(S) = \frac{V_i(S)-V_1(S)}{1/SC} = SC[V_i(S)-V_1(S)] \rightarrow ①$$

$$V_1(S) = I_2(S) \cdot R_1 = R_1[I_1(S) - I_3(S)] \rightarrow ②$$

$$I_3(S) = \frac{V_1(S)-V_0(S)}{R_2} \rightarrow ③$$

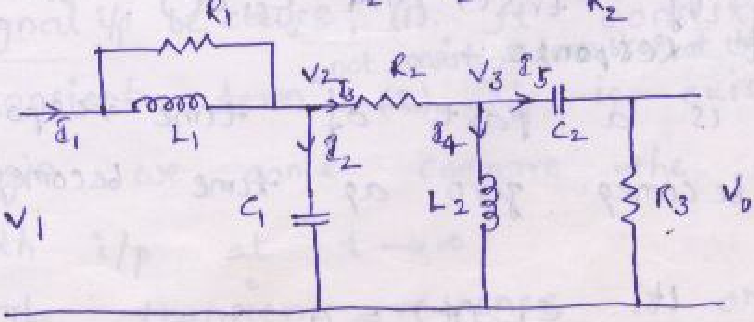
$$V_0(S) = SL \cdot I_3(S) \rightarrow ④$$

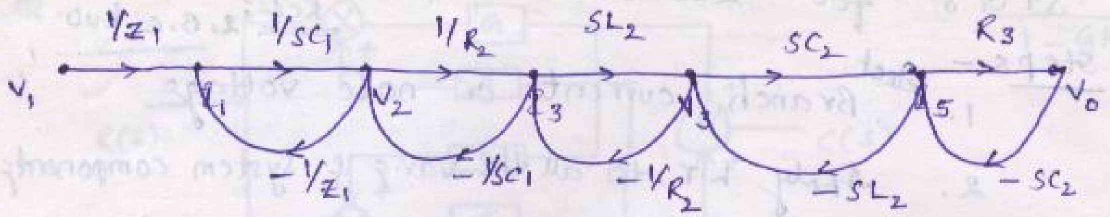
→ whatever the variables along the series branch, they are taken as nodes in SFG.



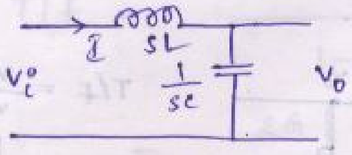
$$\frac{V_0(S)}{V_i(S)} = T/F = \frac{SC R_1 SL \cdot 1/R_2}{1+R_1SC+\frac{R_1}{R_2}+\frac{SL}{R_2}+\frac{R_1SC \cdot SL}{R_2}} =$$

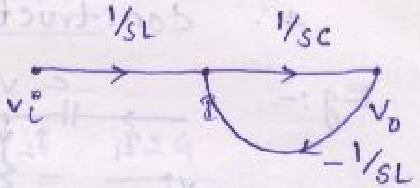
Q.





Q. Draw SFG for,





$$T/F = \frac{1/sc}{1/sc + sL}$$

$$= \frac{1}{1+s^2LC}$$

$$T/F = \frac{1/s^2LC}{1+1/s^2LC}$$

$$= \frac{1}{1+s^2LC}$$

→ TIME DOMAIN ANALYSIS :-

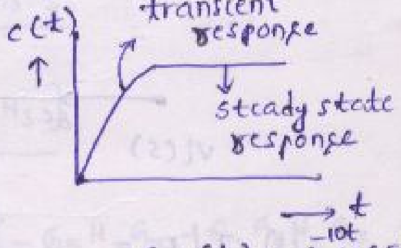
Ref: 1. Nagrath / Gopal

- time domain specifications
- e_{ss}
- Responses

→ Time Response :-

If the response of the system varies w.r.t time then it is called time response.

Time Response $c(t) = c_{tr}(t) + c_{ss}(t)$



Ex:- $c(t) = 5 + 10 \sin 2t$
$+ e^{-10t} \cos 5t + \dots$

Identify $c_{tr}(t)$ and $c_{ss}(t)$.

$c_{tr}(t) = e^{-10t} \cos 5t + \cdot$
$c_{ss}(t) = 5 + 10 \sin 2t$

→ Transient Response :-

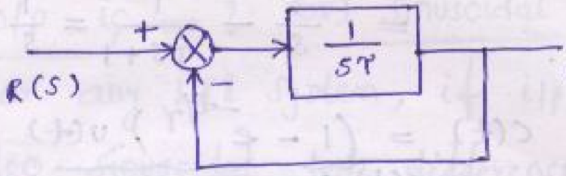
It is a part of time response that becomes zero as time becomes very large.

$$\lim_{t \to \infty} c_{tr}(t) = 0$$

→ Steady state Response :-

It is a part of time response that remains after the transients or (becomes zero) died out.

→ Time response for the I order systems :-

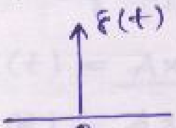


C/L T/F :

$$\frac{C(S)}{R(S)} = \frac{1}{\tau S+1}$$

↳ 1. Impulse response :-

$r(t) = \delta(t)$



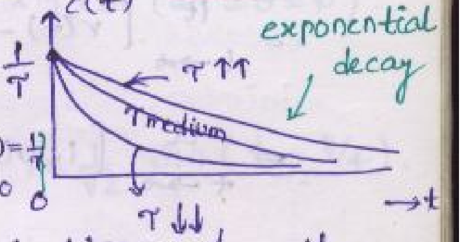
$R(S) = 1$

$$C(S) = \frac{1}{\tau S+1} = \frac{1}{\tau[S+1/\tau]}$$

Response :

$$\Rightarrow c(t) = \frac{1}{\tau} \cdot e^{-t/\tau}$$

$\left\{\begin{array}{l}\text{At } t=0,\ c(t)=\frac{1}{\tau}\\ t\to\infty,\ c(t)=0\end{array}\right.$



* Error is nothing but deviation of the o/p from the ref. i/p.

$e(t) = r(t) - c(t)$

$$e_{ss} = \underset{t\to\infty}{Lt}\ e(t) = \underset{t\to\infty}{Lt}\ [r(t) - c(t)]$$

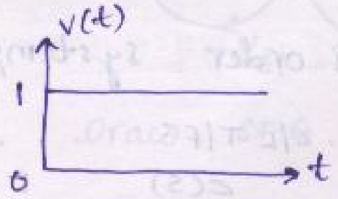
* The impulse response is nothing but a system response or a natural response. It consists only transient terms.

* The e_{ss} are not defined for impulse signal i/p because, (1). It consists only the transient term & not consists ss term b'coz at the ss, there is no i/p exists. (2). I/p is existed only at origin, we can't compare the response with i/p at $t\to\infty$

* The transient response is only due to system time constant and ss response is

only due to i/p.

Unit step input :-



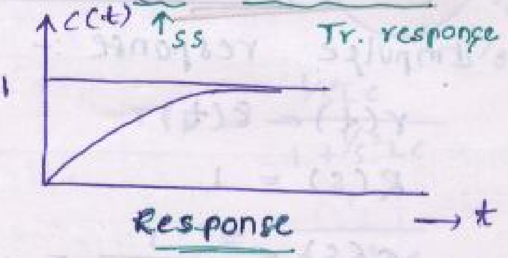
$r(t) = 1.u(t)$

$R(s) = \frac{1}{s}$

$$c(s) = \frac{1}{s(\tau s+1)}$$

$$= \frac{1}{s} - \frac{\tau}{\tau s+1} = \frac{1}{s} - \frac{1}{s+1/\tau}$$

$$c(t) = (\underbrace{1}_{\uparrow ss} - \underbrace{e^{-t/\tau}).u(t)}_{\text{Tr. responce}}$$



Response

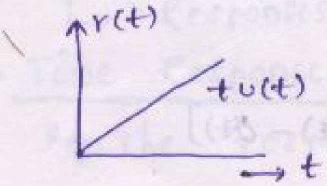
$$e_{ss} = \lim_{t\to\infty} e(t)$$

$$= \lim_{t\to\infty} [r(t) - c(t)]$$

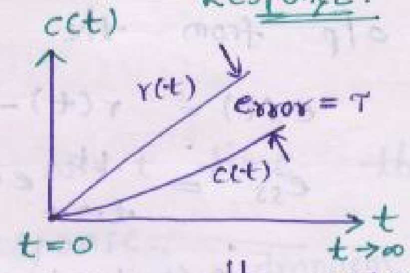
$$= \lim_{t\to\infty} [1.u(t) - 1.u(t) + e^{-t/\tau}.u(t)]$$

$$= 0$$

Unit Ramp input :-



Responce :



$r(t) = t\,u(t)$

$R(s) = 1/s^2$

$$C(s) = \frac{1}{s^2(\tau s+1)}$$

$$= \frac{1}{s^2} - \frac{\tau}{s} + \frac{\tau^2}{\tau s+1}$$

$$= (\underbrace{t - \tau}_{S.S.} + \underbrace{\tau.e^{-t/\tau}}_{Tr.})\,u(t)$$

$$\text{ss. error} = \lim_{t\to\infty} e(t)$$

$$= \lim_{t\to\infty} [t\,u(t) - t\,u(t) + \tau u(t) - \tau e^{-t/\tau} u(t)]$$

$$= +\tau$$

Unit Parabolic input :-

$r(t) = 1.\frac{t^2}{2}\,u(t)$

$R(s) = 1/s^3$

$$C(s) = \frac{1}{s^3(\tau s+1)}$$

purchase : R.K. Kanodia

↳ Sinusoidal Response :

Q. The CL T/F of an unity -f/b system is given by $\frac{C(S)}{R(S)} = \frac{1}{S+1}$. for the i/p $r(t) = \sin t$, the s.s. o/p is — ? (or) sinusoidal response is — ?

* for any LTI system, if i/p is sinusoidal, the o/p also sinusoidal but difference in magnitude & phase shift. The standard form of i/p & o/p as follows.

$r(t) = A\sin(\omega t \pm \theta) \Rightarrow c(t) = A \times M \sin(\omega t \pm \theta \pm \phi)$

$r(t) = A\cos(\omega t \pm \theta) \rightarrow c(t) = A \times M \cos(\omega t \pm \theta \pm \phi)$.

$r(t) = \sin t, \Rightarrow \omega = 1$

Replace $s = j\overset{1}{\cancel{\omega}} = j$.

$\rightarrow \frac{C(S)}{R(S)} = \frac{1}{j+1}$

$\therefore M = \frac{1}{\sqrt{2}}$

$\angle\phi = \frac{\angle 1}{\angle(j+1)} = \frac{0^\circ}{45^\circ} = -45^\circ$

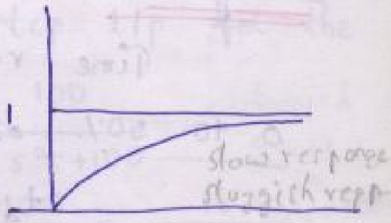
$\therefore c(t) = 1 \times \frac{1}{\sqrt{2}} \cdot \sin(t - \pi/4)$.

Case 2 :- $\xi = 1$:-

$$\frac{C(S)}{R(S)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

$$\Rightarrow C(S) = \frac{\omega_n^2}{s(s+\omega_n)^2}$$

$$\Rightarrow c(t) = 1 - \cancel{\omega_n t \cdot e^{-\omega_n t}} - e^{-\omega_n t}$$

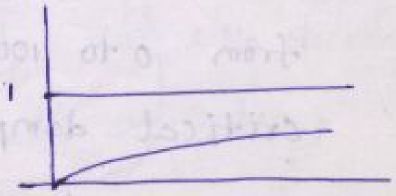


stable

Case 3 :- $\xi > 1$:-

$$C(S) = \frac{\omega_n^2}{s(s+P_1)(s+P_2)}$$

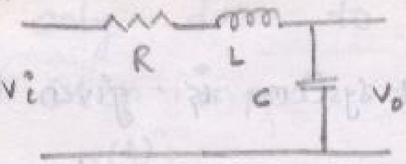
$$c(t) = 1 - k_1 e^{-P_1 t} - k_2 \cancel{e^{-P_2 t}}$$



ξ - damping ratio

$\xi\omega_n$ - actual damping

↳ damping factor.



$$\frac{V_0(s)}{V_i(s)} = \frac{Y_{sC}}{R+sL+Y_{sC}} = \frac{1}{s^2LC+sCR+1}$$

$$= \frac{Y_{LC}}{s^2+\frac{sR}{L}+\frac{1}{LC}}$$

$\omega_n = 1/\sqrt{LC}$

$2\xi\omega_n = \frac{R}{L} \Rightarrow \xi = \frac{R}{2L}\cdot\sqrt{LC}$

$$\Rightarrow \boxed{\xi = \frac{R}{2}\sqrt{\frac{C}{L}}}$$

$\left(R = 2\xi\sqrt{L/C}\right)$

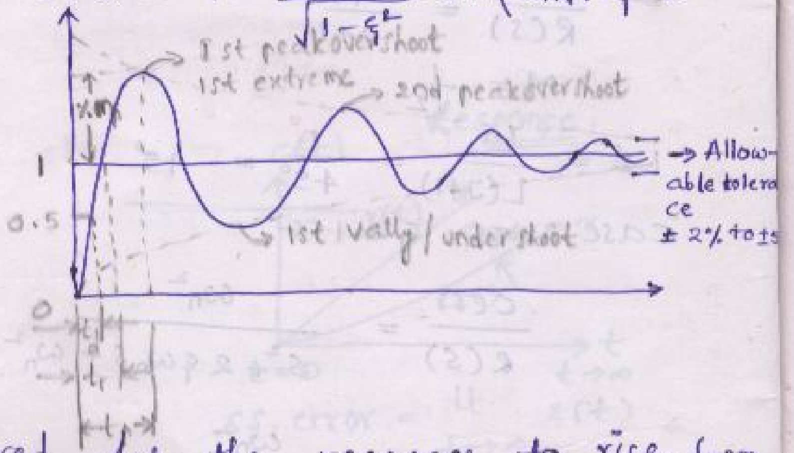
$\frac{IAS}{}$ $\frac{1}{1+sT}$ *

OL BW = $1/T$

CL BW = $\frac{1+k}{T}$ → system gain

Time Domain Specifications:-

For the time domain specifications consider the underdamped system because the rise time and settling time is minimum. for $0<\xi<1$, the unit step response of the system is $c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\sin\left(\omega_n\sqrt{1-\xi^2}\,t + \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi}\right)$ (or) $\cos^{-1}\xi$



* Delay time :-

Time required for the response to rise from 0 to 50% of the final value.

$$t_d = \frac{1+0.7\xi}{\omega_n} \text{ sec}$$

* Rise time :-

Time required for the response to rise from 0 to 100% for underdamped, 5 to 95% for critical damped, 10 to 90% for overdamped.

$$t_r = \frac{\pi - \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi}}{\omega_d} = \frac{\pi - \cos^{-1}\xi}{\omega_d} \text{ sec}$$

radians

* peak time :-

Time required for the response to rise and reach the peaks of the response.

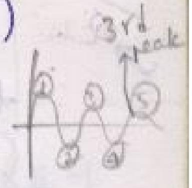
$$t_p = \frac{n\pi}{\omega_d} \quad (\text{for 1st peak } n=1)$$

$$= \frac{\pi}{\omega_d}$$

$$=$$

3rd peak, $t_p = \frac{5\pi}{\omega_d}$

1st vally $t_p = \frac{2\pi}{\omega_d}$



* Settling time :-

Time required to rise and stay within the specified tolerance band %±2 or ± 5%.

$$t_s = 4\tau = \frac{4}{\xi\omega_n} \rightarrow \pm 2\%$$

$$= 3\tau = \frac{3}{\xi\omega_n} \rightarrow \pm 5\%$$

These values are also valid for overdamped and critical damped.

* Peak overshoot :-

It indicates normalized difference b/w s.s. o/p to 1st peak of the time response.

$$\% M_p = \frac{c(t_p) - c(\infty)}{c(\infty)} \times 100$$

$$= (c(t_p) - 1) \times 100$$

$$= e^{-\pi\xi/\sqrt{1-\xi^2}} \times 100$$

Q. find the % M_p for unit step i/p for the given function (i). $\frac{C(s)}{R(s)} = \frac{100}{s^2+100}$ undamped $\xi = 0$ so % M_p = 100%

(ii). $\frac{C(s)}{R(s)} = \frac{16}{s^2+8s+16}$ $\xi = 1$ critical

(iii). $\frac{C(s)}{R(s)} = \frac{16}{s^2+100s+16}$

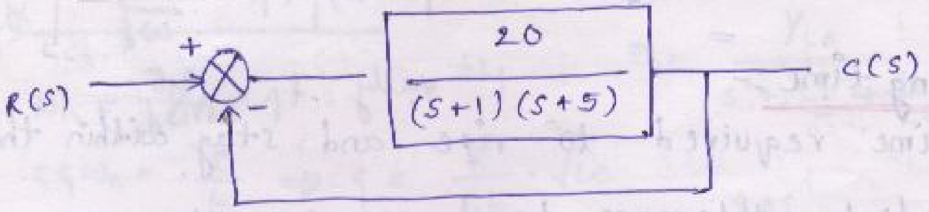
(i). critical damped → % M_p = 0%

As ξ increases from 0 to 1, the % M_p decreases

$\xi > 1$, the system does not ~~exist~~ have oscillations

hence no % M_p and no peak time.

Q. A block diagram is shown in fig. The time period of oscillations before reaching the ss, is — ?



$$T_{oscillations} = \frac{2\pi}{\omega_d} \qquad \frac{20}{s^2+6s+25}$$

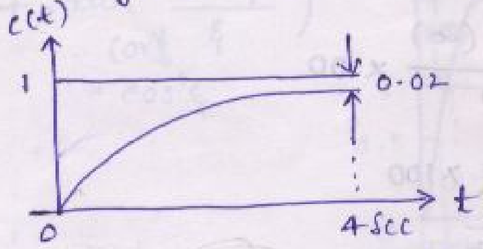
$$\text{So } \omega_d = \omega_n\sqrt{1-\xi^2} \qquad \Rightarrow \omega_n = 5,\ \xi = 0.6$$

$$= 4.$$

Q. find no. of oscillation (or) cycles

$$N = \frac{t_s}{T_{osci}}$$

Q. find the time const of the system for the given unit step response.



$$t_s = 4T$$

$$4 = 4T$$

$$\Rightarrow T = 1 \text{ sec}$$

Q. Given $G(s) = \frac{25}{s(s+4)}$, $H(s) = 1$. find the time domain specifications.

find unit step response for above system.

$$\frac{C(s)}{R(s)} = \frac{25}{s^2+4s+25} \rightarrow \frac{25}{25} \leftarrow \frac{25}{s^2+4s+25}$$

$$\downarrow 1$$

$$\omega_n = 5 \text{ rad/sec}$$

$$\xi = 0.4, \quad \omega_d = \omega_n\sqrt{1-\xi^2} = 4.5 \text{ rad/sec}$$

$$t_d = \frac{1+0.7\xi}{\omega_n} = 0.256 \text{ sec}$$

$$t_r = \frac{\pi - \cos^{-1}\xi}{\omega_d} \leftarrow (\text{radians}) = 0.44 \text{ sec}$$

$$t_p = \frac{\pi}{\omega_d} = 0.69 \text{ sec}$$

$\pm 2\%\ t_s = 2$ sec k=1, unit step i/p,

$\pm 5\%\ t_s = 1.5$ sec k≠1 steady

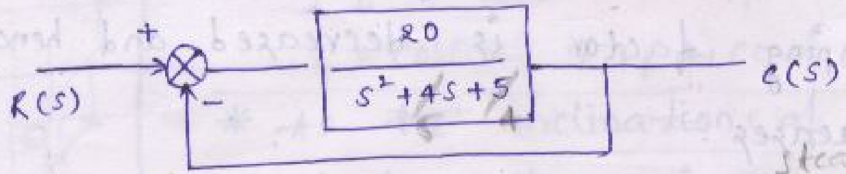
$$\text{(b)}.\ c(t) = 1 - \frac{e^{-0.4\times5t}}{\sqrt{1-0.4^2}}\cdot\sin(4.5t+\cos^{-1}0.4)$$

unit step response

ss value transient It never effect the transient any effect the steady state value

depends on system gain & magnitude of system i/p k

Q. for a system shown in fig. find the time domain specifications when the unit step i/p is applied. find unit step response for above system.



$$\frac{C(S)}{R(S)} = \frac{20}{s^2+5s+24} = \frac{20}{24}\cdot\frac{24}{s^2+5s+24}$$

steady state value

$\omega_n = 4.89$ rad/sec

$\xi = 0.51,\quad \omega_d = 4.2$ rad/sec

$t_d = 0.277$ sec $\pm 2\%\ t_s = 1.6$ sec

$t_r = 0.5$ sec $\%M_p = 15.43\%$

$t_p = 0.74$ sec $c(t) = \frac{20}{24}\left(1 - \frac{e^{-2.5t}}{0.859}\cdot\sin(42t+1.03)\right)$

*→ As ξ increases, the poles moves towards the L.H.S and nearer to the real axis. Hence the frequency of osci are decreases.

As the freq. of osci decreases, the time specifications t_d, t_r, t_p must be increases.

As ξ increases the % M_p must be decreases.

As ξ increases the time const. should be decreases hence the settling time must be decreases. & BW ↓

$\omega_d \uparrow\uparrow$

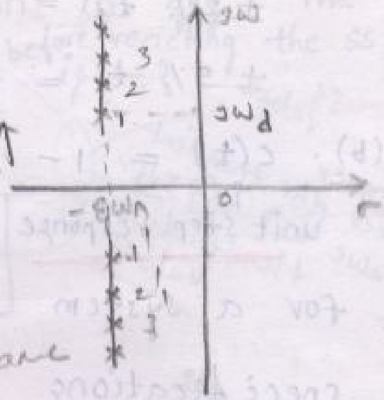
$t_d, t_r, t_p \downarrow\downarrow$

so $\xi\omega_n$ – const
$\downarrow$ must be decreased

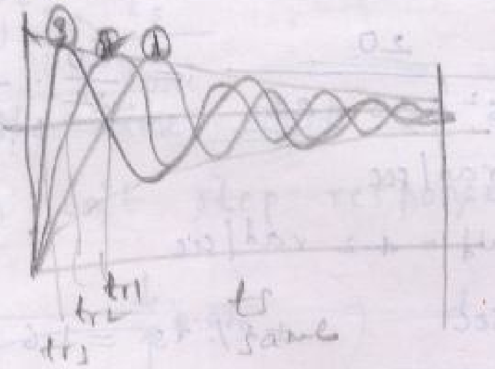
$\xi\omega_n$ const.

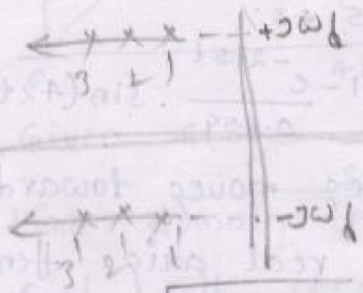
$\omega_n\sqrt{1-\xi^2}\uparrow \Rightarrow \sqrt{\underset{\uparrow}{\omega_n^2} - \underset{\text{const}}{\xi^2\omega_n^2}}\uparrow$

$\xi\downarrow \Rightarrow \%M_p\uparrow$ t_s – same



→ * As poles moves vertically ll^{el} to jω axis, the damping factor is decreased and hence % Mp increases.





ω_d const;

$t_p = \frac{\pi}{\omega_d} \to$ const.

$\simeq t_r, t_d$

$\omega_d = \sqrt{\omega_n^2 - \omega_n^2\xi^2}$, ξ increases as well as ω_n increases

$t_d = \frac{1+0.7\xi\uparrow}{\omega_n\uparrow}$ (approximately const) slight variation

$t_r \downarrow = \frac{\pi - \cos^{-1}(\xi\uparrow)\downarrow}{\omega_d\uparrow}$ slightly

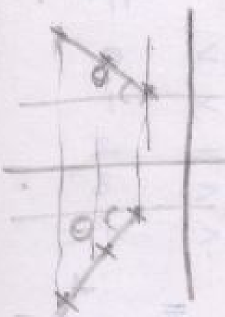
$\tau\downarrow\downarrow \to t_s\downarrow\downarrow$

$\xi\uparrow\uparrow \to \%M_p\downarrow\downarrow$

* As ω_d is constant, the t_p is same. Even t_r, t_d are approximately constant.

As the poles moves towards L.H.S., the time constant decreases hence t_s decreases.

As ξ increases, the $\% M_p$ decreases.



$\omega_d \uparrow\uparrow \rightarrow t_d, t_r, t_p \downarrow\downarrow$

$\tau \downarrow \rightarrow t_s \downarrow\downarrow$

$\xi = \cos\theta \rightarrow \% M_p \rightarrow$ const

* As the inclination of the poles is constant, the ξ is constant. hence the $\% M_p$ is constant.

Q. find the time domain specifications for unit step i/p. for the given system.

$$\frac{d^2y}{dt^2} + 4 \cdot \frac{dy}{dt} + 8y = 8x$$

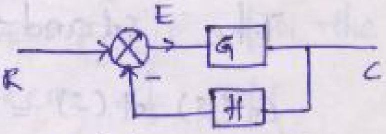
Ans:- $\frac{y(s)}{x(s)} = \frac{8}{s^2 + 4s + 8}$

steady state errors:-

It is the deviation of o/p from the reference i/p at the steady state $[t \rightarrow \infty]$

* $e_{ss} = \lim_{t \rightarrow \infty} e(t) = \lim_{s \rightarrow 0} s\,E(s)$

$$= \lim_{s \rightarrow 0} \frac{s \cdot R(s)}{1 + G(s) \cdot H(s)}.$$



$$\frac{E(s)}{R(s)} = \frac{1}{1 + GH}$$

* The SSE are depends on

(1). type of i/p (ie) R(s) (2). type of system ie G(s) H(s)

Type of i/p :- (R(S)) :-

$\xrightarrow{}$ step i/p :- $r(t) = A\,u(t) \Rightarrow R(s) = \frac{A}{s}$ $\nearrow A.t^{0}u(t)$ $\leftarrow$ order. for step i/p

$$e_{ss} = \lim_{s\to 0} s \cdot \frac{A/s}{1+G(s).H(s)} = \frac{A}{1+\lim_{s\to 0} G(s).H(s)} = \frac{A}{1+k_p}$$

k_p = static position error const

$$= \lim_{s\to 0} G(s).H(s) \Rightarrow k_p$$

$$\therefore e_{ss} = \frac{A}{1+k_p}$$

$\xrightarrow{}$ Ramp i/p :-

$$r(t) = At\,u(t) \Rightarrow R(s) = A/s^2$$

$$\therefore e_{ss} = \frac{A}{k_v} \quad \because e_{ss} = \lim_{s\to 0} s \cdot \frac{A/s^2}{1+G(s)H(s)} = \frac{A}{\underbrace{\lim_{s\to 0} s\,G(s)H(s)}_{k_v}}$$

$\xrightarrow{}$ Parabolic i/p :-

$$r(t) = At^2/_2\,u(t) \Rightarrow R(s) = A/s^3$$

$$e_{ss} = \lim_{s\to 0} \frac{s \cdot A/s^3}{1+G(s).H(s)} \simeq \frac{A}{k_a} = \frac{A}{\lim_{s\to 0} s^2\,(G(s).H(s))}$$

Type of Systems :- System is represented as $G(s).H(s) = \frac{K(1+sT_1)(1+sT_2)\cdots}{s^{n}(1+sT_a)(1+sT_b)\cdots}$,

* Type is nothing but no. of poles at origin. $\uparrow$ Type – n System

* Order is nothing but total no. of poles in s-plane.

$$\text{Type} = i/p \xrightarrow{e_{ss}} \text{constant}$$

$$\longrightarrow \boxed{\text{Type} > i/p \xrightarrow{e_{ss}} 0}$$

$$\text{Type} < i/p \xrightarrow{e_{ss}} \infty$$

The standard form of the system is

$$G(s).H(s) = \frac{K(1+sT_1)(1+sT_2)\cdots}{\underset{\text{Type}}{\underset{\downarrow}{s^{n}}}(1+sT_a)(1+sT_b)\cdots}$$

Type		i/p ① (step)(A)	e_{ss}
0	=	0	$\frac{A}{1+K}$ const.
1	>	0	0
2	>	0	0
3	⋮	0	⋮
4	⋮	0	⋮
⋮		⋮	

Type		i/p ① Ramp(At)	e_{ss}
0	<	1	∞
1	=	1	$\frac{A}{K}$ const
2	>	1	0
⋮	⋮	⋮	⋮

$K_p = \lim_{s\to 0} GH$

$K_v = \lim_{s\to 0} sGH$

$K_a = \lim_{s\to 0} s^2GH$

Type		i/p ② parabolic($A\frac{t^2}{2}$)	e_{ss}
0	<	2	∞
1	<	2	∞
2	=	2	A/K (const)
3	>	2	0
4	>	2	0
⋮	⋮	⋮	⋮

Q. Given $G(s) = \frac{10(s+2)}{s^2 * (s+4)(s+10)}$. find the e_{ss} for the i/p $r(t) = 1 + 4t + t^2/2$, $H(s) = 1$.

Ans:-

Type		i/p	e_{ss}
2	>	0	0 +
2	>	1	0 +
2	=	2	$A/K = \frac{1}{20/40} = 2$

Q. Given $G(s).H(s) = \frac{10}{s(s+2)}$. find the e_{ss} for the following i/ps (1). 4t u(t) (2). t^2u(t) (3). 2u(t) (4). $(1+t+t^2)$u(t)

Ans:-

Type	i/p	e_{ss}
1		

$4t \rightarrow \frac{A}{k} = \frac{4}{10/2}$

$t^2 \rightarrow \infty$

$2u(t) \rightarrow 0$

$1+t+t^2 \rightarrow \infty$

$0 + k + \infty$

Q. find the e_{ss}, for unit ramp i/p for the given unity f/b control system of T/f

$\frac{10}{s^3 + 20s^2 + 10} = \frac{C(s)}{R(s)}$ (e_{ss} valid for CL stable system)

Ans:- The given T/f for closed loop is unstable hence the e_{ss} are not valid. ('s' term is missing)

Q. $\frac{C(s)}{R(s)} = \frac{10s+10}{s^3+20s^2+10s+10}$

$\rightarrow$ e_{ss} are calculated to CL stable system, by using open loop (OL) T/F.

Ans:- OL T/F $= \frac{10s+10}{s^3+20s^2+10s+10-10s-10}$

$= \frac{10s+10}{s^3+20s^2} = \frac{10s+10}{s^2(s+20)}$

Type	i/p	e_{ss}
2	1	0

Q. Given $G(s) = \frac{k(s+2)}{s(s^3+7s^2+12s)}$, $H(s) = 1$. find The e_{ss} for the i/p $\frac{R}{2}t^2$.

Ans:- $G(s) = \frac{k(s+2)}{s^2(s^2+7s+12)}$

$e_{ss} = \frac{A}{K} = \frac{R}{2k/12} = \frac{6R}{k}$.

Q. The OL T/F of the system is $\frac{k}{s(s+1)(s+2)}$ Determine the value of k, show that $e_{ss} = 0.1$ for unit ramp i/p.

Ans:- $e_{ss} = \frac{A}{K} = \frac{1}{k/2} = 0.1$

$\Rightarrow k = 20$

Q. find the e_{ss} for OL T/F of a unity f/b control system $G(s) = \frac{1}{(s+10)(s+20)}$. for the following i/ps (1). $10\,u(t)$ (2) $10t\,u(t)$ (3). $(10+10t+10t^2)\,u(t)$

(1) $\downarrow$ $\frac{A}{1+k} = \frac{10}{1+\frac{1}{200}}$

(2) $\downarrow$ ∞

(3) $\downarrow$ ∞

Ans:- $\frac{10t^2}{2}$ x^2 $\frac{20}{2}$ $\rightarrow 5$

for $G(s) = \frac{(s+1)}{s^2(s+10)(s+20)}$ i/p's

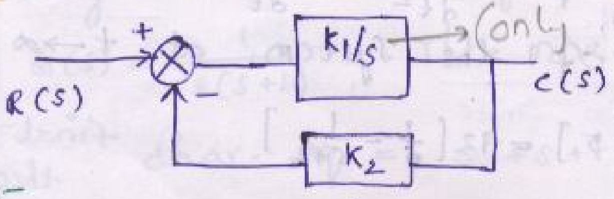
(1) $10\,u(t) \rightarrow 0$

(2) $10t\,u(t) \rightarrow 0$

(3) $10t^2 u(t)$ $\frac{A}{k} = \frac{20}{1/200}$

(4) $10+10t+10t^2 \rightarrow$

Q. For the following system, the ss gain = 2 τ = 0.4 sec, the values of k_1 and k_2 are



(only 1 pole → 1st order)

$$\frac{C(s)}{R(s)} = \frac{k_1/s}{1+k_1/s \cdot k_2} = \frac{k_1}{s+k_1k_2} = \frac{1/k_2}{1+s\left[\frac{1}{k_1k_2}\right]}$$

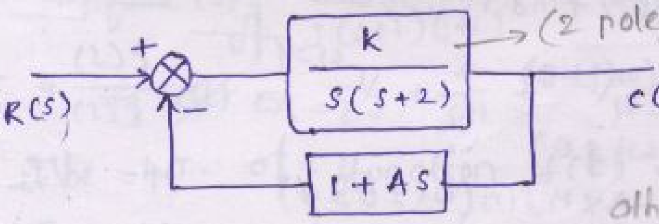
Ans:-

standard form $\frac{C(s)}{R(s)} = \frac{K}{1+s\tau}$ 1st order

$\therefore \quad 2 = \frac{1}{k_2} \Rightarrow k_2 = 0.5$ $\quad \left\{ \text{SS gain} \ \cancel{\times} = K = 2 \right\}$

$0.4 = \frac{1}{k_1k_2} \Rightarrow k_1 = 5$

Q. For the system shown in fig. with $\xi = 0.7$ and undamped natural freq. $\omega_n = 4$ rad/sec. The values of k and A are —?



→ (2 poles ⇒ 2nd order system) If ss gain k, then $\frac{C(s)}{R(s)} = \frac{k \cdot \omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$ other wise like this problem,

char. equation : $s^2 + 2\xi\omega_n s + \omega_n^2 = 0$

(1+GH = 0)

$$1 + \frac{k}{s(s+2)} \cdot (1+As) = 0$$

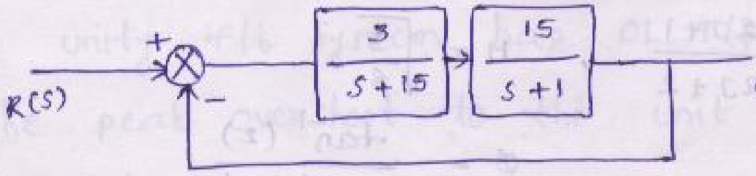
$$\Rightarrow \quad s^2 + s(2+kA) + k = 0$$

$\omega_n^2 = k = 16$

$2\xi\omega_n = 2 + kA$

$\Rightarrow A = 0.225$

Q. A block diagram shown in fig. gives a unity f/b CL control system. The ss error to the unit step i/p is —?



$$GH = \frac{45}{(s+1)(s+15)} =$$

$$\frac{A}{1+k} = \frac{1}{1+\frac{45}{15}} \times 100$$

$$= 25\%.$$

$\frac{20}{1500}$

Q. A control system is defined by the folloing mathematical exp. $\frac{d^2x}{dt^2} + 6.\frac{dx}{dt} + 5x = 12(1-e^{-2t})$ The responce of the system at $t \to \infty$ – ?

$$x(s)[s^2+6s+5] = 12\left[\frac{1}{s} - \frac{1}{s+2}\right]$$

final value theorem

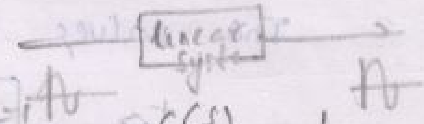
$$\underset{t\to\infty}{lt}\, f(t) = \underset{s\to0}{lt}\, s \cdot F(s)$$

Initial, value th

$$\underset{t\to0}{lt}\, f(t) = \underset{s\to\infty}{lt}\, sF(s)$$

Q. If the CL T/F of a control system is given by $\frac{C(s)}{R(s)} = \frac{1}{s+1}$ for the i/p $R(t) = \sin t$, the ss value of $c(t) = ?$

Sol:- (or) finding o/p (or) find responce



$r(t) = A\sin(\omega t \pm \theta)$

$= A\cos(\omega t \pm \theta)$

$\omega = 1$

$s = j\omega$

$\frac{C(s)}{R(s)} = \frac{1}{J+1}$

$c(t) = A \times M \sin(\omega t \pm \theta \pm \phi)$

$M = 1/\sqrt{2}$ $\angle\phi = \frac{1}{45°}$

$= 1/\sqrt{2}\, \sin(t - \pi/4)$

$\angle\phi = -\pi/4$

Q. for any linear system if i/p is a sinusoidal, the o/p also a sinusoidal but diff. in magnitude and phase angle. The standard form of i/p can be represented as

Sol: $\frac{C(s)}{R(s)} = \frac{s+1}{s+2}$, $r(t) = 10\cos(2t + 45°)$

$\Rightarrow \frac{2J+1}{2J+2}$, $M = \sqrt{\frac{5}{8}}$

$\phi = \frac{\tan^{-1}(2)}{\tan^{-1}(2/2)}$

$=$

$c(t) = 10 \times \sqrt{\frac{5}{8}} \cos(2t + 63.45°)$

Q. Consider the unit step response of a unity f/b control system of OL T/f is $G(s) = \frac{1}{s(s+1)}$. The max. overshoot = ?

Sol: char. eq = $s^2+s+1=0$

$\omega_n = 1, \quad \xi = 0.5$

$\therefore \% M_p = e^{-\pi\xi/\sqrt{1-\xi^2}} \times 100$

$= 0.163.$

Q. The CL T/f of a control system $\frac{C(s)}{R(s)} = \frac{2(s-1)}{(s+1)(s+2)}$ for a unit step i/p, the o/p is — ?

(1) 0 (2) ∞ ✓(3) $-3e^{-2t}+4e^{-t}-1$ (4) $-3e^{-2t}-4e^{-t}+1$

Sol. $C(s) = \frac{2(s-1)}{s(s+1)(s+2)}$

$\rightarrow$ ~~c(s)~~ $= -1/s + \frac{4}{s+1} + \frac{-3}{s+2} = -1+4e^{-t}-3e^{-2t}$

Q. ✓ The L.T. of function $f(t) = F(s)$, $F(s) = \frac{\omega}{s^2+\omega^2}$ $\downarrow$ $f(t) = \sin\omega t$

The final value of $f(t)$ = — ?

(1) ∞ (2) 0 (3) 1 ✓(4) None

for sinusoidal signal the final value is None (does not have final value)

Q. A unity f/b system has OL T/f G(s), the SS error is zero for ~~(a)~~ step i/p type-1

(b). Ramp i/p type-1 (c) step i/p type-0

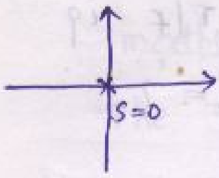
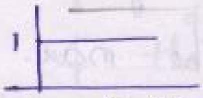
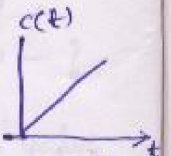
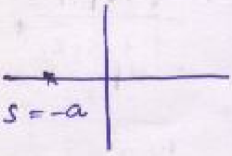
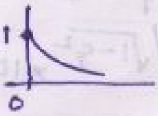
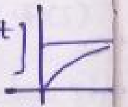
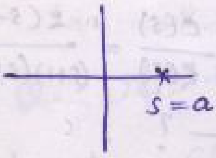
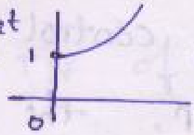
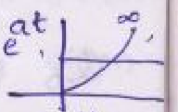
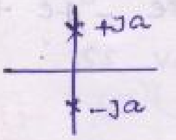
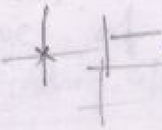
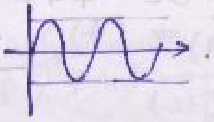
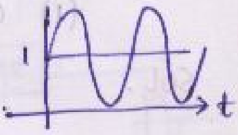
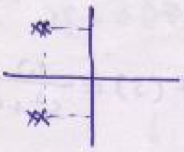
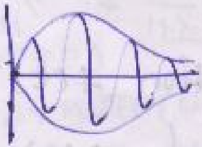
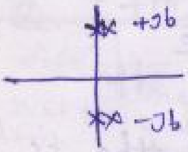
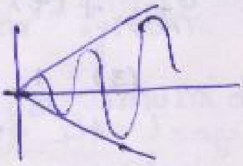
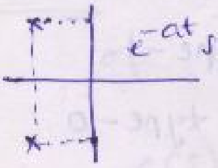
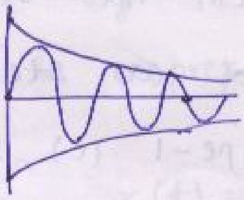
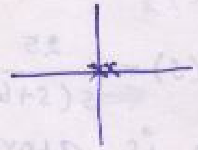
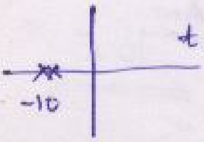
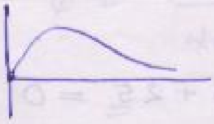
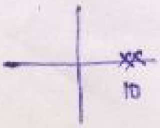
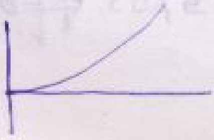
(d). Ramp i/p type-0.

Q. A unity f/b system has OL T/f $G(s) = \frac{25}{s(s+6)}$. The peak overshoot to the unit step is approximately

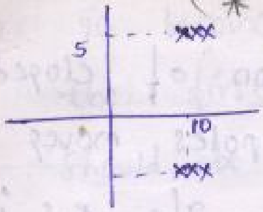
Any 10%

$s^2+6s+25=0$

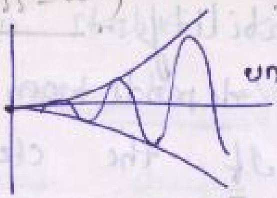
$\omega_n = 5, \quad \xi = 0.6$

poles location	T/F	Impulse response $R(S)=1$	stability	Unit step response $R(S)=\frac{1}{S}$	stability
 $S=0$	$\frac{1}{S} \rightarrow 1$	 $t \rightarrow \infty$, finite stable	Marginally stable	$\frac{C(S)}{R(S)} = \frac{1}{S}$ $C(S) = \frac{1}{S^2}$	 unstable
 $S=-a$	$\frac{1}{S+a} \rightarrow e^{-at}$		stable	$\frac{1}{S(S+a)} = \frac{1}{a}\left[1-e^{-at}\right]$	 stable
 $S=a$	$\frac{1}{S-a} \rightarrow e^{+at}$		unstable	$\frac{1}{S(S-a)} = -\frac{1}{a} + \frac{1}{a}e^{at}$	 unstable
			stable		
	$te^{-At}\sin\omega t$ 		stable		
	$t\sin t$ 		unstable		
	$e^{-at}\sin bt$		stable		
	$\frac{1}{S^2} \rightarrow t$		unstable		
	te^{-10t} 		stable		
	te^{10t} 		unstable		

(* for unstable $e_{ss} = \infty$)



$\frac{t^2}{2} \cdot e^{10t} \sin 5t$



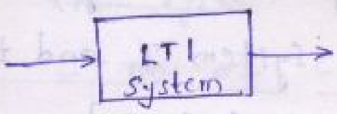
As type increases system stability decreases and ess improves

Stability :-

06-06-07

RH — ④
RL — ②
BP — ③
NP — ①

① CL stability
②. no. of CL poles
③. k
④ k marginal, ω_{max}
⑤. Relative stability τ, t_s
⑥. TF → (RL) — Rules
(OP NC) → Gm, Pm



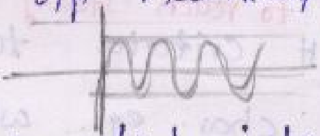
* A linear Time Invarient system is said to be stable, if the following conditions are satisfied.

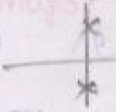
(1). If the i/p is bounded to the system, the o/p must be bounded.

(2). If the i/p to the system is zero, the o/p must be zero, irrespective of all the initial condi. s.

Marginal / critical / Limitedly stable system :-

A LTI system said to be marginal, if for the bounded i/p the o/p maintains const. freq and amplitude.





The stability is classified into 2 ways.

1. Absolutely stable system
2. conditional " "

Absolutely s. system :-
Here the system is stable for all the values of system parameters ie from k, 0 to ∞.

Conditional s. system :-
Here the system is stable for certain range of system parameters. ie k>0, k<10, 20..

Relative stability :-

R.S. depends on the position of closed loop poles. If the closed loop poles moves towards LHS, the R.S. improves. The R.S is applicable for only closed loop stable systems.

* The R.S. is used to find system τ and t_s. [how fast the transients are died out]

R.H. criteria:-

1. To find closed loop system stability.
2. To find no. of CL poles in the right half of s-plane.
3. To find range of k. value to find system stability.
4. To find $K_{marginal}$ value.
5. If the system is marginal stable to find the frequency of the oscillations. ($\omega_{marginal}$)
6. To find the relative stability ie τ & t_s & time required to reach steady state

* In R.H criteria to find a CL system stability we use char. eq. where as in Root locus, BP, and NP uses OL T/F.

The n-order general form of char. eq. is

$$a_0 s^n + a_1 s^{n-1} + a_2 s^{n-2} + \cdots a_{n-1} s^1 + a_n s^0 = 0$$

$$\begin{array}{c|ccc}
s^n & a_0 & a_2 & a_4 \cdots \\
s^{n-1} & a_1 & a_3 & a_5 \cdots \\
s^{n-2} & \dfrac{a_1 a_2 - a_0 a_3}{a_1} & \dfrac{a_1 a_4 - a_0 a_5}{a_1} & \\
\vdots & \vdots & & \\
s^0 & a_n & &
\end{array}$$

1. To become a system stable all the coe. in the first column must be +ve. and no coe. should be zero.
2. If sign changes occurs in 1st column then the system is unstable. the no. of sign changes

* = no. of CL poles in the right half of the s- plane.

stable

Q. Identify the system stability, for (1). $s^2+5s+10=0$

m.s.

(2). $s^3+10s^2+3s+30=0$ (3). $s^3+4s^2+5s+5=0$.

stable

(4). $s^3+8s^2+4s+100=0$ (5). $s^3+5s^2+10=0$

unstable missing 's' → unstable

* for $s^2+bs+c=0$, $b, c > 0 \rightarrow$ stable

$b=0 \rightarrow$ m.s. (Marginal)

* for $as^3+bs^2+cs+d=0$, $ad=bc \rightarrow$ M.S

$bc > ad \rightarrow$ stable

missing term → unstable

$bc < ad \rightarrow$ unstable

Q. find the no. of poles in the right half of s-plane, for (i). $s^4+2s^3+6s^2+8s+10=0$

s^4	1	6	10
s^3	2	8	
s^2	2	10	
s^1	-2		
s^0	10		

2 sign changes so 2 poles on right half of s-plane

(ii). $s^4+2s^3+3s^2+4s+5=0$

s^4	1
s^3	
s^2	
s^1	
s^0	

(iii). $s^4+2s^3+2s^2+4s+8=0$

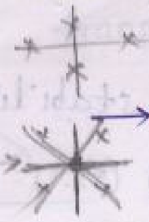
s^4	1	2	8
s^3	2	4	
s^2	$\overset{\epsilon}{\cancel{0}}$	8	
s^1	$\frac{4\epsilon-16}{\epsilon}$		
s^0	8		

If any zero occurs in the first column, replace zero by smallest +ve const and conti. Routh tabular form. finally substitute $\epsilon = 0$ and check the no. of sign changes.

(iv). $s^5 + s^4 + 2s^3 + 2s^2 + 3s + 15 = 0$

(v). $s^5 + s^4 + 3s^3 + 3s^2 + 2s + 2 = 0$

s^5	1	3	2
s^4	$1s^4$	$3s^2$	$2s^0$
s^3	$\cancel{0} \to 4$	$\cancel{0} \to 6$	0
s^2	3/2	2	
s^1	2/3		
s^0	2		



* whenever the poles are located symmetrical about original then the row of zero's occur.

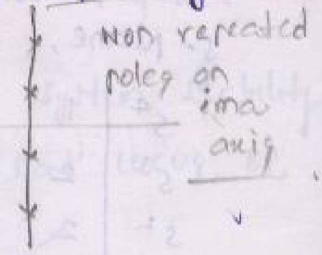
* whenever in Routh table, only once rows of zero are occured and all the coe. in 1st column +ve, then the CL poles must be on ima. axis which are symmetrical about origin.

* The auxillary eq. gives the location of the CL poles. The AE contains only even power of s- terms.

$AE \Rightarrow s^4 + 3s^2 + 2 = 0$

$\Rightarrow (s^2+2)(s^2+1) = 0$

$\Rightarrow s = \pm J\sqrt{2}, \pm J$. System is m.s.



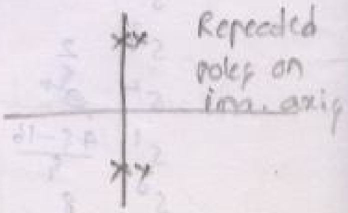
(vi). $s^6 + 3s^5 + 4s^4 + 6s^3 + 5s^2 + 3s + 2 = 0$

s^6	1	4	5	2
s^5	3	6	3	
s^4	$2s^4$	$4s^2$	$2s^0$	
s^3	$\cancel{0} \to 8$	$\cancel{0} \to 8$	0	$\longrightarrow$ ①
s^2	$2s^2$	$2s^0$		
s^1	$\cancel{0} \to 4$	0		$\longrightarrow$ ②
s^0	2			

$2s^4 + 4s^2 + 2 = 0$

$s^4 + 2s^2 + 1 = 0$

$(s^2+1)^2 = 0 \Rightarrow s \pm J1$



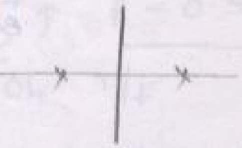
* whenever many times rows of zeros occurs and all the coe-s in the 1st column are +ve then the roots are repeated on ima. axis and which are symmetrical about origin and the system is unstable.

(vii). find the no. of CL poles in the left half of s-plane for $s^4 + s^3 - s - 1 = 0$.

s^4	1	0	-1
s^3	1	-1	
s^2	$1s^2$	-1	
s^1	$\cancel{0}^{2} \longrightarrow$ ①		
s^0	-1		

AE : $s^2 - 1 = 0$

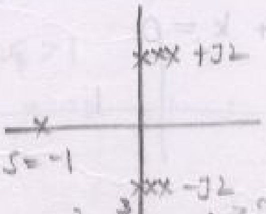
$s = \pm 1$



* whenever in the Routh table, row of zero's occurs and sign changes then the roots are located on the real axis which are symmetrical about origin.

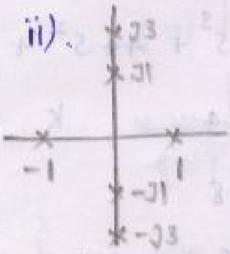
(viii). find the Routh table for the given different poles location.

i).



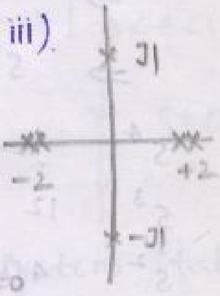
$(s^2+4)^3(s+1)=0$ need not to take

ii).

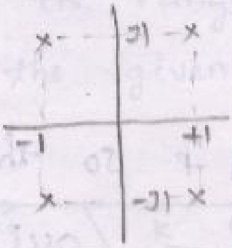


$(s^2+1)(s^2-1)(s^2+9)=0$

iii).



iv).



$(s^2+2s+2)(s^2-2s+2)=0$

Q. a). find the range of k value of system stability

b). find the k value to become the system m.s.

c). If the system is m.s. find the freq of oscillations.

$$s^3 + 9s^2 + 4s + k = 0 \Rightarrow 0 < k < 36 \text{ (range)}$$

$m = 36 \rightarrow$ m.s.

for freq. of oscillations,

even power of s terms $= 0$

$$\Rightarrow 9s^2 + 36 = 0 \overset{s \pm j2}{\Rightarrow} s = \pm j\boxed{2} \rightarrow \overset{2}{\text{rad/sec}}$$

(ii). $$G(s) \cdot H(s) = \frac{k}{s(s+2)(s+4)(s+6)} + k$$ char. eq

for $(s+1)(s+2)(s+3) = 0$ expansion

product of s terms	Addition of all const.	Σ of product of 2 const	Σ of π of 3 const.
s^3	$+ 6s^2$	$+ 11s$	$+ 6$

char. eq $\Rightarrow 1 + GH = 0$

$$\Rightarrow s(s+2)(s+4)(s+6) + k = 0$$

$$\Rightarrow s^4 + 12s^3 + 44s^2 + 48s + k = 0$$

s^4	1	44	k
s^3	12	48	
s^2	40	k	
s^1	$\frac{40 \times 48 - 12k}{40}$		
s^0	k		

Q. Determine the value of k and p so that the system T/f $G(s) = \frac{k(s+1)}{s^3 + ps^2 + 3s + 1}$ oscillates at a freq. of 2 rad/sec.

Sol. If freq. of oscillations are given so the system is m.s.

y. char. eq. $\Rightarrow s^3 + ps^2 + 3s + 1 + k(s+1) = 0$

$$\Rightarrow s^3 + ps^2 + s(3+k) + 1 + k = 0$$

$s^3 \quad 1 \quad 3+k$

$s^2 \quad p \quad k+1 \leftarrow AE$

$s^1 \quad \frac{p(3+k)-(k+1)}{p} \rightarrow 0 \quad (M.S) \rightarrow = 0 \Rightarrow p = \frac{k+1}{k+3}$

$s^0 \quad k+1$

AE : $ps^2 + (k+1) = 0$

$s = j\omega = j2 ; \Rightarrow p = \frac{k+1}{4} \uparrow \quad k = 1$

$\Rightarrow s^2 = -4 ; \Rightarrow -4p + (k+1) = 0 \quad$ $p = 0.5$

Q. A unity f/b control system has an OL T/F $G(s) = \frac{k(s+13)}{s(s+3)(s+7)}$. find the value of k for system stability ~~(b)~~. Determine the value of ξ, >, < $\alpha = 1$ when $k=1$.

Sol. $s^3 + 10s^2 + (21+k)s + 13k = 0$

$210 + 10k > 13k$

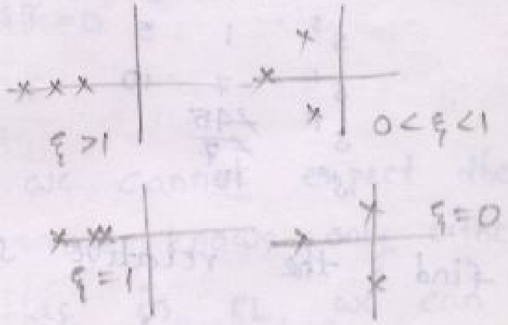
$210 > 3k$

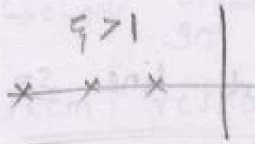
$\Rightarrow 0 < k < 70$.

when $k = 1$,

$s^3 + 10s^2 + 22s + 13 = 0$

$\Rightarrow s = -1, -1.7, -7.2$





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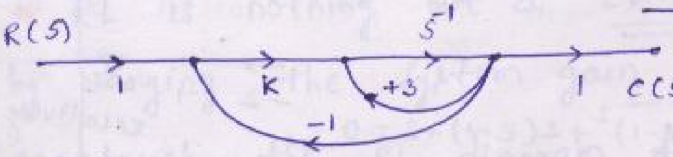
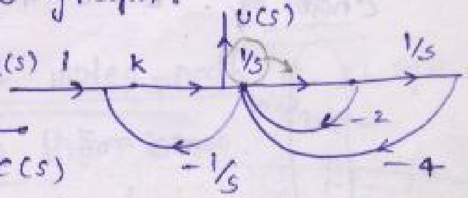
Q. find the range of k- value for system stability for the given signal flow graph.



$\frac{C(s)}{R(s)} = \frac{k/s}{1 - 3/s + k/s} = \frac{k}{s-3+k}$

$s^1 \quad 1$

$s^0 \quad k-3 > 0 \Rightarrow k > 3.$

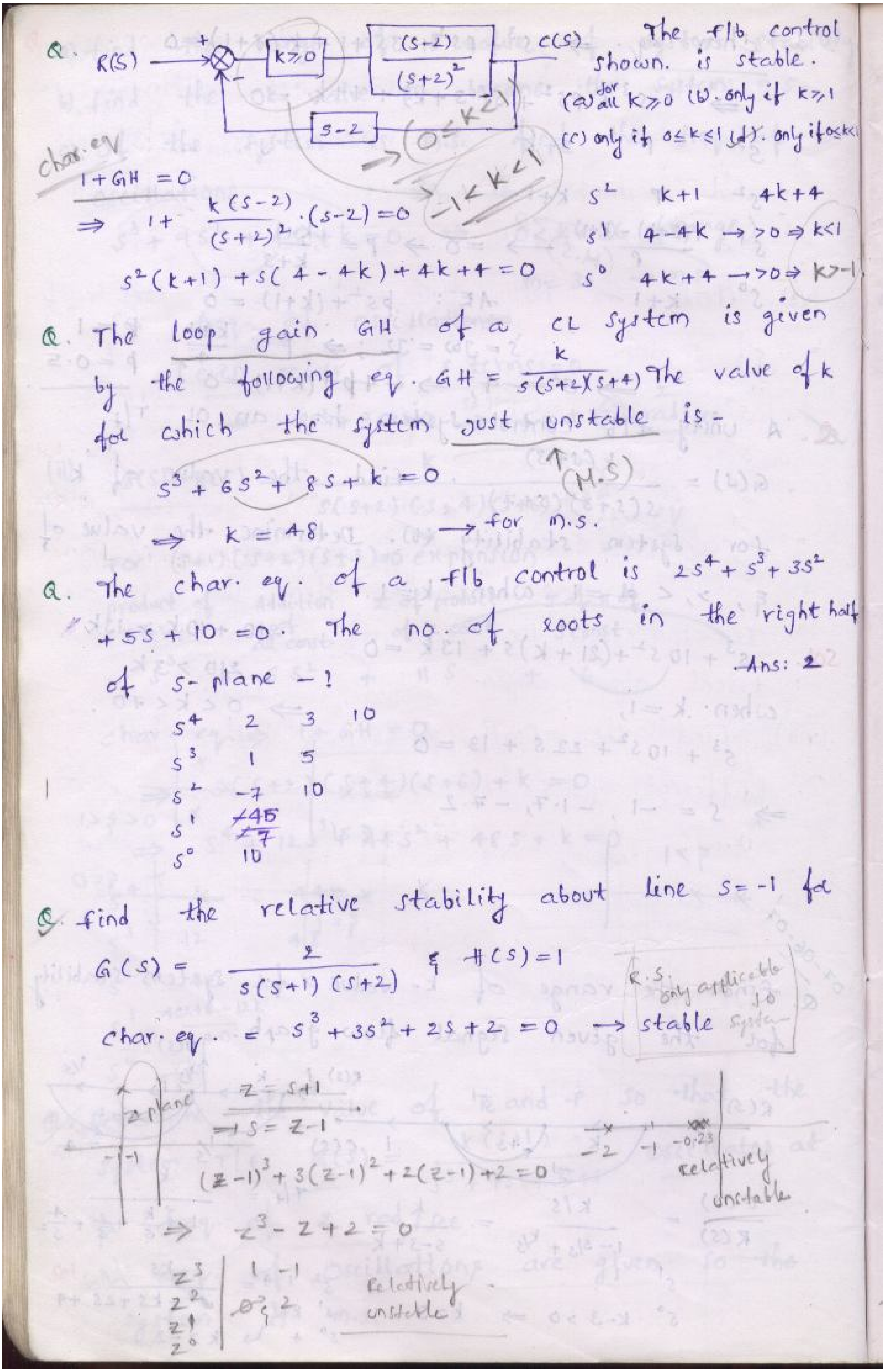


$T/F = \frac{k}{1 + \frac{k}{s} + \frac{2}{s} + \frac{4}{s^2}} = \frac{ks}{s^2 + ks + 2s + 4}$

$s^2 \quad 1 \quad 4$

$s^1 \quad k+2$

$s^0 \quad 4 \quad \hookrightarrow k > -2 .$



Q. The f/b control shown. is stable.
(a) for all k≥0 (b). only if k≥1
(c) only if 0≤k≤1 (d). only if 0≤k<

char. eq
1+GH = 0

$\Rightarrow 1 + \frac{k(s-2)}{(s+2)^2}\cdot(s-2) = 0$ $\to$ $-1<k<1$

$s^2(k+1) + s(4-4k) + 4k+4 = 0$

s^2	$k+1$	$4k+4$
s^1	$4-4k \to >0 \Rightarrow k<1$	
s^0	$4k+4 \to >0 \Rightarrow k>-1$	

Q. The loop gain GH of a CL system is given by the following eq. $GH = \frac{k}{s(s+2)(s+4)}$ The value of k for which the system just unstable is –

↑ (M.S)

$s^3 + 6s^2 + 8s + k = 0$

$\Rightarrow k = 48 \to$ for M.S.

Q. The char. eq. of a f/b control is $2s^4 + s^3 + 3s^2 + 5s + 10 = 0$. The no. of roots in the right half of s-plane – ? Ans: 2

s^4	2	3	10
s^3	1	5	
s^2	−7	10	
s^1	$\frac{45}{7}$		
s^0	10		

Q. find the relative stability about line s = −1 for $G(s) = \frac{2}{s(s+1)(s+2)}$ & $H(s) = 1$

char. eq. = $s^3 + 3s^2 + 2s + 2 = 0 \to$ stable

R.S. only applicable to stable system

$z = s+1$
$\to s = z-1$

$(z-1)^3 + 3(z-1)^2 + 2(z-1) + 2 = 0$

$\Rightarrow z^3 - z + 2 = 0$

relatively unstable

z^3	1	−1
z^2	$0 \approx \epsilon$	2
z^1		
z^0		

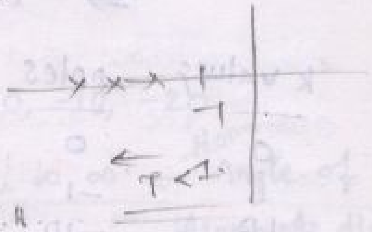
Relatively unstable

Q. check whether the τ is greater or lesser or equal to 1 sec. for $s^3 + 7s^2 + 25s + 39 = 0$.

$s = -\frac{1}{\tau}$

$s = -1$

(or)



$s = z-1$ sub. and then solve using R.H.

Q. If the RH criteria applicable, is applicable for sine & cosine terms -?

* The RH criteria not applicable for trigonometric terms and exponential terms b'coz gives infinite series. but approximate soln. can be obtained for exponential terms. transpotation delay system

Q. find the system stability for $G(s) = \frac{e^{-sT}}{s(s+1)}$, $H(S)=1$

Transpotation delay system not effect the magnitude it effects

$$e^{-sT} \simeq (1-sT)$$

$$G(s) = \frac{(1-sT)}{s(s+1)}$$

char. eq $= s^2 + s + 1 - sT = 0$

s^2 1 1

s^1 $1-T \rightarrow > 0$

s^0 1 $\Rightarrow T < 1$ Sec

Root Locus:-

↳ CL pole ↳ path → k = 0 to ∞

In RH criteria we cannot expect the system response because we knows only either poles LHS or RHS where as in RL, we can find the system response by observing the CL poles path.

* RL is nothing but a CL poles path by varying the system gain from 0 to ∞.

RL

① . CL syst

② <k<

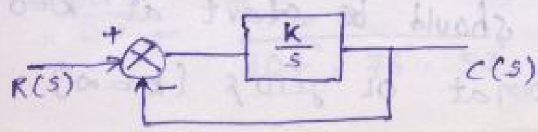
③ kmar/ωmar

④ . k→ ondamped/ and/critical

⑤ . KS.

⑥ . $\xi = 1$ inclination = cos ξ

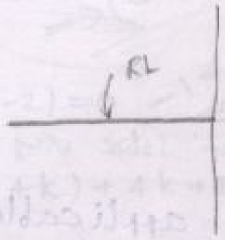
Q. construct the RL diagram for the following block diagrams.



char. eq ⇒ $1 + GH = 0$

⇒ $1 + \frac{k}{s} = 0$ → $s + k = 0$ ⇒ CL poles $s = -k$

k values	poles
0	0
1	−1
10	−10
⋮	⋮
∞	−∞



for $G = k/s^3$

$s^3 + k = 0$

$s = \sqrt[3]{k}$

$= -1\sqrt[3]{k}$

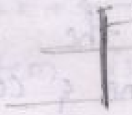
for $G = k/s^2$

$1 + k/s^2 = 0$

$s^2 + k = 0$

$s = \pm j\sqrt{k}$

k	s
0	0
1	±j1
10	±j10



* As order increases drawing the RL diagram with char. eq. becomes very difficult hence OL T/f is used to draw a RL.

⇒ Relationship b/w OL T/f poles and zero's to CL T/f poles.

OL T/f $G(s) \cdot H(s) = \frac{k \cdot N(s)}{D(s)}$ → ①

OL zero's $N(s) = 0$

OL poles $D(s) = 0$

CL poles $1 + G(s) \cdot H(s) = 0$

$1 + k \cdot \frac{N(s)}{D(s)}$ → (0 to ∞)

⇒ $D(s) + k\,N(s) = 0$

* CL poles are nothing but a sum of OL poles and OL zero's with system gain k.

* case 1: k = 0

⇒ $D(s) = 0$. → CL poles

when k = 0, the OL poles must be equal to CL poles.

* case 2: k = ∞, N(s) must be zero. $N(s) = 0$

so OL zero's = CL poles $k = -\frac{D(s)}{N(s) \to 0}$

* The RL diagram should be start at k = 0 [at OL poles] and ends at OL zero's [k = ∞]

Q. find where the RL diagram starts and ends.

$$G(s)\cdot H(s) = \frac{k(s+5)}{s(s+10)(s+20)}$$

starts : OL poles $k=0$, $s = 0, -10, -20$

Ends : OL zero's $k=\infty$, $s=-5, \infty, \infty$ ← Along Angle of Asymptote dire.

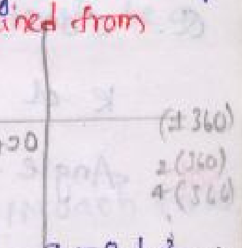
⇒ Angle & Magnitude condition :-

* The CL system stability is given by char. eq. $1+GH=0$.

The construction rules of RL are obtained from angle & magnitude condition.

$$\Rightarrow G(s)\cdot H(s) = -1+j0.$$

But the RL diagram drawn for O/L T/f ie $GH = -1+j0$ (± 180)



Angle condition : $\angle G(s)\cdot H(s) = \angle -1+j0$

$$= \pm(2q+1)180^\circ,\ q=0,1,2..$$

$$= \text{odd multiples}(\pm 180)$$

purpose :-

To check any point existing on RL or not that means all the points on RL must satisfy the angle condition.

Q. Check whether the following points lies on root locus or not for $GH = \frac{k}{s(s+2)(s+4)}$

① $s=-0.75$ ② $s=-1+j4$

$$\angle GH = \frac{\angle k}{\angle s\ \angle s+2\ \angle s+4}\Big|_{s=-0.75}$$

$$= \frac{\angle k}{\angle -0.75\ \angle 1.25\ \angle 3.24} = \frac{0^\circ}{\pm 180^\circ \cdot 0^\circ \cdot 0^\circ}$$

tan0 = 0

tan 180 = 0

?

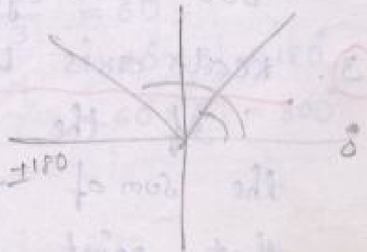
b'coz of -ve sign

$= \pm 180^\circ$ satisfies angle condi. so the given point on RL.

for $s=-1+j4$

$$\angle GH = \frac{\angle k}{\angle(-1+j4)\ \angle(1+j4)\ \angle(3+j4)}$$

$$= \frac{0}{104^\circ \cdot 76^\circ \cdot 53^\circ}$$ not satisfieq, so the given point not on RL.



⇒ Magnitude condition :- $|G(S).H(S)| = 1.$

$G(S).H(S) = -1+J0$

The magnitude of GH at a point which is on the Root locus ~~which there~~ that means the magnitude condi. is valid only when the given point is on the RL.

purpose:- To apply mag. condi. 1st we've to verify angle condi.

To find the system gain at any point which is on the RL.

Q. Consider the system with $GH = \frac{k}{s(s+4)}$. Find k of system gain at a point $s = -2+J5$.

Sol: Angle condi. $\angle GH = \frac{\angle k}{\angle(-2+J5)\,\angle(+2+J5)} = -180^\circ$

satisfies angle condi. so the given point is on RL.

To find k, magnitude condi.

$$M.C. \quad \frac{k}{\sqrt{4+25}\,\sqrt{4+25}} = 1 \Rightarrow k = 29.$$

Rules for constructing RL :-

① Symmetrical :-

The RL diagrams are symmetrical about real axis because the loc. of poles and zero's are symmetrical about real axis.

② No. of RL branches / Loci :- (N)

proper T/F → If the poles $P > Z \Rightarrow$ no. of RL branches = P

Improper T/F → $P < Z \Rightarrow$ " = Z

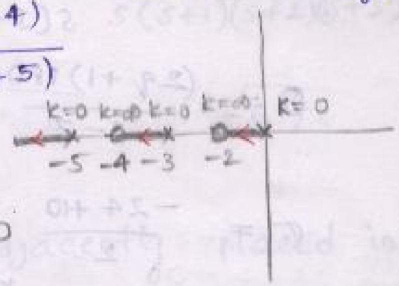
But actually $N = P = Z$. ← strictly proper T/F.

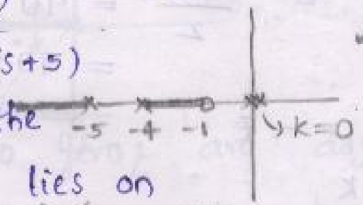
③. Real axis loci :-

If the point exists on real axis RL branch the sum of the poles and zero's to the RHS of that point should be odd.

Q. find the sections of real axis which belongs to RL. (1). $GH = \frac{k(s+2)(s+4)}{s(s+3)(s+5)}$

(2). $GH = \frac{k(s+1)}{s^2(s+4)(s+5)}$





check whether the following points lies on RL or not (a). 0 ~~no~~ yes b). -1, yes c). -4 yes d). -5 yes e). -2, yes f). -∞ yes

* At the initial position of p & z's there must be a RL branch.

④ Asymptode Angles :-

Asymptotes are RL branches which approach to ∞.

* The no. of asymptotes $= p - z$.

* Angle of asymptote $= \frac{(2q+1)180}{p-z}$, $q = 0, 1, \ldots (p-z-1)$.

→ The angle of asymptote gives the direction of the zeros when the $p > z$.

→ The asymptotes are symmetrical about real axis.

⑤ Centroid :-

centroid gives the intersection point of asymptotes on the real axis.

$$\sigma = \frac{\text{sum of real part of poles} - \Sigma R.P(z's)}{p-z}$$

The 'σ' may be located any where on real axis.

Q. Calc. the angle of asymptotes and σ for

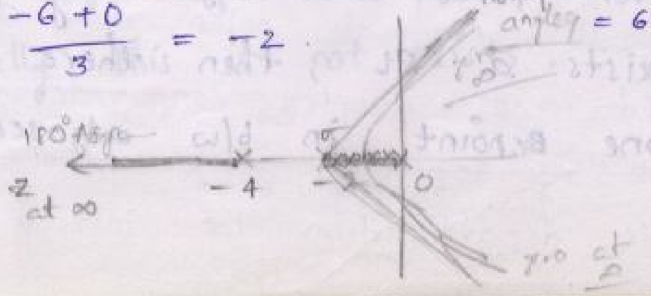
$$GH = \frac{k}{s(s+2)(s+4)}$$

$$\theta = \frac{(2q+1)180}{p-z} \rightarrow \frac{180}{p-z} = \frac{180}{\boxed{3}} = 60^\circ$$

$$= 60^\circ \times \boxed{3} = 180^\circ$$

no. of angles $= 60^\circ \times \boxed{5} = 300^\circ$

$$\sigma = \frac{-6+0}{3} = -2$$

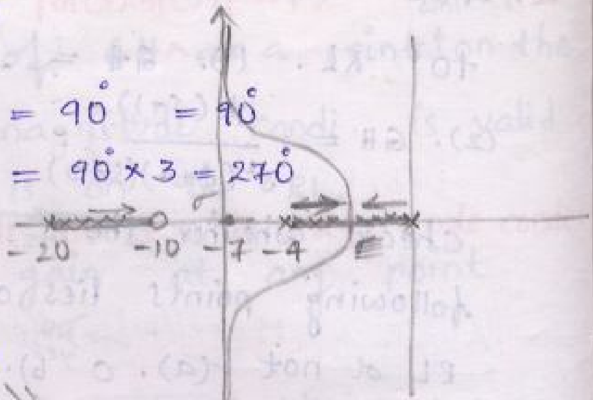


(2). $GH = \frac{K(s+10)}{s(s+4)(s+20)}$

$\theta = \frac{(2q+1)180}{p-z} = \frac{180}{2} = 90^\circ = 90^\circ$

$= (90^\circ \times 3 = 270^\circ$

$\sigma = \frac{-24+10}{2} = -7$



(3). $GH = \frac{K}{s(s+1)(s+2)(s+3)}$

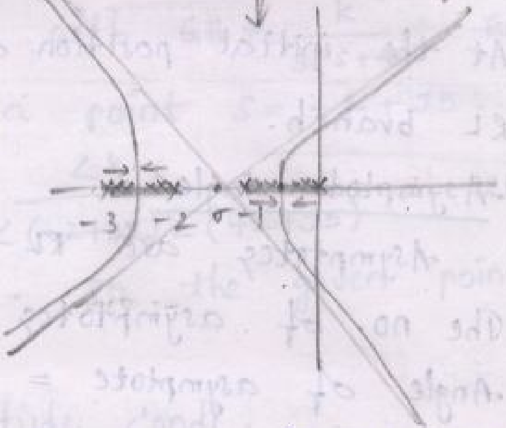
$\theta = \frac{180}{4} = 45^\circ$

$45 \times 3 = 135^\circ$

$45 \times 5 = 225^\circ$

$45 \times 7 = 315^\circ$

$\sigma = \frac{-6}{4} = -1.5$



⑥. Break points :-

A point where the RL meets, intersection point of RL branches. It is point where RL branches leaves or enter into the real axis.

→ The point where RL branches leaves the real axis - break out point

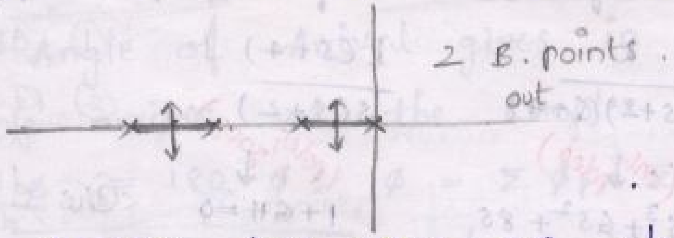
→ The point where RL branches enter into the real axis - break in point.

* The RL branches enter or leaves real axis with an angle of ± $\frac{180^\circ}{n}$ where 'n' is no. of RL branches [no. of poles at the break point].

→ pointing the existance of B.points :-

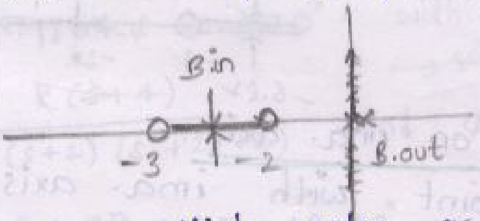
(1). whenever poles are adjasently placed in b/w there exists a RL, then there should the min. one Br.point in b/w adjasecently placed poles.

Q. find the B. points for $GH = \frac{k}{s(s+1)(s+2)(s+3)}$



(2) whenever two zeros are adjacently placed in b/w there exists the RL branch then there should be the min. one B.in point in b/w adj. placed zero's.

Q. find the no. of B. points for $GH = \frac{k(s+2)(s+3)}{s^2}$



whenever multiple poles or zeros located at a particular loc. then there must be the atleast one break away at break in point at that loc.

(3) whenever zero exists left most side on real axis, to the left of that zero there exists a root locus branch then there should be the min. one B.in point to the left of that zero. {only $p > z$}

$GH = \frac{k(s+10)}{(s+1)(s+2)}$

(4) when pole lies on the real axis to the left of that pole there exists a RL branch there should be the min one B.away point to the left of that pole when $p < z$ only. practically this is not exists.

Q. Determination of co-or. of B. points :-

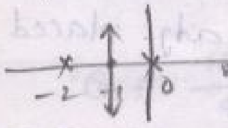
GH $\frac{k}{s(s+2)}$, $\frac{k}{s(s+2)(s+4)}$ $\frac{k(s+4)}{s(s+2)}$.

(only poles)

$s^2+2s=0$

diffe. $\Rightarrow 2s+2=0$

$\Rightarrow s=-1$

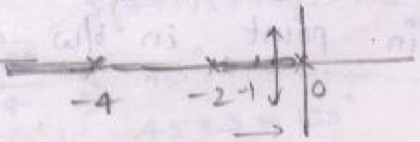


(only poles)

s^3+6s^2+8s

$\xrightarrow{d/ds} 3s^2+12s+8=0$

$\Rightarrow s=-0.84, -3.15$



(poles & zeros)

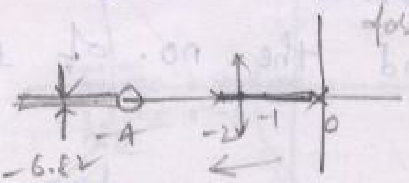
$1+GH=0$

$\Rightarrow GH=-1$

$\therefore -1 = \frac{k(s+4)}{s(s+2)}$

$\frac{dk}{ds} = \frac{(-2s-2)(s+4)+s^2+2s}{(s^2+2s)^2} = 0$

$\Rightarrow s = -1.17, -6.82$



① CE

② Rewrite CE in the form k=f

③ $\frac{dk}{ds}=0$

④ Roots of $\frac{dk}{ds}$ gives valid & invalid B. point. valid B. point must be on RL for which k → +ve

⑦. Intersection point on ima. axis :-

Intersection point with ima. axis given by R-H criteria.

when $k_{marginal}$ → +ve, there will be B. points.

Eg :- $GH = \frac{k}{s(s+1)(s+3)(s+5)}$

$\longrightarrow$ CE $= s^4+9s^3+23s^2+15s+k=0$

$$\begin{array}{llll} s^4 & 1 & 23 & k \\ s^3 & 9 & 15 & \\ s^2 & 21.3 & k & \\ s^1 & \frac{21.3\times15-9k}{21.3} & & \\ s^0 & k & & \end{array}$$

$\frac{21.3\times15-9k}{21.3}=0 \Rightarrow k=35.5$

Make even powers to zero

$21.3s^2+35.5=0$

$\Rightarrow s = \pm j.1.29$

① CE

② RH criteria

③ k_{mar}

④ from AE

⑤ Roots AE

⑧ Angle of departure and arrival:-

Angle of departure should be calculated at complex conjugate poles and angle of arrival calculated at complex conjugate zero's.

* Angle of departure gives with what angle the pole depart from the initial position.

$\phi_d = 180 - \phi$; $\phi = \Sigma\phi_p - \Sigma\phi_z$

add

* Angle of Arrival gives in what dirc. the pole arrives at the complex zero.

$\phi_a = 180 + \phi$; $\phi = \Sigma\phi_p - \Sigma\phi_z$

Q. find the angle of arrival for the system

$$GH = \frac{K(s^2+2s+2)}{s(s+2)}$$

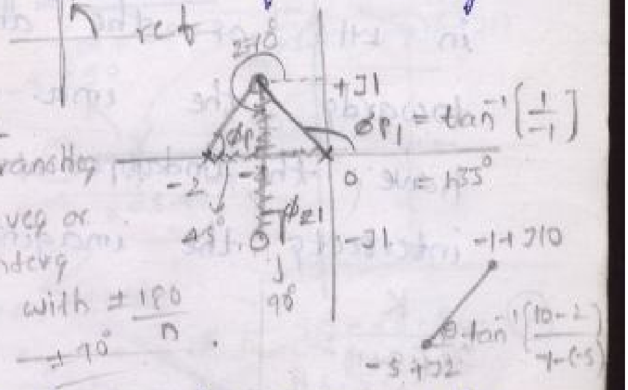
$$\phi = 135 + 45 - 90$$

$$= 90$$

$$\phi_a = 180 + \phi = 270°$$

RL branches leaves or enters with $\pm\frac{180}{n}$

$\pm 90°$



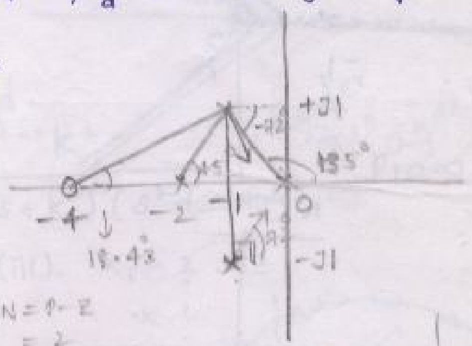
Q. $GH = \frac{K(s+4)}{s(s+2)(s^2+2s+2)}$ find ϕ_d at conju. poles.

$$\phi = 135 + 90 + 45 - 18.43°$$

$$= 251.5°$$

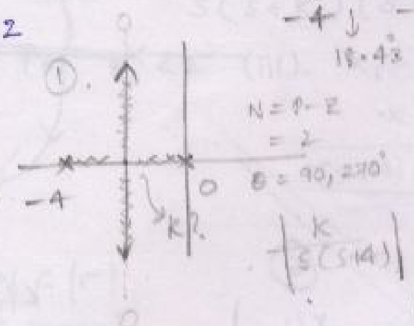
$$\phi_d = 180 - 252$$

$$= -72°.$$



find out equivalent RL

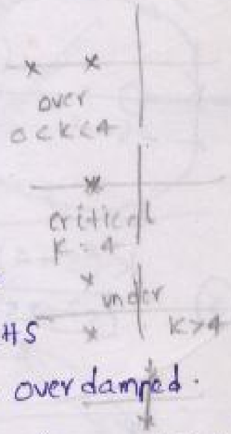
1. $GH = \frac{K}{s(s+4)}$



N = P - Z
= 2
θ = 90, 270°

$\left|\frac{K}{s(s+4)}\right| = 1$

K = 4



2. $\frac{K}{s(s+1)^2}$

3. $\frac{K(s+5)}{s^2}$

4. $\frac{K(s^2+2s+2)}{(s+4)(s+6)}$

5. $\frac{K(s+4)(s+6)}{s^2+2s+2}$

6. K/s, K/s^2, K/s^3, K/s^4

7. $\frac{K}{s(s+1)^2(s+2)}$

8. $\frac{K(s+1)^2}{s(s+2)}$

9. $\frac{Ks}{s^2+4}$ 10. $\frac{K}{s(s^2+2s+2)}$

* whenever the system poles are located in LHS at different loc.s → overdamped.

In the above system when $0<K<4$ then the poles are in the -ve real axis at diff loc.s, system is overdamped.

* whenever the system having B. point or roots meet at a particular point then the system is critical damped.

In the above system when $K=4$ both poles met at $s=-2$.

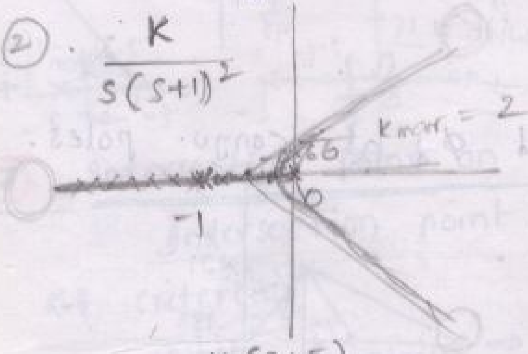
- * whenever the RL branches leaves or enters into the real axis, the system should have the under damped nature.
- * whenever the angle of asymptotes <90° and σ in LHS or the angle of departure and arrival towards the ima axis then the system should have the undamped nature. The RL branches meets or intersects the imaginary axis.

$s(s^2+2s+1)=0 \Rightarrow s^3+2s^2+s=0$

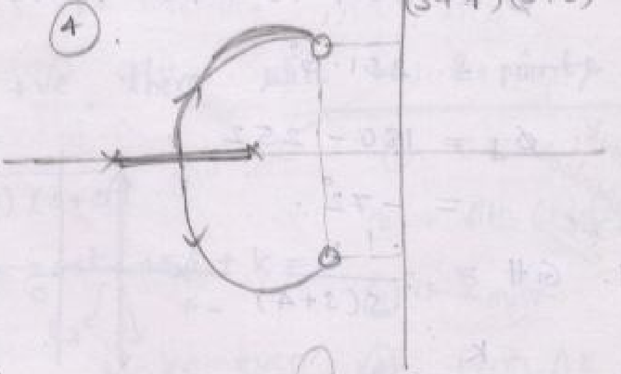
$3s^2+4s+1=0$

$s=-1, -0.33$

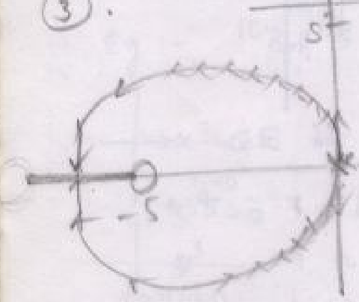
② . $\frac{K}{s(s+1)^2}$



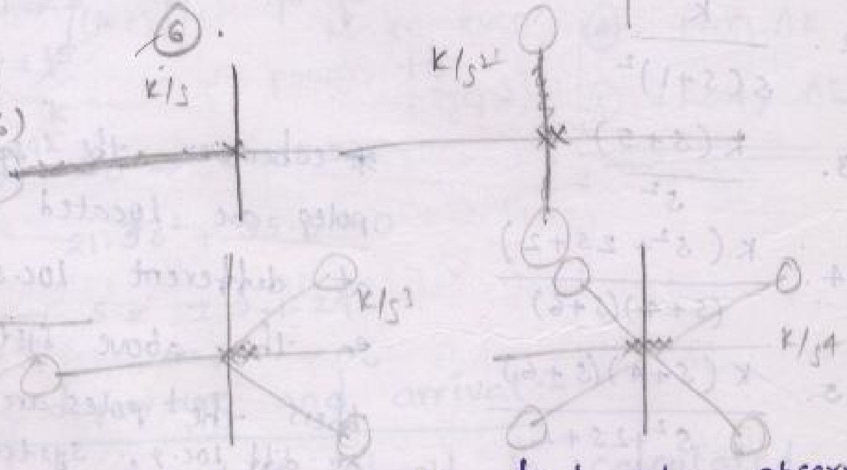
④ . $\frac{K(s^2+2s+2)}{(s+4)(s+6)}$



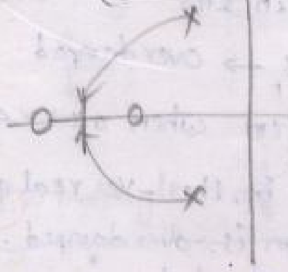
③ . $\frac{K(s+5)}{s^2}$



⑥ .

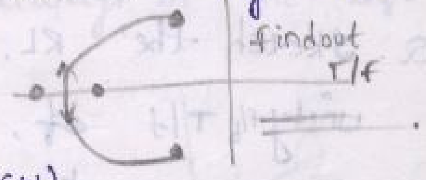


⑤ $\frac{K(s+4)(s+6)}{(s^2+2s+2)}$

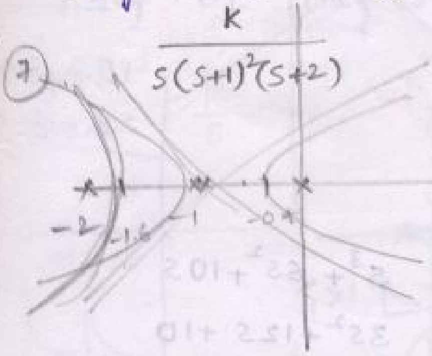


→ when given a RL diagram, to find T/F, observe the direction of RL branches. If the RL branch away from point then the point is pole. If the RL branch inside the point or towards the point then the point is zero.

✳ whenever the T/F consists only poles at origin the RL diagrams are nothing but angle of asymptotes.

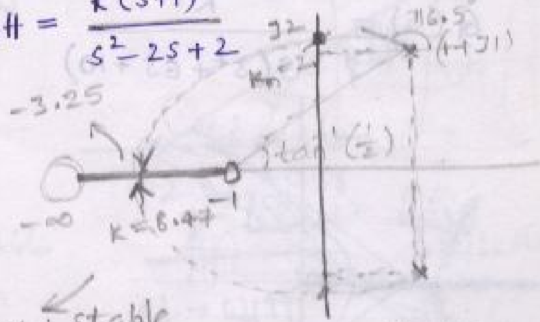


7) $\frac{K}{s(s+1)^2(s+2)}$



11).

$$GH = \frac{K(s+1)}{s^2-2s+2}$$



when k>2; stable

undamped k=2 m.s.

under damped ← 2<k<8.47

k=8.47 → critical

k>8.47 → over damped.

$\phi = 90-26.56$

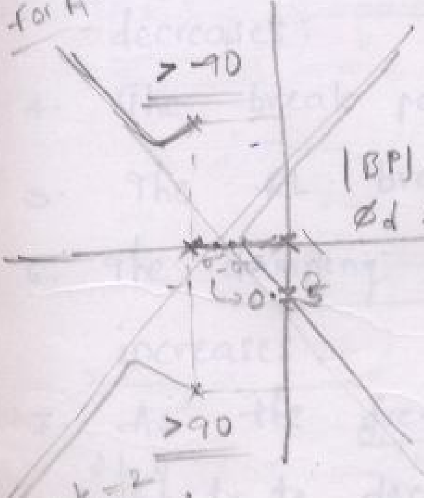
$\phi_d = 116.5°$

Q. The OL T/f $GH = \frac{K}{s(s+k_1)(s^2+2s+2)}$. Draw RL

Refer once again

for (i) $k_1 > 2$ (ii). $k_1 < 2$ (iii). $k_1 = 2$.

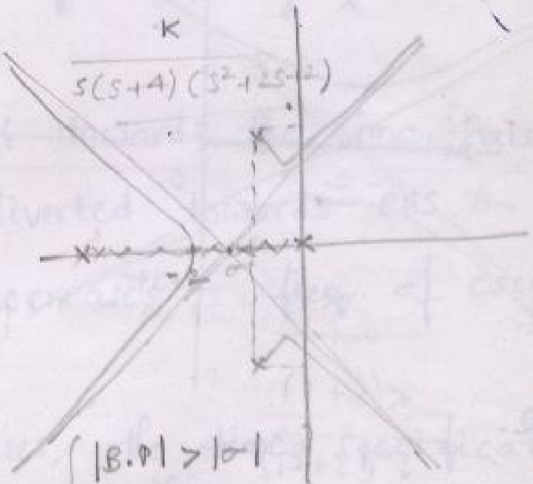
for $k_1=1$



$|BP| < |\sigma|$

$\phi_d > \mp 90°$

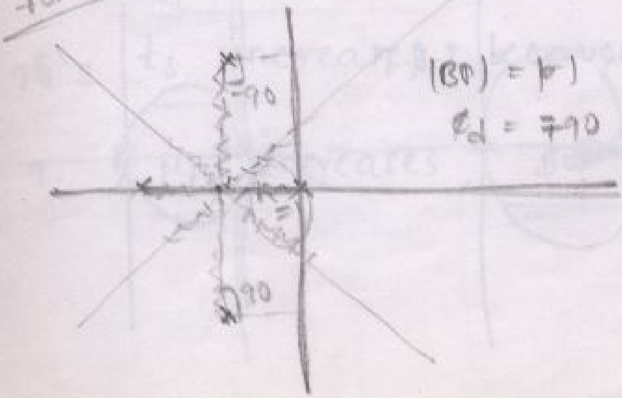
$\frac{K}{s(s+4)(s^2+2s+2)}$



$|B.P| > |\sigma|$

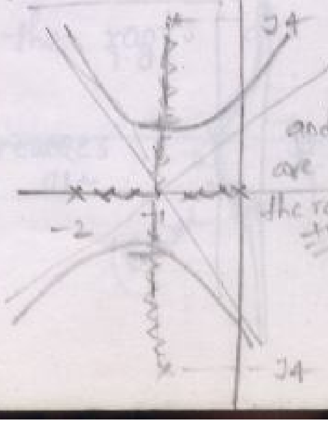
$\phi_d < \mp 90$

for $k_1=2$



$|BP| = |\sigma|$

$\phi_d = \mp 90$



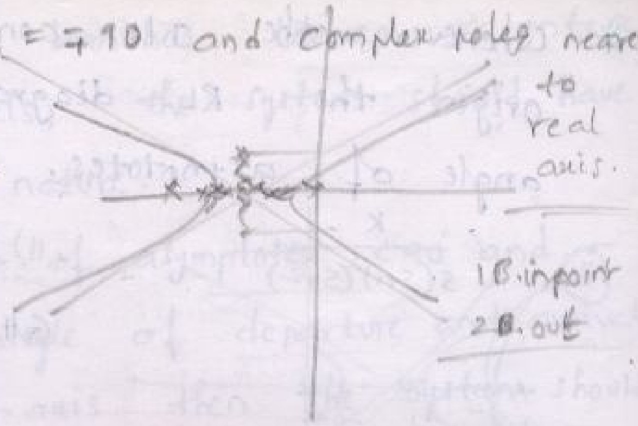
$|BP| = |\sigma|$

$\phi_d = \mp 90$

and complex poles are away from the real axis

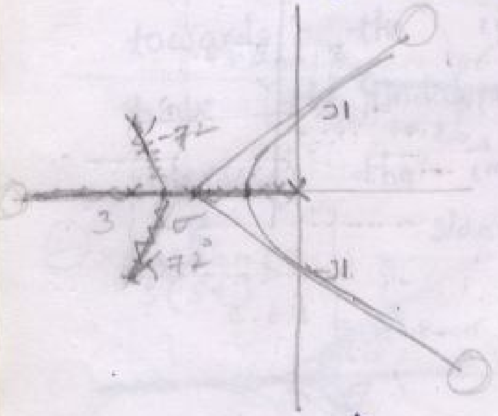
$\frac{\pm 180}{2} = \pm 90$

when $|\beta P| = |\sigma|$, $\phi_d = \mp 90$ and complex poles nearer to real axis.



Q. Sketch the RL for unity f/b T/f of,

$$G(s) = \frac{K}{s(s^2+6s+10)}$$



$s^3 + 6s^2 + 10s$

$3s^2 + 12s + 10$

$-1.18, -2.81$

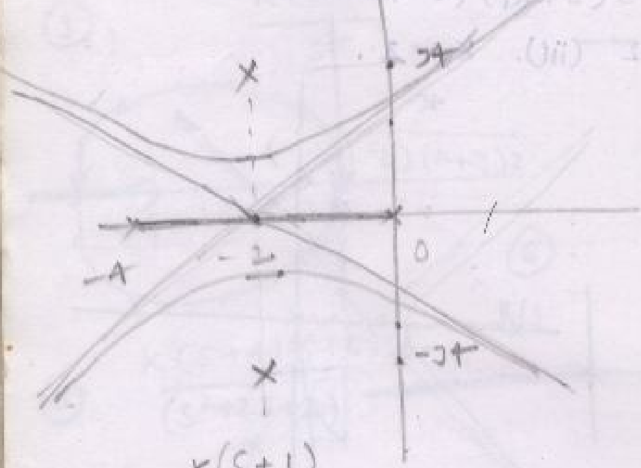
$\sigma = \frac{-6}{3}$

$= -2$

$\phi_p = 90 + \tan^{-1}(\frac{1}{}$

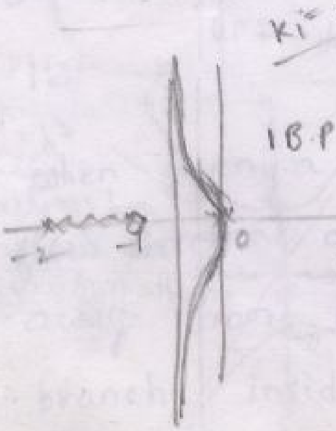
$\phi_d = -72°$

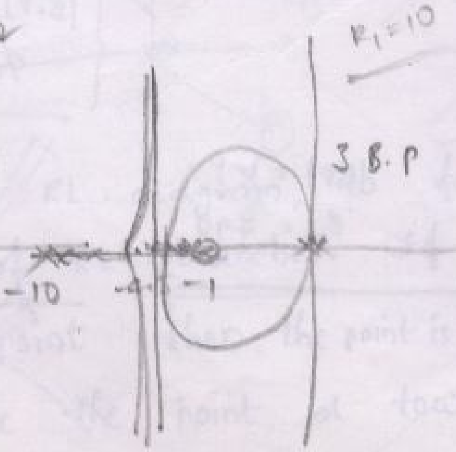
Q. $GH = \frac{K}{s(s+4)(s^2+4s+20)}$

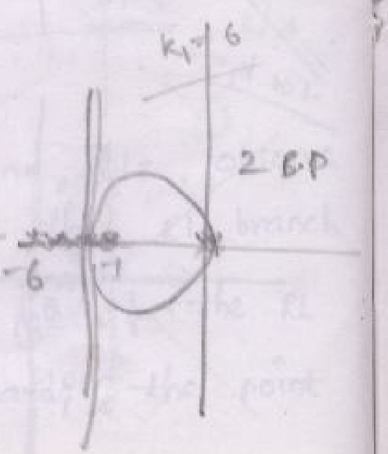


$-2 \pm j2.45$

Q. $\frac{K(s+1)}{s^2(s+k_1)}$

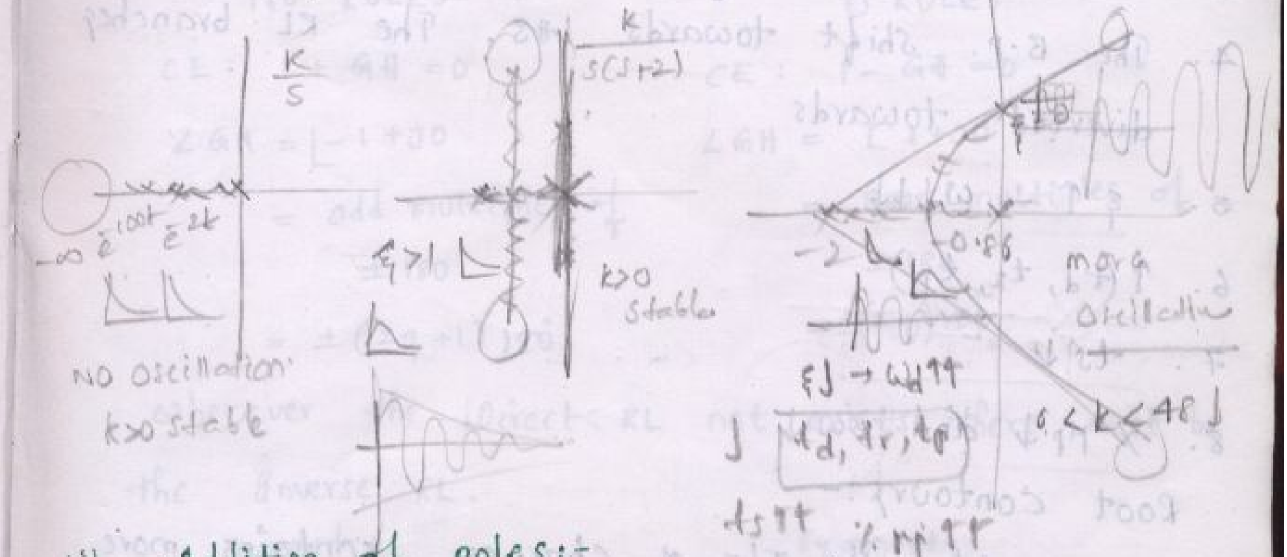






Effects of Addition of poles & zero's:-

The addition of P/ Z's always in the left half of the s-plane.



(1). Addition of poles:-

1. The system becomes more oscillatory.
2. The system relative stability decreases.
3. The range of k-value for the system stability decreases.
4. The break point shift towards the ima. axis.
5. The RL branches diverted towards RHS.
6. The damping factor decreases, freq. of osci. increases.
7. As the freq. increases, the time specification td, tr, tp decreases.
8. ts increases because the roots are moving
9. %Mp increases, BW increases. $(BW \propto \frac{1}{tr})$

(ii). Addition of zero's :-

1. The system becomes less oscillatory.
2. The range of k value's for system stability increases
3. The relative stability increases.
4. The B.P. shift towards LHS. The RL branches diverted towards
5. $\xi \uparrow - \omega_d \downarrow\downarrow$
6. $\uparrow (t_d, t_r, t_p)$
7. $t_s \downarrow$
8. $\% M_p \downarrow$ and $BW \downarrow$

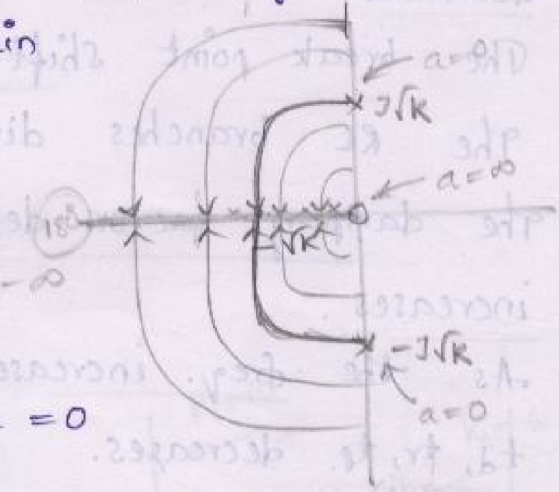
Root contours :-

If the T/F or char. eq. contains more than one unknown parameter, varying all the parameters from 0 to ∞, and drawing a RL diagram is nothing but a RC.

Draw the RC for the following CE : $s^2+as+k=0$

Assume a : system gain

k : const.



$$GH = \frac{as}{s^2+k}$$

$$-1 = \frac{as}{s^2+k}$$

$$a = \frac{-s^2-k}{s} \qquad \frac{da}{ds} = 0$$

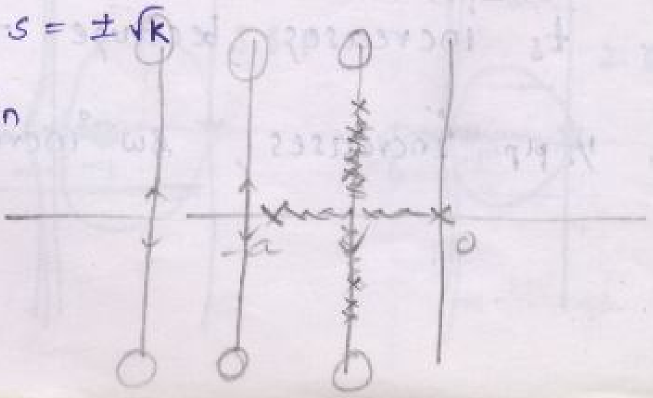
$$\Rightarrow s = \pm\sqrt{k}$$

Assume :

k : system gain

a : const.

$$GH = \frac{\overset{k \to 0 \text{ to } \infty}{k}}{s(s+a)}$$



Difference b/w direct RL and Inverse RL :-

Direct RL	Inverse RL
1. $k \rightarrow 0$ to ∞	$k \rightarrow -\infty$ to 0
180° RULES	0° RULES
CE : $1 + GH = 0$	CE : $1 - GH = 0$
$\angle GH = \angle -1 + J0$	$\angle GH = \angle 1 + J0$
= odd multiples of ± 180	= Even multiples of ± 180°
$= \pm(2q+1)180°$	$= \pm(2q)180°$

wherever the Direct RL not exists there must be the Inverse RL.

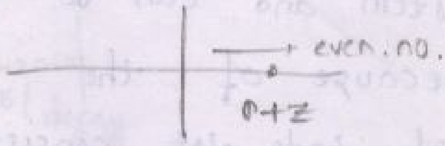
	symmetry	symmetry

2. no. of loci

$P > Z \Rightarrow N = P$

$P < Z \Rightarrow N = Z$

3. Real axis loci.

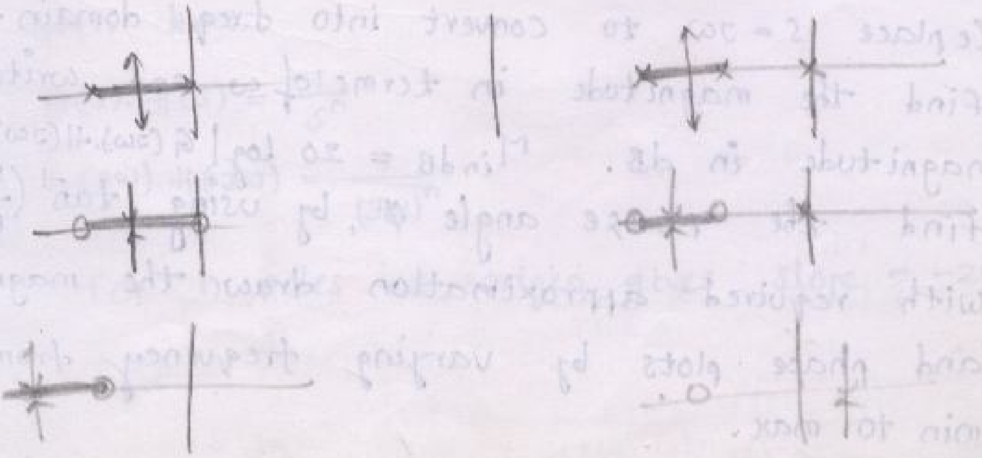


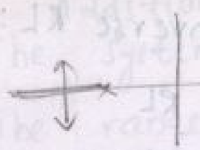
4. Asymptotes :-

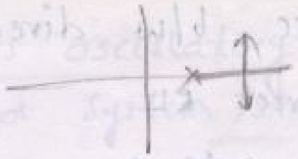
$$\theta = \frac{(2q+1)180}{P-Z} \qquad \theta = \frac{(2q)180}{P-Z}$$

5. $$\sigma = \frac{\Sigma \text{ R.P. poles} - \Sigma \text{ R.P. zero's}}{P-Z}$$

B. Points





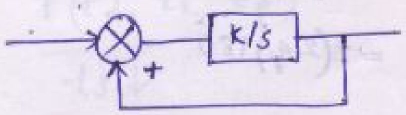


$\phi_d = 180 - \phi$ $\phi_d = 0° - \phi$

$\phi_a = 180 + \phi$ $\phi_a = 0° + \phi$

ℓr with

$K_{(max)}$ +ve $K_{(max)}$ −ve



$(1 - GH = 0)$

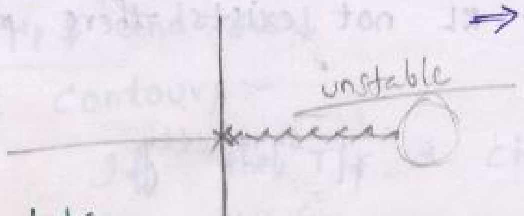
CE : $s - k = 0$

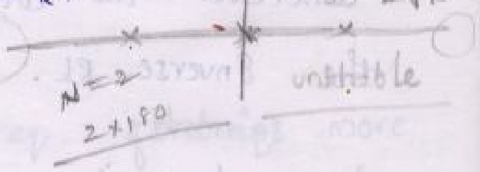
$\Rightarrow s = k.$

K/s^2

$s^2 - k = 0$

$s = \pm\sqrt{k}$





Bode plots:-

1. we can draw the bode plot for any higher order system and can be find the CL system stability. because of the logarithmic scale.
2. The bode plot consists magnitude & phase plots.

purpose:

1. freq. response OL T/f
2. CL system stability
3. Gm & Pm

procedure to draw Bode plots:-

1. Replace $s = j\omega$ to convert into freq. domain.
2. find the magnitude in terms of ω and write magnitude in dB. $M_{in\,dB} = 20 \log|G(j\omega).H(j\omega)|$
3. find the phase angle φ, by using $\tan^{-1}\left(\frac{Ima.Part}{Real\,part}\right)$
4. with required approximation draw the magnitude and phase plots by varying frequency from min to max.

Q. $G(s)H(s) = k$

$G(j\omega)H(j\omega) = k$

$M = k$

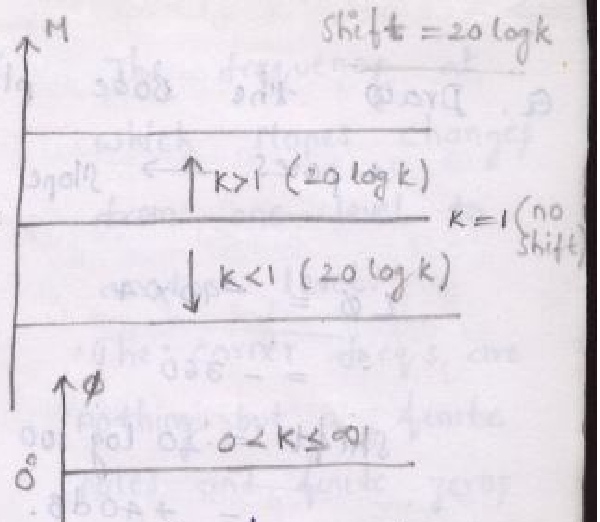
$M_{in\,dB} = 20\log k \xrightarrow{} \text{slope} = 0$

$k = 1, \quad M = 0\,dB$

$k = 10, \quad M = +20\,dB$

$k = 0.1, \quad M = -20\,dB$

$\angle G(j\omega).H(j\omega) = \angle k \longrightarrow$



* The phase plot is always ind. of k value, where as the shift in Magnitude plots depends on k - value.

Q: $G(s).H(s) = \frac{1}{s}$

$G(j\omega).H(j\omega) = \frac{1}{j\omega}$

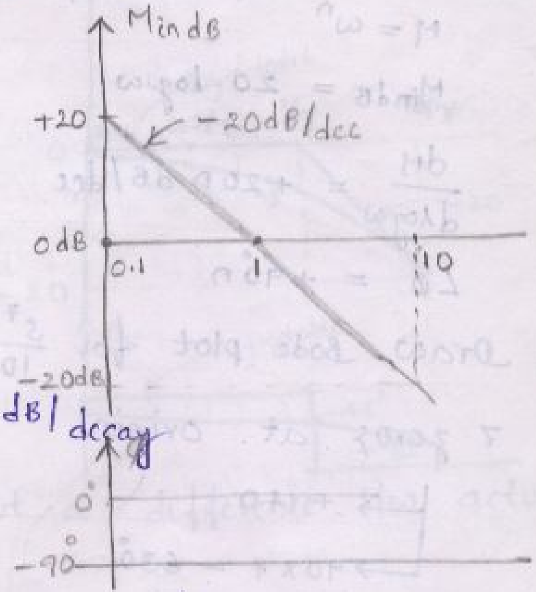
$M = 1/\omega$

$M_{in\,dB} = 20\log\frac{1}{\omega}$

$= -20\log\omega$

$\text{slope} = \frac{dM}{d\log\omega} = -20\,dB/\text{decay}$

$\angle\phi = \frac{\angle 1}{\angle j\omega} = -90^\circ$



✓NOTE:- whenever the T/f consists of poles and zeros at the origin then the plot start at opposite sign of the slope and intersect 0dB line at $\omega = 1$, when $k = 1$.

$G(s).H(s) = \frac{1}{s^n}$

$G(j\omega)H(j\omega) = \frac{1}{(j\omega)^n}$

for n poles at origin gives slope $= -20\times n\,dB/\text{decay}$

Q. Draw the Bode plot for $\frac{100}{s^4}$.

4 poles $\longrightarrow$ slope $= -20 \times 4$

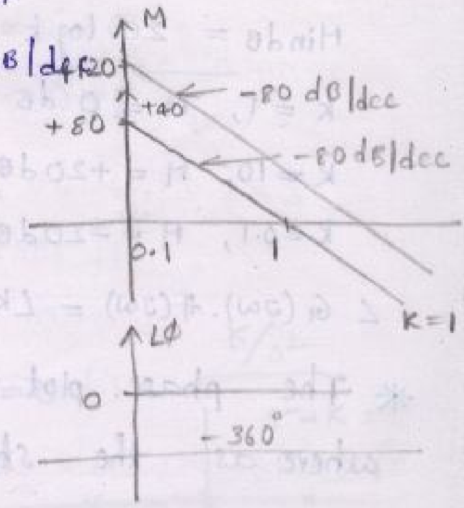
$= -80$ dB/dec

$\angle\phi = -90 \times 4$

$= -360^\circ$

shift $= 20 \log 100$

$= +40$ dB.



Q. $G(s).H(s) = s^n$

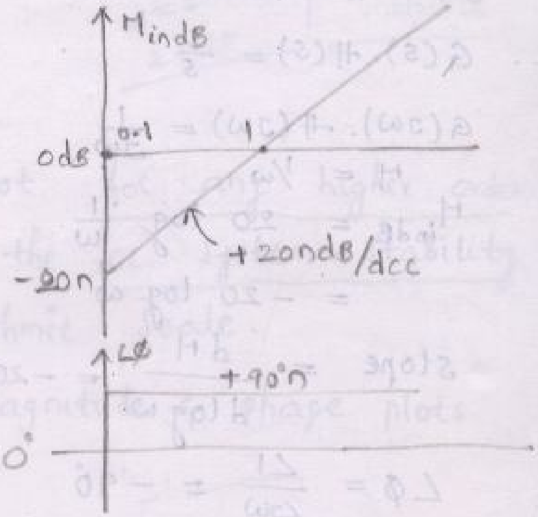
$G(j\omega).H(j\omega) = (j\omega)^n$

$M = \omega^n$

$M_{indB} = 20 \log \omega$

$\frac{dM}{d\log\omega} = +20n$ dB/dec

$\angle\phi = +90^\circ n$



Q. Draw Bode plot for $\frac{s^7}{10}$

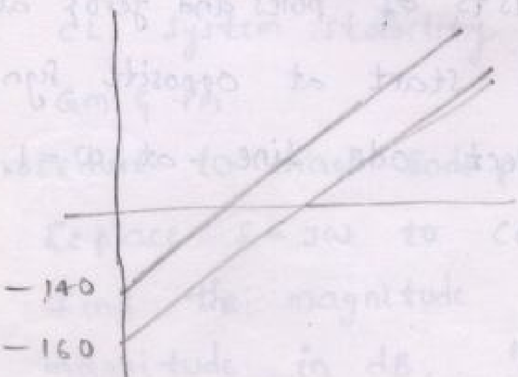
7 zeros at origin.

$\hookrightarrow + 140$

$\hookrightarrow 90 \times 7 = 630^\circ$

shift $= 20 \log \frac{1}{10}$

$= -20$ dB



Q. $GH = \frac{1}{1+sT}$

$G(j\omega).H(j\omega) = \frac{1}{1+j\omega T}$

$M = \frac{1}{\sqrt{1+(\omega T)^2}}$

$M_{indB\,Actual} = -20 \log \sqrt{1+(\omega T)^2}$; $\phi_{Actual} = -\tan^{-1}(\omega T)$

Asymptotic / Approx.

case 1: $\omega\tau < 1$, neglect $\omega\tau$ $\quad \omega < 1/\tau$

$M = 0\,dB$, slope $= 0$

$$\angle\phi = \frac{\angle 1}{\angle 1} = 0^\circ$$

case 2: $\omega\tau > 1$, neglect $\quad \omega > \frac{1}{\tau}$

$$M_{asy} = -20\log(\omega\tau)$$

$$\frac{dM}{d\log\omega} = -20\,dB/dec$$

$$\phi_{asy} = \frac{\angle 1}{\angle j\omega\tau} = -90^\circ$$

The frequency at which slopes changes from one level to another level.

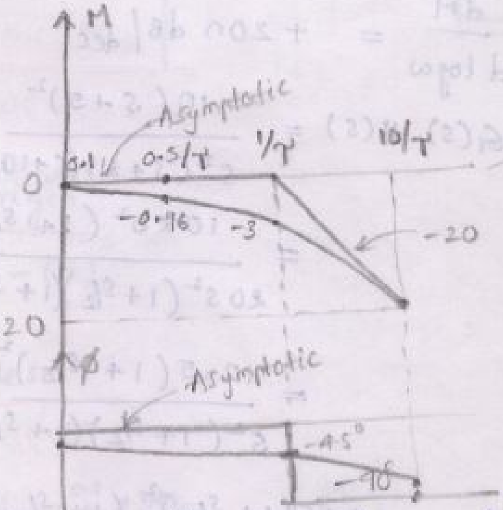
The corner freq's are nothing but a finite poles and finite zeros in the magnitude form.

for one finite poles

	s	ϕ
$<$ cf $\rightarrow$	0	0
$>$ cf $\rightarrow$	$-20\,dB/dec$	-90°

for 'n' finite poles

	s	ϕ
$<$ cf	0	0
$>$ cf	$-20n\,dB/dec$	$-90^\circ n$



Error at corner frequency :-

Error is nothing but a difference b/w actual and asymptotic value.

$\omega\tau = 1$, at $\omega = \frac{1}{\tau}$, $M_{asy} = 0\,dB$

$$M_{actual} = -20\log\sqrt{1+(\omega\tau)^2} = -20\log\sqrt{2}$$

$$= -3\,dB$$

$$E = 3\,dB$$

$$M_{asy}\left(\omega = \frac{0.5}{\tau}\right) = 0\,dB$$

$$M_{act} = -20\log\sqrt{1+0.5^2} = -0.96\,dB \; ; \; E = 0.96\,dB$$

Error is maximum at corner freq. On either side of cf, the error decreases symmetrically

$$\phi_{act} = -\tan^{-1}\omega\tau$$

At $\omega = \frac{1}{\tau}$, $\phi_{act} = -\tan^{-1} 1 = -45^\circ$; $\phi_{asy} = 0^\circ$ & -90°

$E = 45^\circ$

$\Rightarrow G(s).H(s) = (1+sT)^n$

$M_{indB} = +20n \log \sqrt{1+(\omega T)^2}$

$\phi_{act} = +n \cdot \tan^{-1}(\omega T)$

case1: $\omega T < 1$, neglect ωT

$M_{asy} = 0, \quad \phi_{asy} = 0$

case 2: $\omega T > 1$, neglect 1,

$M_{asy} := +20n \log \omega T$

$= +20n \log \omega + \overset{0}{\cancel{20n \log T}}$

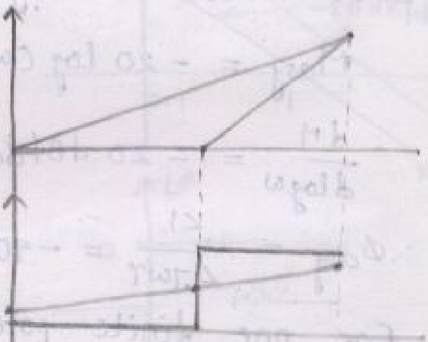
$$\frac{dM}{d\log\omega} = +20n \ dB/dec$$

$\phi = \angle j\omega \ldots\ldots$ n-times

$= 90^\circ$

for n- finite zeros

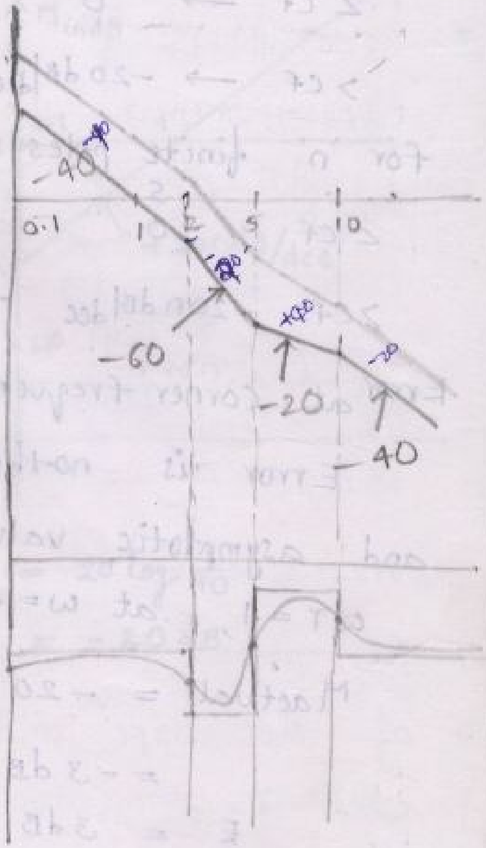
$$\begin{array}{lcc} & s & \phi \\ < cf \Rightarrow & 0 & 0 \\ > cf \Rightarrow & +20n dB/dec & +90^\circ n \end{array}$$



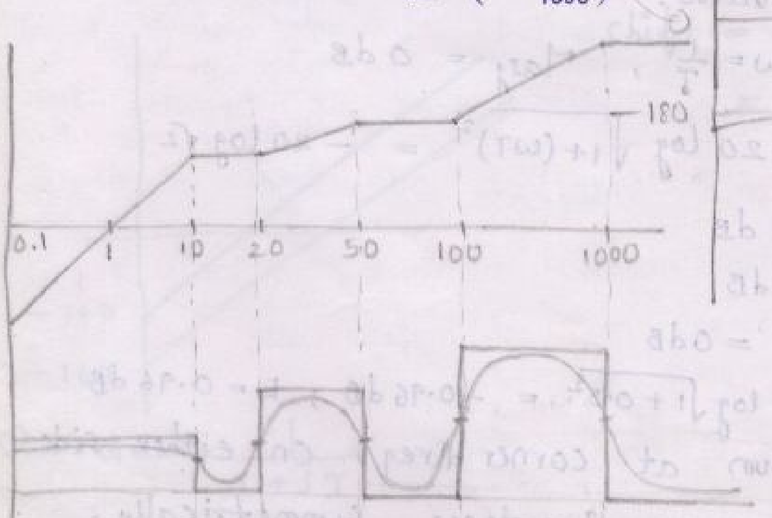
Q. $G(s).H(s) = \dfrac{10(s+5)^2}{s^2(s+2)(s+10)}$

$= \dfrac{10 \times 5^2 (1+s/5)^2}{20 s^2 (1+s/2)(1+s/10)}$

$= \dfrac{12.5(1+s/5)^2}{s^2(1+s/2)(1+s/10)}$



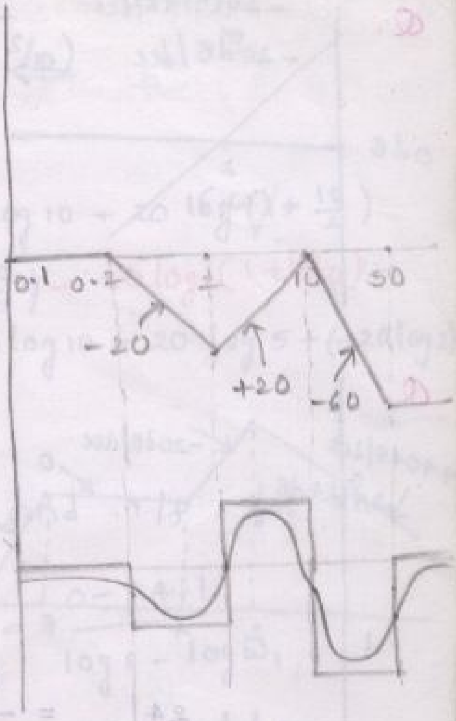
Q. $G(s)H(s) = \dfrac{\overset{1.0}{0.1} s (1+s/20)^2 \cdot (1+s/100)^3}{(1+s/10)(1+\frac{s}{50})^2(1+\frac{s}{1000})^3}$



The change in slope at cf is nothing but poles and zero's at that point.

Q. $G(s) \cdot H(s) = \dfrac{1 \cdot (1 + s/2)^2 (1 + s/50)^3}{(1 + s/0.2)(1 + s/10)^4}$

The

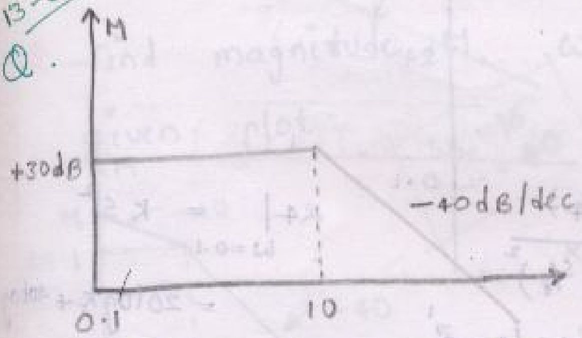


NOTE :-

The change in slope at corner frequency is nothing but poles & zeros at that point.

13-06-07!

Q.



Initial slope → P/Z → origin

change in slope = $-40 - (0)$

$= \underline{-40}$ ↳ poles

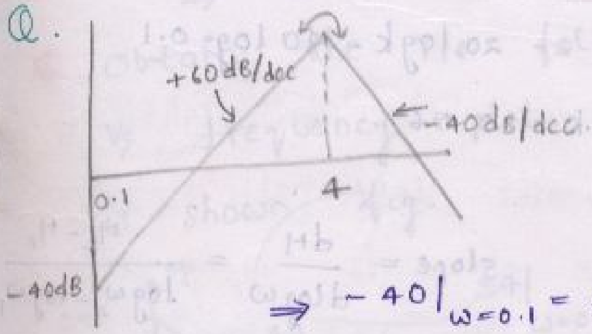
$$30\Big|_{\omega=0.1} = \frac{k}{(1+s/10)^2}$$

$$\Rightarrow 30 = 20 \log k - 40 \log(1 + \overset{0.1}{\cancel{s}}/10)^{\text{neg}}$$

Refer this once again..

$$\Rightarrow 30 = 20 \log k \Rightarrow k = 10^{1.5} =$$

Q.



$$\frac{k s^3}{(1 + s/4)^5} = -40\Big|_{\omega=0.1}$$

Change in slope = $-40 - (+60)$

$= -100$ dB/dec.

↓ 5 poles

$$\Rightarrow -40\Big|_{\omega=0.1} = 20 \log k + 60 \log 0.1$$

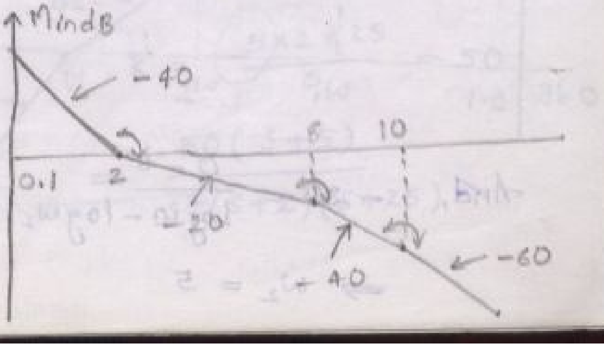
$$\Rightarrow k = \underline{10}$$

Q.

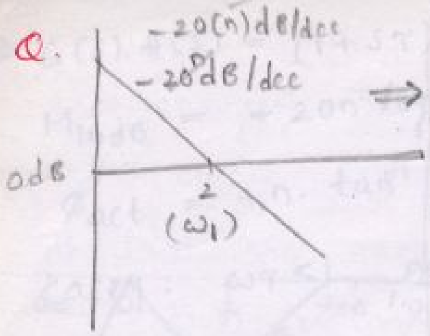
$$\frac{k(1 + s/2)}{s^2(1 + \frac{s}{8})(1 + s/10)}$$

$$0\Big|_{\omega=2} = 20 \log k - 40 \log 2$$

$\Rightarrow k = 4$



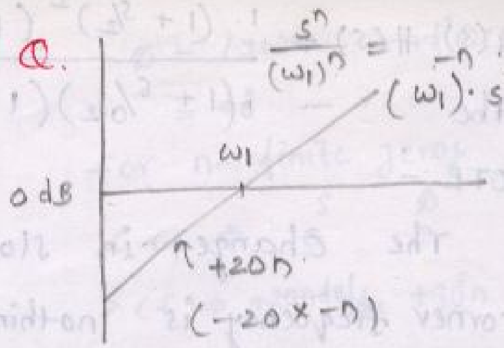
Q.



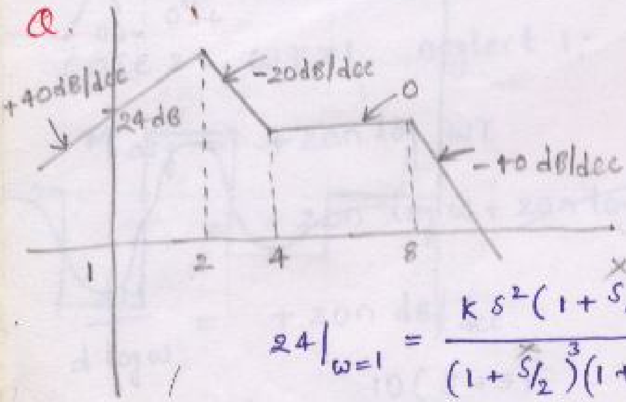
$\Rightarrow \frac{(\omega_1)^n}{s^n}$

Q.

$\frac{s^n}{(\omega_1)^n} = (\omega_1)^{-n}\cdot s^n$



Q.

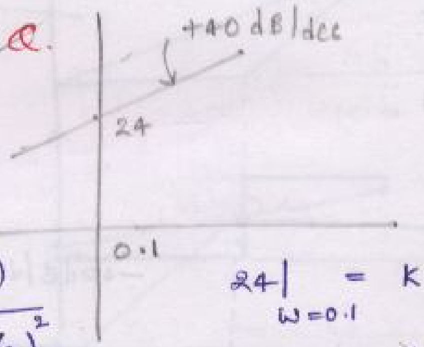


$$24|_{\omega=1} = \frac{K s^2(1+s/4)}{(1+s/2)^3(1+s/8)^2}$$

$24 = 20\log k + 40\log \cancel{1}$

$\Rightarrow k = 15.8$

Q.

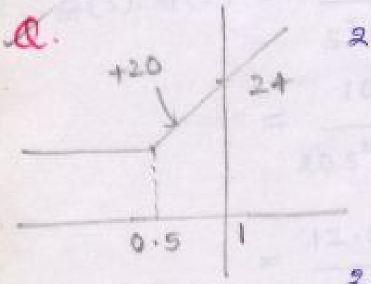


$24|_{\omega=0.1} = K s^2$

$= 20\log k + 40\log 0$

$\Rightarrow K = 1584.8$

Q.

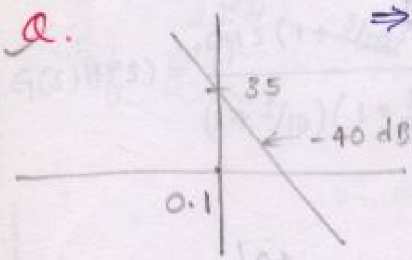


$24|_{\omega=1} = K(1+s/0.5)$

$24 = 20\log k + 20\log\left(1+\frac{1}{0.5}\right)$

$\Rightarrow k = 7.9$

Q.

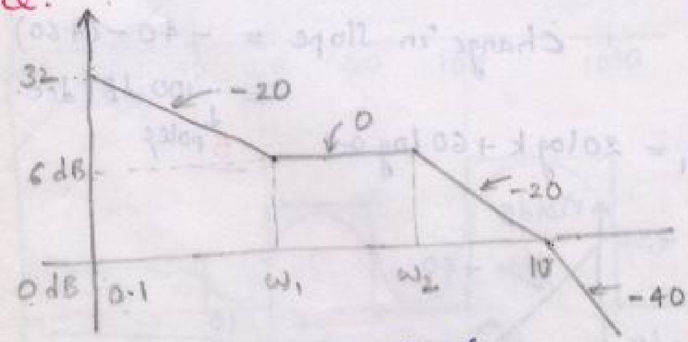


$35|_{\omega=0.1} = \frac{k}{s^2}$

$35 = 20\log k - 40\log 0.1$

$\Rightarrow k = 0.56.$

Q.



$$\text{slope} = \frac{dM}{d\log\omega} = \frac{M_2 - M_1}{\log\omega_2 - \log\omega_1}$$

$$\Rightarrow -20 = \frac{6-32}{\log\omega_1 - \log 0.1}$$

$\Rightarrow \omega_1 = 2$

And, $-20 = \frac{0-6}{\log 10 - \log\omega_2}$

$\Rightarrow \omega_2 = 5$

$$T/F = \frac{K(1+s/2)}{s(1+s/5)(1+s/10)}$$

check, $32|_{0.1} = \frac{k(1+s/2)}{s(1+s/5)(1+s/10)}$

also for $6|_{\omega=2}$

$6|_{\omega=5}$

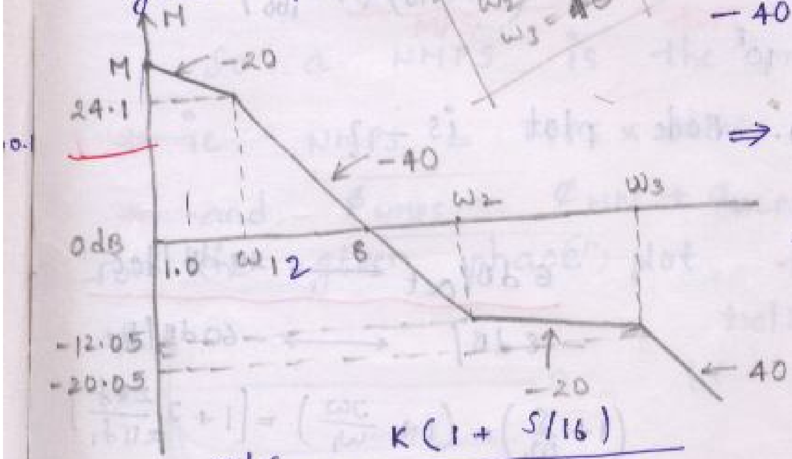
$$0|_{\omega=10} = 20\log k - 20\log 10 + 20\log(1+\tfrac{10}{2}) - 20\log(1+\tfrac{10}{5}) - 20\log(1+10/10)$$

$$0 = 20\log k - 20\log 10 + 20\log 5 + (-20\log 2)$$

$\Rightarrow k = 4$

Q. find magnitude M, $\omega_1, \omega_2, \omega_3$ and T/f for the given plot.

$\omega_2 = 16$
$\omega_3 = 40$



$$-40 = \frac{0-24.1}{\log 8 - \log \omega_1}$$

$\Rightarrow \omega_1 = 2$ rad/sec.

$$-20 = \frac{24.1-M}{\log 2 - \log 1}$$

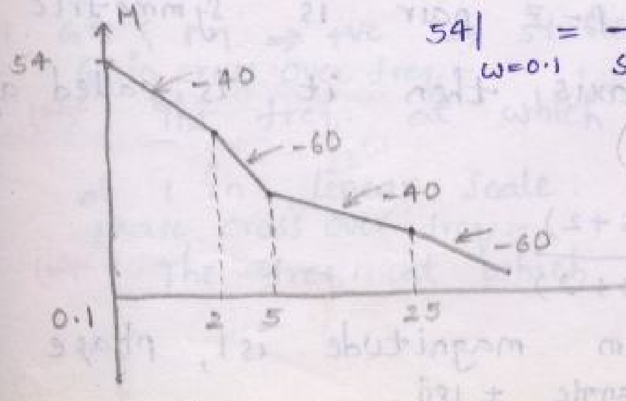
$\Rightarrow M = 30.12$ dB.

$$T/f = \frac{K(1+s/16)}{s(1+s/2)(\frac{s}{40}+1)}$$

$$30.12|_{\omega=1} = 20\log k - 20\log 1$$

$\Rightarrow k = 32$

Q. obtain the T/f for the given log magnitude vs frequency plot of a min. phase system is shown fig.

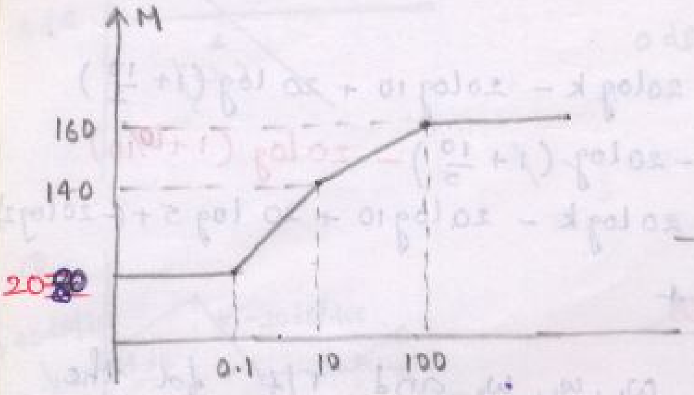


$$54|_{\omega=0.1} = \frac{K(1+s/5)}{s^2(1+s/2)(1+s/25)}$$

$$\therefore k = \frac{5\times2\times25}{5} = 50$$

$$= \frac{50(s+5)}{s^2(s+2)(s+25)}$$

Q. The approx. Bode plot of a min. ph. system shown in fig. A. T/f of the system is – ?



2 dec ⟶ 120

1 dec ⟶ 60 dB/dec

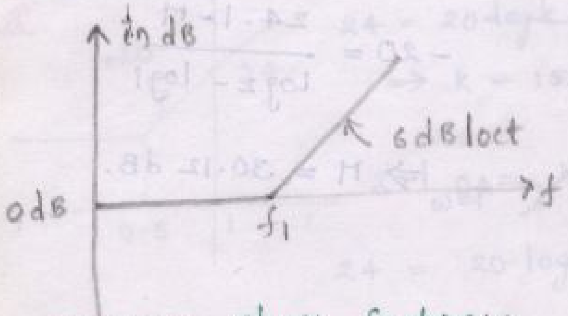
⟶ So, 0.1 to 10 ⟶ +60 dB/dec

10 – 100 ⟶ +20 dB/dec

$$20\Big|_{\omega=0.1} = \frac{K\left(\frac{s}{0.1}+1\right)^3}{\left(1+s/_{10}\right)^2\left(1+\frac{s}{100}\right)}$$

$$K = \frac{10\times10^2\times100}{(0.1)^3} = 10^8$$

Q. The fcn. corr. to. Bode plot is – ?



(?) 6 dB/oct ⟷ 20 dB/dec

–18 dB/ ⟷ –60 dB/dec

$$\left(1+\frac{s}{\omega_1}\right) = \left(1+\frac{j\omega}{\omega_1}\right) = \left[1+j\frac{2\pi f}{2\pi f_1}\right]$$

$$= \left[1+j\,f/_{f_1}\right]$$

Minimum phase system:-

A system in which all the poles and zeros in the LHS then it is called min. phase system.

Ex: $\dfrac{(s+1)}{(s+2)(s+3)}$

ALPHAS System:-

A system in which zeros lies on Right of s-plane, poles lies on the left of s-plane and the loc. of p – z pair is symmetric about imaginary axis then it is called as Alpha system.

Ex: $\dfrac{(s-2)(s^2-2s+2)}{(s+2)(s^2+2s+2)}$

* for Alpha system magnitude is 1, phase angle ± 180°.

The control systems are low pass system.

Non-minimum phase system:-

A system in which one & more z's located in right side of s-plane and remain all p's & z's are in LHS then it is called a Non-minimum phase system.

Ex:- $\dfrac{(s-1)(s+4)}{(s+2)(s+3)}$

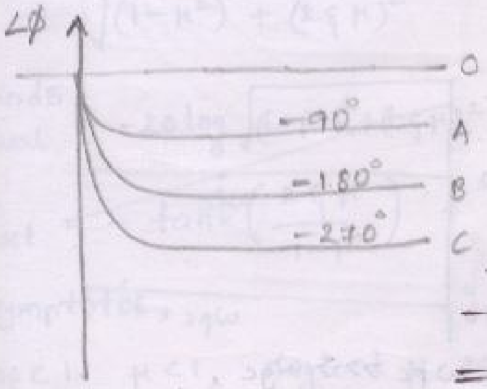
$$= \underbrace{\frac{(s+1)(s+4)}{(s+2)(s+3)}}_{MPS} \cdot \underbrace{\frac{(s-1)}{(s+1)}}_{ALPS}$$

So a NMPS is the product of MPS & ALPS

* ie NMPS = MPS × ALPS

* and $\phi_{NMPS} = \phi_{MPS} + \phi_{ALPS}$.

Q. for the given phase plot, the A, B, C plots are_



A: MPS

B: ALPS

C: NMPS

$-90° - 180°$

$= -270°$

Stability conditions:- → To find c/L system stability.

c/L system stability given by Char. eq: ie $1+G(s)H(s)=0$

Gain margin $GM = \frac{1}{M}\Big|_{\omega=\omega_{pc}}$

phase margin $PM = 180 + \angle GH\big|_{\omega=\omega_{gc}}$

1. GM & PM ⇒ +ve → stable. ; $\omega_{pc} > \omega_{gc}$, $GM > 1$.

Gain cross over freq:-

(ω_{gc}) The freq. at which the magnitude = 0 dB or 1 in linear scale.

phase cross over freq:-

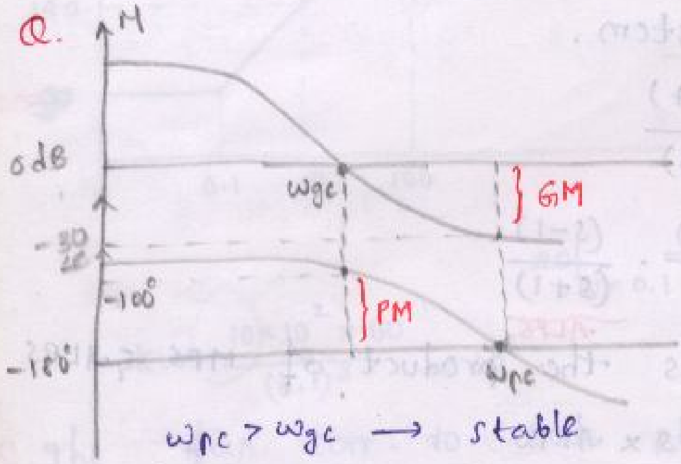
(ω_{pc}) The freq. at which ph. angle is $-180°$.

2. $\omega_{pc} = \omega_{gc} \longrightarrow GM = PM = 0, \longrightarrow$ m.s.

$GM = 1$

3. $\omega_{pc} < \omega_{gc} \Rightarrow GM$ -ve <1, PM -ve } unstable.

Q.



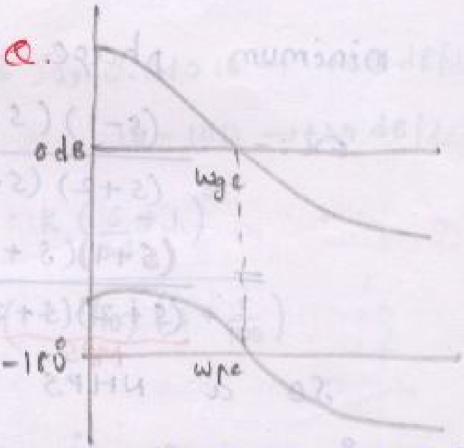
$\omega_{pc} > \omega_{gc} \longrightarrow$ stable

$GM = -20\log(M)_{\omega=\omega_{pc}}$

$GM = -(-30dB) = 30 dB$

$PM = 180 - 100 = 80$

Q.

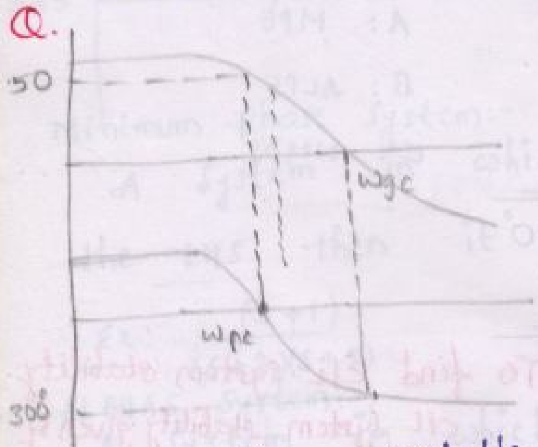


$\omega_{gc} = \omega_{pc} \longrightarrow$ m.s

$GM = -(0dB) = 0 dB$

$PM = 180 - 180 = 0$

Q.

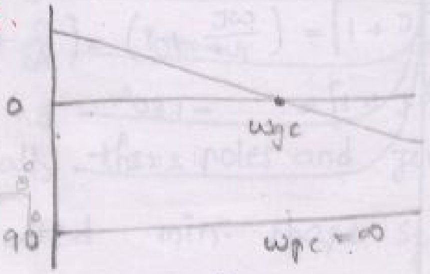


$\omega_{pc} < \omega_{gc} \longrightarrow$ unstable

$GM = -(+50) = -50 dB$

$PM = 180 - 300 = -120°$

Q.



$\omega_{pc} >>> \omega_{gc}$,

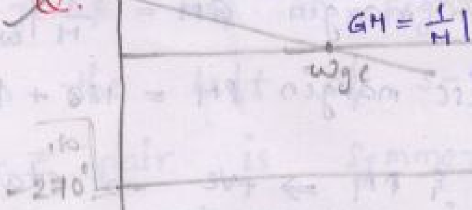
(?) Absolutely stable

Q.

$GH = 1/s$ $\quad M = \frac{1}{\omega}\Big|_{\omega=\omega_{pc}\infty} = 0$

$GM = \frac{1}{M}\Big|_{\omega=\omega_{pc}} = \infty$

$PM = 180 - 90 = 90°$



$\omega_{pc} << \omega_{gc} \rightarrow$ unstable

$\omega_{pc} = 0$

$PM = 180 - 270 = -90$

BODE PLOTS FOR COMPLEX P/Z's :-

complex poles

$$GH = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$$

$0 \le \xi \le 1$

$$= \frac{\omega_n^2}{(j\omega)^2+2\xi\omega_n j\omega+\omega_n^2}$$

$$= \frac{1}{-\frac{\omega^2}{\omega_n^2}+j2\xi\frac{\omega}{\omega_n}+1}$$

$$= \frac{1}{1-\frac{\omega^2}{\omega_n^2}+2j\xi\frac{\omega}{\omega_n}}$$

$$= \frac{1}{(1-\mu^2)+j2\xi\mu}$$

$$M = \frac{1}{\sqrt{(1-\mu^2)^2+(2\xi\mu)^2}}$$

M in dB Actual $= -20\log\sqrt{(1-\mu^2)^2+(2\xi\mu)^2}$

$$\phi_{act} = -\tan^{-1}\left(\frac{2\xi\mu}{1-\mu^2}\right)$$

Asymptotic,

case 1: $\mu<1$, neglect μ, $2\xi\mu$

$$M_{asy} = -20\log 1$$
$$= 0$$

$$\phi_{asy} = 0$$

case 2: $\mu>1$, neglect 1,

$$M_{asy} = -20\log\sqrt{\mu^4}$$
$$= -40\log\frac{\omega}{\omega_n}$$
$$= -40\log\omega + 40\log\omega_n$$

$$\text{slope} = \frac{dM}{d\log\omega} = -40 \text{ dB/dec}$$

complex zeros

$$\frac{s^2+2\xi\omega_n s+\omega_n^2}{\omega_n^2}$$

$$= \frac{-\omega^2+2\xi\omega_n j\omega+\omega_n^2}{\omega_n^2}$$

$$= -\frac{\omega^2}{\omega_n^2}+j2\xi\frac{\omega}{\omega_n}+1$$

$$= \left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]+j\left[2\xi\frac{\omega}{\omega_n}\right]$$

$$= (1-\mu^2)+(2\xi\mu)j \qquad \because \text{let } \mu = \frac{\omega}{\omega_n}$$

$$M = \sqrt{(1-\mu^2)^2+(2\xi\mu)^2}$$

M in dB Act. $= 20\log\sqrt{(1-\mu^2)^2+(2\xi\mu)^2}$

$$\phi_{act} = \tan^{-1}\left(\frac{2\xi\mu}{1-\mu^2}\right)$$

$$M_{asy} = 0 \text{ dB}$$

$$\phi_{asy} = 0$$

$$M_{asy} = +40\log\mu^{\omega/\omega_n}$$

$$\text{slope} = +40 \text{ dB/decay}$$

$$\phi_{asy} = -\tan^{-1}\left(\frac{\overset{\text{very small}}{2\xi\mu}}{\underset{\text{neglect}}{1}-\mu^2}\right)$$

$$= -\tan^{-1}(-0 \text{ very small})$$

$$= -(180 - \overset{0}{\tan^{-1}0})$$

$$= -180^\circ$$

$$\phi_{asy} = +180^\circ$$

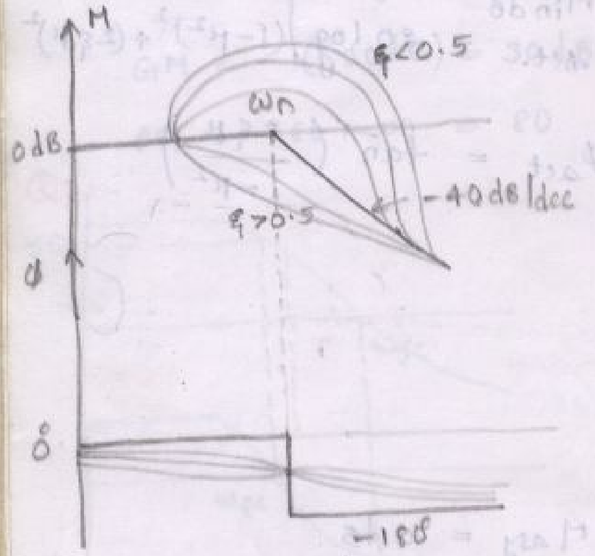
< cf	0	0
> cf	−40 dB/dec	−180°

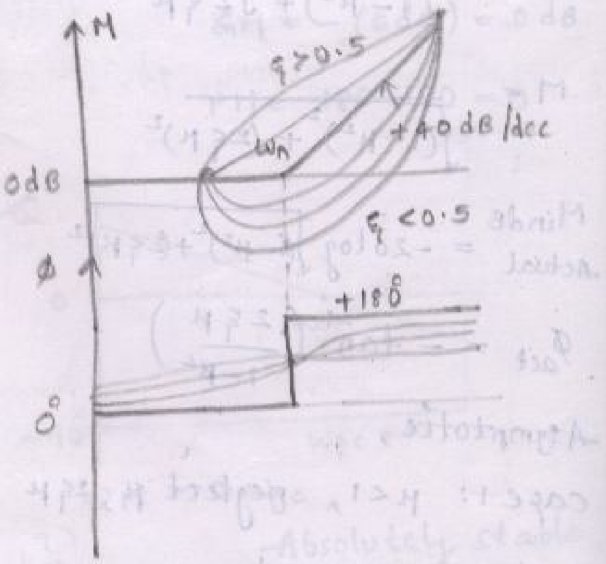
< cf	0	0
> cf	+40 dB/dec	+180

for n- complex poles

< cf	0	0
> cf	−40n dB/dec	−180°n

< cf	0	0
> cf	+40n dB/dec	+180°n





correction of cf :-

$$M_{act} = -20 \log \sqrt{(1-\mu^2)^2 + (2\xi\mu)^2}$$

$\omega = \omega_n$

$\Rightarrow \mu = 1 \quad M_{correction} = -20 \log 2\xi$

$\xi = 0.1, \quad M = -20 \log 0.2 =$

$\xi = 0.8, \quad M = -20 \log 1.6 =$

$$\phi = -\tan^{-1}\left(\frac{2\xi\mu}{1-\mu^2}\right)$$

$$= -90$$

for 'n' no. of,

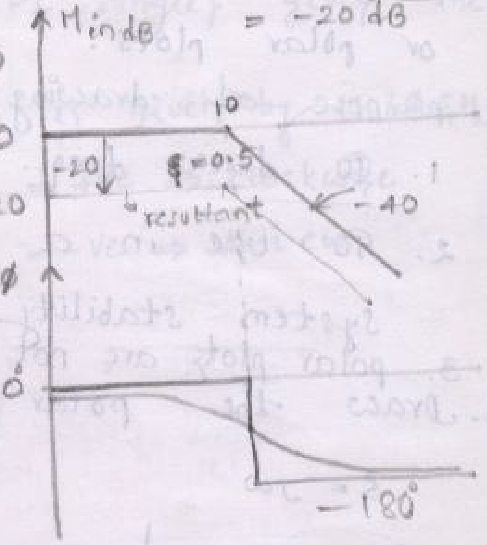
$M_{correction} = -20n \log 2\xi$ $M_{corr.} = 20n \log 2\xi$

Q. Draw the Bode plot for $\frac{10}{s^2+10s+100}$

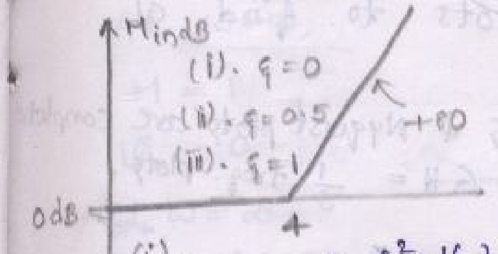
Shift $= 20 \log 0.1$ $= -20$ dB

$GH(s) = 0.1\left[\frac{100}{s^2+10s+100}\right]$

Refer once again $\omega_n = 10$



Q. $GH(s) = \frac{\quad}{\quad}$?



(i) $T/f : K\left[\frac{s^2+16}{4}\right]^2$

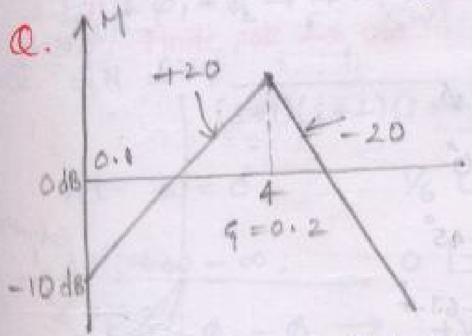
$0|_{\omega=4} = 20 \log k$

$\Rightarrow k = 1$

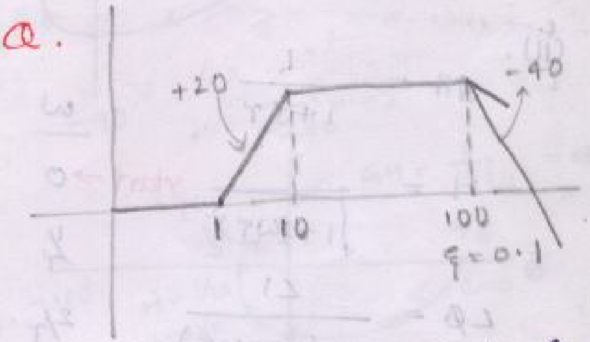
(ii). $T/f : \frac{(s^2+4s+16)^2}{16}$

(iii). $T/f : \frac{(s^2+8s+16)^2}{16}$

Q.



Q.



$-10|_{\omega=0.1} = \frac{KS\ 16}{s^2+16s+16}$

$-10 = 20 \log k + 20 \log 0.1$

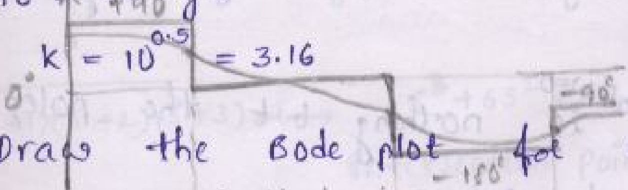
$10 = 20 \log k$

$\Rightarrow k = 10^{0.5} = 3.16$

$0|_{\omega=1} = \frac{K(1+s/1).10^4}{(1+s/10).(s^2+20s+10^4)}$

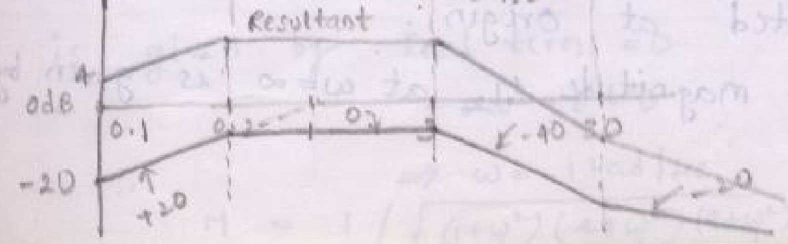
$0 = 20 \log k$

$\Rightarrow k = 1$



Q. Draw the Bode plot for $G(s)H(s) = \frac{16s(1+s/30)}{(1+s/0.2)(1+\frac{s}{3}+\frac{s^2}{9})}$

Shift $= 20 \log 16$ $= 24$ dB



Limitation of Bode plot :-

Drawing 2 plots to find the CL system stability

This can be avoided by drawing Nyquist plots. or polar plots.

purpose of drawing Polar plot :-

1. To draw freq. response of OL T/f ..
2. To use in a Nyquist plots to find CL system stability
3. polar plots are not complete plots & Nyquist plots are complete freq. plots.

Q. Draw the polar plot for $GH = \frac{1}{s}$

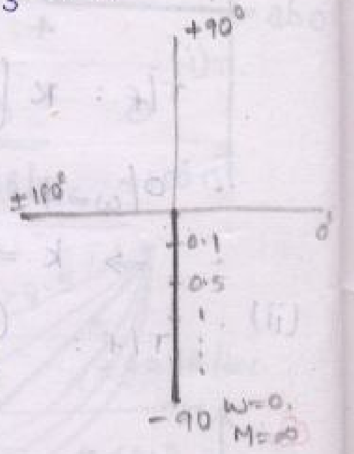
$s = j\omega$

$\Rightarrow GH = \frac{1}{j\omega}$

$M = \frac{1}{\omega}$

$\angle\phi = \frac{\angle 1}{\angle j\omega} = -90^\circ$

	ω	M	∠φ
start	0	∞	-90°
	1	1	-90°
	2	0.5	-90°
	10	0.1	⋮
	⋮	⋮	⋮
end	∞	0	-90°



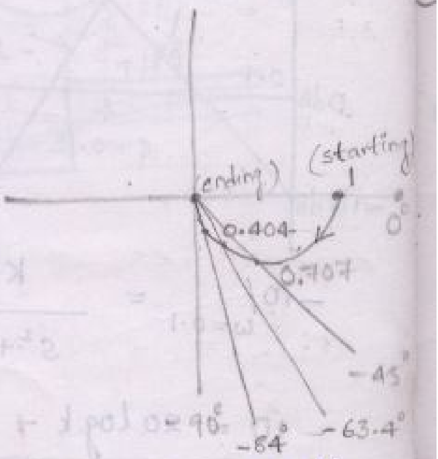
(ii). $GH = \frac{1}{1+sT}$

$M = \frac{1}{\sqrt{1+(\omega T)^2}}$

$\angle\phi = \frac{\angle 1}{\angle(1+j\omega T)}$

$= -\tan^{-1}(\omega T)$

	ω	M	∠φ
start →	0	1	0°
	$1/T$	0.707	-45°
	$2/T$	0.404	-63.4°
	$10/T$	0.1	-84°
	⋮	⋮	⋮
end →	∞	0	-90°



- ✳ At ω = 0, the magnitude M_1 is given by substituting $s=0$.
- ✳ The ph. angle ϕ_1 at ω=0 is nothing but the poles and zero's located at origin.
- ✳ The ending magnitude M_2 at $\omega=\infty$ is given by sub. $s=\infty$.

* for ending phase angle at $s=\infty$, consider the $-90°$ for each pole and $+90°$ for each zero. The algebraic sum of angles gives the ending angle.

* Direction is given by $M_1 \angle \phi_1$ & $M_2 \angle \phi_2$

$\phi_1 - \phi_2 = +ve \Rightarrow$ clockwise

$= -ve \Rightarrow$ Anti cw.

Q. $GH = \dfrac{1}{s+1}$

At $\omega = 0$,

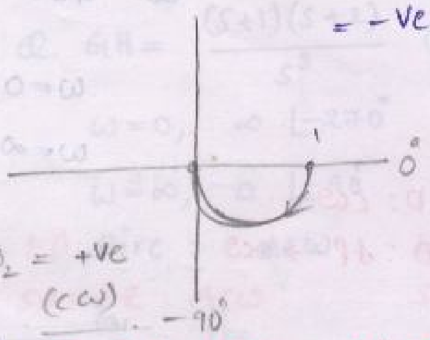
$M = 1$

$\angle\phi = 0°$

At $\omega = \infty$,

$0 \angle -90°$

$\phi_1 - \phi_2 = +ve$ (cw)



Q. $GH = \dfrac{1}{(s+1)(s+2)}$

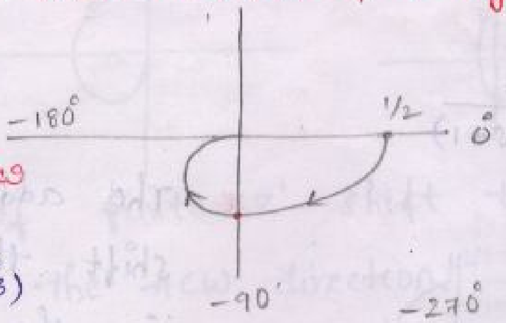
* Intersection point is nothing but magnitude

At $\omega = 0°$; $\frac{1}{2} \angle 0°$

At $\omega = \infty$, $0 \angle -180°$

ED: $\phi_1 - \phi_2 \rightarrow +ve \rightarrow$ cw (cw)

SD: finite pole $\rightarrow$ cw



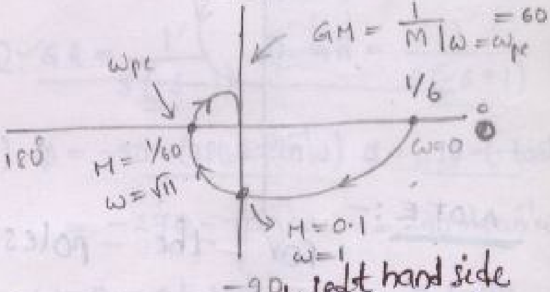
Q. $GH = \dfrac{1}{(s+1)(s+2)(s+3)}$

At $\omega = 0°$; $\frac{1}{6} \angle 0°$

At $\omega = \infty$; $0 \angle -270°$

ED: $\phi_1 - \phi_2 \rightarrow +ve \rightarrow$ cw (cw)

SD: finite pole $\longrightarrow$ cw.



* The addition of each finite pole in the left hand side shift the ending angle by $-90°$ in the cw direction.

$$\frac{1}{(s+1)(s+2)(s+3)} \longrightarrow s^3 + 6s^2 + 11s + 6$$

Intersection point with ima. axis is given by Real terms $= 0$

$-6\omega^2 + 6 = 0$

$\Rightarrow \omega = 1$ rad/sec

$M = 1 / \sqrt{(1+\omega_1^2)(4+\omega_1^2)(9+\omega_1^2)} = \frac{1}{\sqrt{2 \times 5 \times 10}} = \frac{1}{10}$.

Intersection point with Real axis ⇒ Ima. part = 0

⇒ $-j\omega^3 + 11j\omega = 0$ Intersection point with -ve real axis = ω_{pc}.

⇒ $\omega = \sqrt{11}$ rad/sec

$$M = \frac{1}{\sqrt{12\times15\times20}} = \frac{1}{60}$$

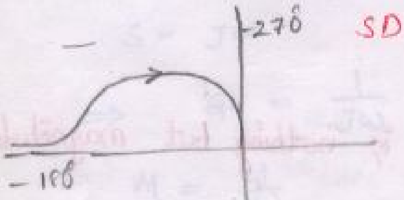
Q. $GH = \frac{1}{s^2(s+1)}$

$\omega = 0$; $\infty \angle -180^\circ$

$\omega = \infty$; $0 \angle -270^\circ$

$\overline{}$ cw   ED: ccw

SD: 4P → ccw



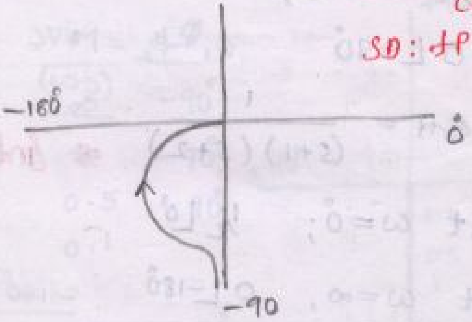
Q. $GH = \frac{1}{s(s+1)}$

$\omega = 0$; $\infty \angle -90^\circ$

$\omega = \infty$; $0 \angle -180^\circ$

cw   ED: +ve ccw

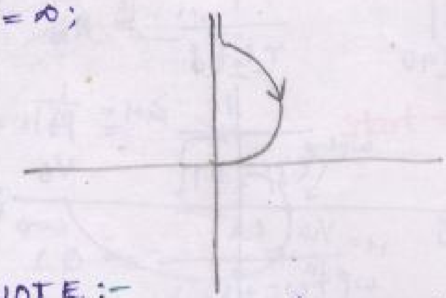
SD: 4P: ccw



Q. $GH = \frac{1}{s^3(s+1)}$

$\omega = 0$;

$\omega = \infty$;



* The addition of pole at origin shift the total plot by -90° in the cw direction.

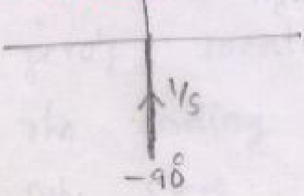
NOTE:- for the poles and z's at origin the polar plot is nothing but a angle lines.

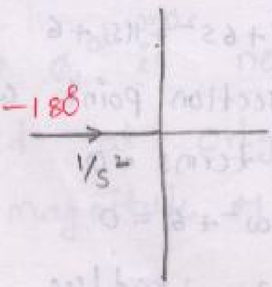
[T/F should not consists any finite p's and z's].

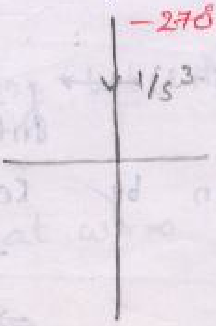
$GH = \frac{1}{s}, \frac{1}{s^2}, \frac{1}{s^3}, \frac{1}{s^4}$. and s, s^2, s^3, s^4.

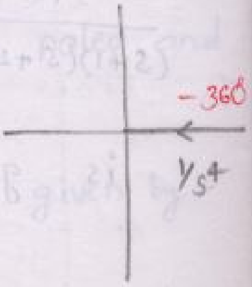
$\omega = 0, \rightarrow 0\angle -90^\circ$

$\omega = \infty \rightarrow \infty \angle -90^\circ$





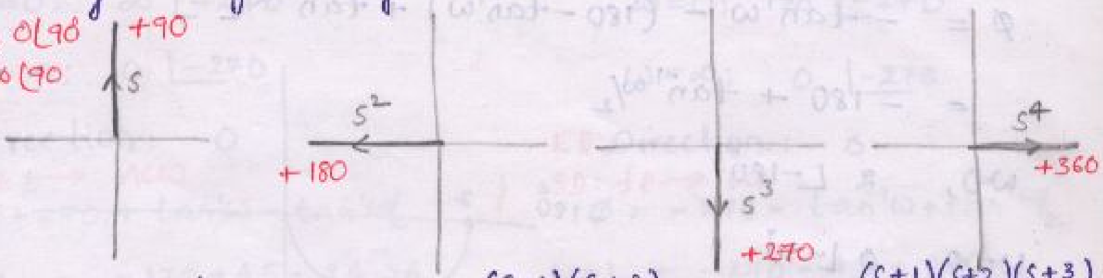




* The addition of z at origin shift the (total plot) ending angle by +90° in ACW direction.

$\omega=0,\ 0\angle 90^\circ$

$\omega=\infty,\ \infty\angle 90$



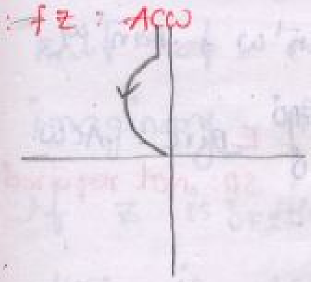
Q. $GH = \frac{s+1}{s^3}$

$\omega=0;\ \infty\angle -270^\circ$

$\omega=\infty,\ 0\angle -180^\circ$

ED: dire: ACW

SD: fz: ACW



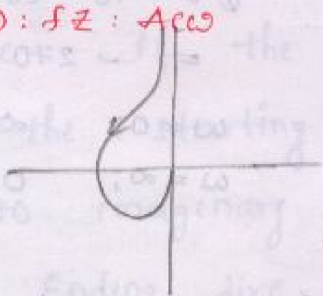
Q. $GH = \frac{(s+1)(s+2)}{s^3}$

$\omega=0,\ \infty\angle -270^\circ$

$\omega=\infty,\ 0\angle -90^\circ$

ED: Dire: ACW

SD: fz: ACW



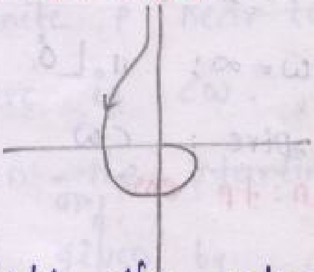
Q. $GH = \frac{(s+1)(s+2)(s+3)}{s^3}$

$\omega=0,\ \infty\angle -270^\circ$

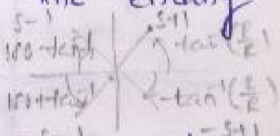
$\omega=\infty,\ 1\angle 0^\circ$

ED: Dire: ACW

SD: fz: ACW



* The addition of finite z' shift the ending angle by 90° in the ACW direction.



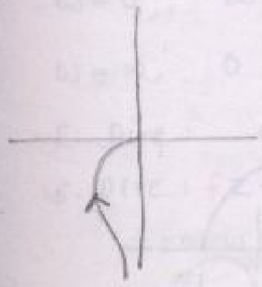
Q. $GH = \frac{1}{s(s+1)}$

$\phi = -90-\tan^{-1}\omega$

$\omega=0,\ \infty\angle -90^\circ$

$\omega=\infty,\ 0\angle -180^\circ$

E.Dire: CW



Q. $GH = \frac{1}{s(s-1)}$

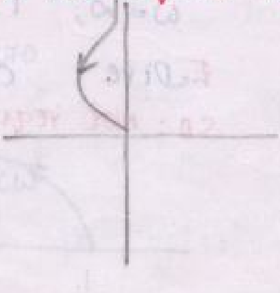
$\phi = -90-(180-\tan^{-1}\omega)$

$= -270+\tan^{-1}\omega$

$\omega=0,\ \infty\angle -270^\circ$

$\omega=\infty,\ 0\angle -180^\circ$

E.Dire: ACW



Q. $GH = \frac{1}{s(-s-1)}$

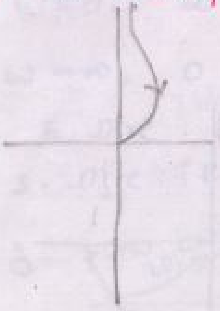
$\phi = -90-(180+\tan^{-1}\omega)$

$= -270-\tan^{-1}\omega$

$\omega=0,\ \infty\angle -270^\circ$

$\omega=\infty,\ 0\angle -360^\circ$

E.Dire: CW



Q. $GH = \frac{1}{s(-s+1)}$

$\phi = -90-(-\tan$

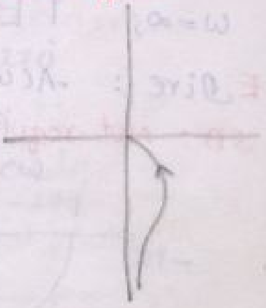
$= -90+\tan^{-1}\omega$

$\omega=0,\ \infty\angle -90$

$\omega=\infty,\ 0\angle 0^\circ$

E.Dire: ACW

(SD: Not required b'coz TF consits -ve sign)



Q. $GH = \frac{(s+2)}{(s+1)(s-1)}$

$\phi = -\tan^{-1}\omega - (180 - \tan^{-1}\omega) + \tan^{-1}\omega/2$

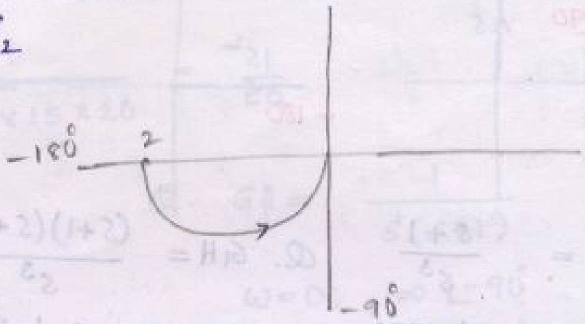
$= -180 + \tan^{-1}\omega/2$

$\omega = 0, \quad 2 \angle -180°$

$\omega = \infty, \quad 0 \angle -90°$

E. Dire : ACW

SD : Not required



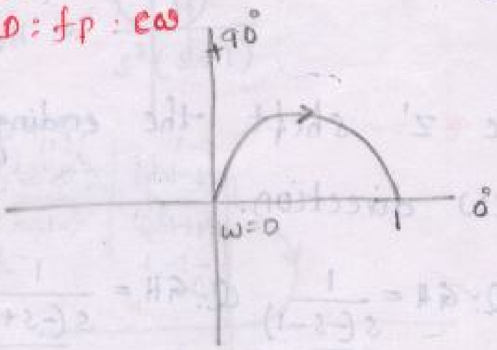
Q. $GH = \frac{s}{s+1}$

$\omega = 0; \quad 0 \angle +90°$

$\omega = \infty; \quad 1 \angle 0°$

E. Dire : CW

SD : fp : CW



Q. $GH = \frac{s+3}{s(s-1)}$

$\phi = -90 - (180 - \tan^{-1}\omega) + \tan^{-1}\omega/3$

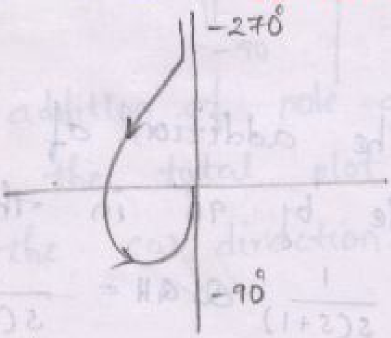
$= -270 + \tan^{-1}\omega + \tan^{-1}\omega/3$

$\omega = 0; \quad \infty \angle -270°$

$\omega = \infty; \quad 0 \angle -90°$

E.Dire : ACW

SD: Not required



Q. $GH = \frac{s+2}{s-2}$

$\phi = \tan^{-1}\omega/2 - (180 - \tan^{-1}\omega/2)$

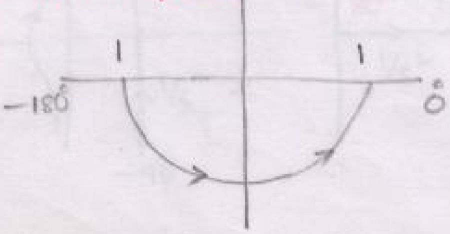
$= -180 + 2\tan^{-1}\omega/2$

$\omega = 0, \quad 1 \angle -180°$

$\omega = \infty, \quad 1 \angle 0°$

E.Dire : ACW

SD : Not required



Q. $GH = \frac{s-2}{s+2}$

$\phi = -\tan^{-1}\omega/2 + (180 - \tan^{-1}\frac{\omega}{2})$

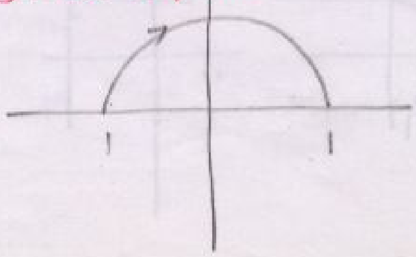
$= 180° - 2\tan^{-1}\omega/2$

$\omega = 0; \quad 1 \angle 180°$

$\omega = \infty, \quad 1 \angle 0°$

E.Dire : CW

SD : Not required.



Q. $GH = \frac{s+1}{s^3(s+2)}$

$\omega = 0; \quad \infty \angle -270$

$\omega = \infty; \quad 0 \angle -270$

ED: Direction : 0

SD: fZ ⟶ ACW

$\phi = -270 + \tan^{-1}\omega - \tan^{-1}\omega/2$

$\omega = 1, \quad = -270 + 45 - 26.56$

$\Longrightarrow \quad > -270$

Q. $GH = \frac{s+2}{s^3(s+1)}$

$\omega = 0; \quad \infty \angle -270$

$\omega = \infty; \quad 0 \angle -270$

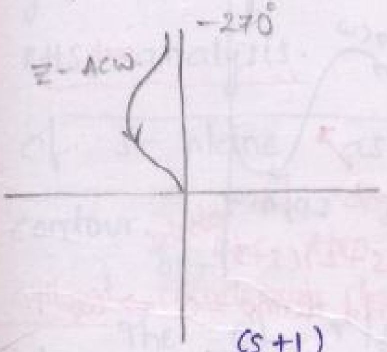
ED: Direction : 0

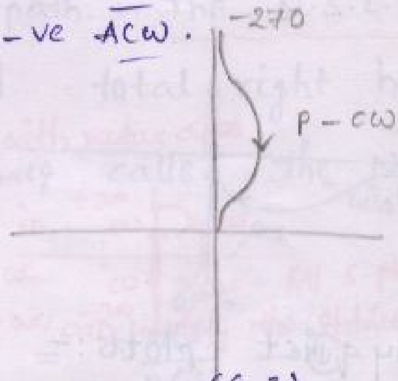
SD: fP ⟶ CW

$\phi = -270 - \tan^{-1}\omega + \tan^{-1}\omega/2$

$\omega = 1, \quad = -270 - 45 + 26.56$

→ If the T/f consists the finite P and Z's are all in the Left half of s-plane then the starting dire. is given by finite P's and Z's which are left half of s-plane. If the finite P near to imaginary then the starting dire is CW. If Z is near to imaginary then the starting dire is ACW. Ending dire. is given by angle direction → $(\phi_1 - \phi_2)$, +ve CW
-ve ACW.





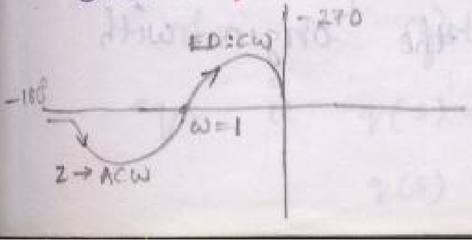
Q. $GH = \frac{(s+1)}{s^2(s+2)(s+3)}$

$\omega = 0, \quad \infty \angle -180°$

$\omega = \infty, \quad 0 \angle -270$

E. Dire : CW

S. Dire : fZ → ACW



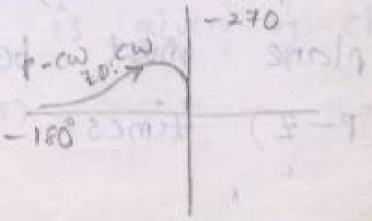
Q. $GH = \frac{(s+3)}{s^2(s+1)(s+2)}$

$\omega = 0, \quad \infty \angle -180°$

$\omega = \infty, \quad 0 \angle 270°$

E. Dire : CW

S. Dire : fP → CW



$$\frac{(s+1)}{s^2(s+2)(s+3)} \xrightarrow[\text{numerator}]{\text{bring to}} -s^2(2-s)(3-s)(s+1)$$

$$= -s^2(6-5s+s^2)(s+1)$$

$$= -s^5-4s^4-s^3-6s^2$$

$$\frac{1}{s-2} = \frac{1}{-(2-s)} = -1(2+s)$$

Intersection ima point with real axis:

(odd terms=0) $s^5 - s^3 = 0$

$s \to j\omega$ $\quad -j\omega^5 + j\omega^3 = 0$

$\omega = 1$

Verification

$\phi = -180 - \tan^{-1}\omega/3 - \tan^{-1}\omega/2 + \tan^{-1}\omega$

$\phi = -180°$

$$\frac{(s+3)}{s^2(s+1)(s+2)} \longrightarrow -s^2(1-s)(2-s)(s+3)$$

$$= -s^5+7s^3-6s^2$$

ima. $-s^5+7s^3=0$

(odd power terms=0)

$s \to j\omega$ $\quad \omega = \pm j\sqrt{7}$ (invalid point).

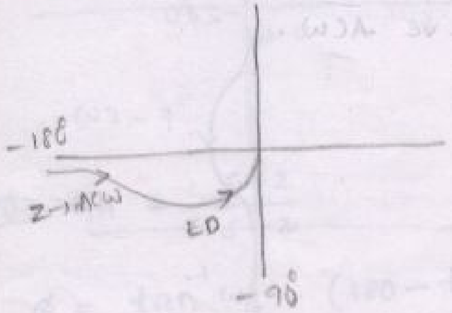
Q. $GH = \dfrac{(s+1)(s+2)}{s^2(s+3)}$

$\omega=0; \quad \infty \angle -180°$

$\omega=\infty, \quad 0 \angle -90°$

E.Dire : ACW

SD: fZ → ACW



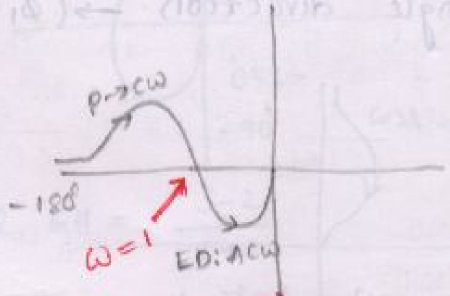
Q. $GH = \dfrac{(s+2)(s+3)}{s^2(s+1)}$

$\omega=0, \quad \infty \angle -180°$

$\omega=\infty, \quad 0 \angle -90°$ ✓

E.Dire : ACW

SD: fP → CW



$-s^2(1-s)(s+2)(s+3)$

→ Making odd terms = 0 ⇒ ω=1.

Nyquist plots:-

Nyquist stability criteria depends on the principle of arguments:-

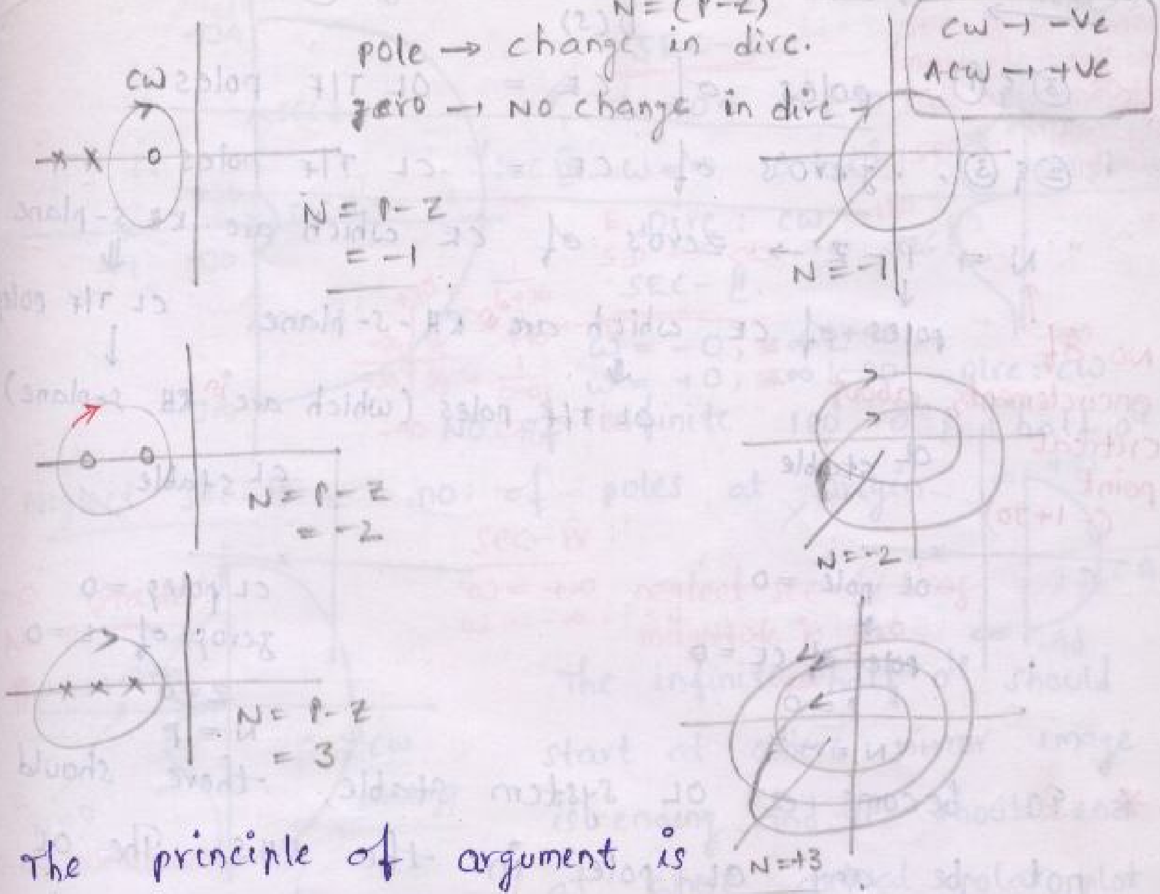
principle of arguments states that if there are P poles, Z zeros are enclosed by the s-plane ~~at~~ closed path, Then the corr. G(s) H(s) plane must be encircles the origen with (P−Z) times.

s - plane G(S)H(S) - plane

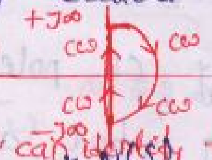
$N = (P-Z)$

pole → change in dirc.

zero → No change in dirc.



The principle of argument is applied to the total right half of s-plane by selecting as a closed path. The N.S.C. is RHS plane analysis. They selected total right half of s-plane as a closed path with radius of ∞ is called the Nyquist contour. selected If any pole is enclosed in N.C. then in G(s)H(s) plane, we will get encirclements. based on encirclements we can identify the stability.



p-z configuration:

The OL T/f $G(S)H(S) = K\frac{N(S)}{D(S)}$ → ①

CL T/f $G(S)/(H(S)G(S)+1) = \frac{C(S)}{R(S)}$

$$= \frac{G(S)}{1 + K\frac{N(S)}{D(S)}}$$

$$\frac{C(S)}{R(S)} = \frac{G(S).D(S)}{D(S) + K.N(S)} \rightarrow ②$$

The CL system stability is given by char. eq. ie $q(S) = 1 + G(S).H(S)$

$$q(S) = 1 + K.\frac{N(S)}{D(S)}$$

$$CE \longrightarrow q(s) = \frac{D(s)+K\,N(s)}{D(s)} \longrightarrow ③$$

compare ③ & ①, poles of CE = OL T/F poles

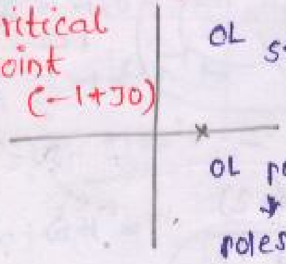
② & ③, zero's of CE = CL T/F poles

$N = P - Z \rightarrow$ zero's of CE which are RH s-plane ⇓ CL T/F poles

↑ No. of encirclements about critical point (−1+J0)

↓ poles of CE which are in RH-s-plane ⇓ OL T/F poles (which are in RH s-plane)

OL stable ×

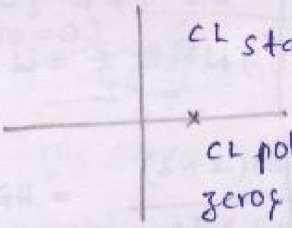


OL pole = 0 ⇓ poles of CE = 0

P = 0

N = −Z

CL stable ✓



CL poles = 0

zeros of CE = 0

Z = 0

N = P

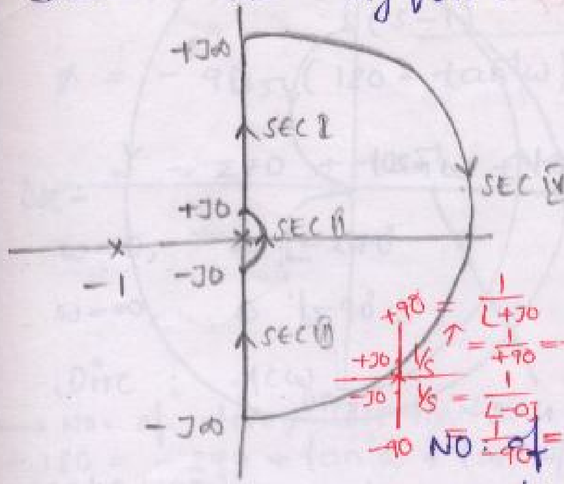
* To become a OL system stable, there should not be any OL pole in the RHS. The OL poles are nothing but poles of char. eq which must be zero on RHS. ie P must be '0' and N = −Z.

* To become a CL system stable, there should not be any CL pole in the RHS. The CL pole is nothing but a zero's of CE in the RHS which must be '0' ie Z = 0, N = P.

Nyquist stability Criteria:-

It states that the no. of encirclements about the critical point (−1+J0) in the G(s)H(s) plane must be = to no. of p's of CE : [OL T/F p's which are in the RH - s- plane]. ie N = P

Q. Draw the Nyquist plot for $G(s).H(s) = \frac{1}{s(s+1)}$



SEC-I

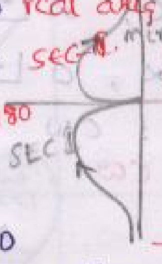
$\omega = 0+$ $\infty \angle -90$

$\omega = \infty$ $0 \angle -180$

E. Dire : cw

S.D → cw

SEC-III is mirror image of sec-I wrt real axis and the dire. of mirror image is continuous.



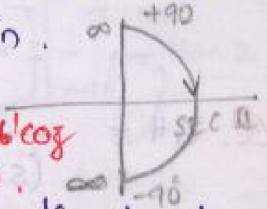
SEC-II.

$\omega = -0$, $\infty \angle +90$

$\omega = +0$, $\infty \angle -90$ Dire : cw

NO. of infinite 180° o^le^s [half o^le]

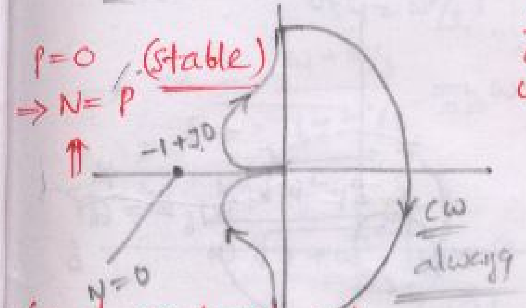
Neglect sec-IV = no. of poles at origin.



SEC-IV :

$\omega = +\infty$; neglect sec-IV b'coz

$\omega = -\infty$; magnitude is zero.



P=0 (stable)

⇒ N = P

(No. of encirclements = 0)

The infinite half o^le should start at where mirror image is ending and it should be end at where actual polar plot is started.

* The dire. of infinite half o^le is always cw. irrespective of location of p's and z's.

$\frac{1}{s(s+1)} \longrightarrow P = 0$ N = P, CL stable.

Q $GH = \frac{10}{s+5}$

$\omega = 0$, $2 \angle 0°$

$\omega = \infty$, $0 \angle -90$

Dire : cw

Q. $GH = \frac{10}{(s+1)(s+2)}$

$\omega = 0$, $5 \angle 0°$

$\omega = \infty$, $0 \angle -180$

Direc : cw

[No poles & zeros at origin so we don't get infinite rad. half o^le]

Q. $GH = \frac{10}{s^2 (s+1)(s+2)}$ ← 2 half o'les.

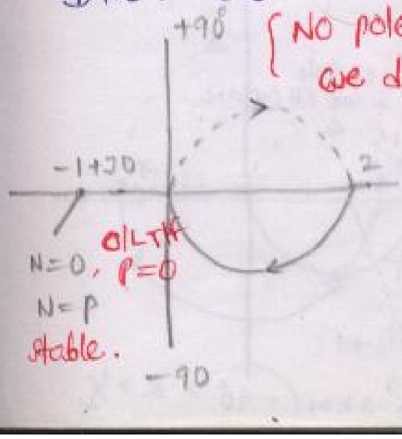
$\omega = 0$, $\infty \angle -180$

$\omega = \infty$, $0 \angle -360$

Dire : cw

N = P - Z ⇒ -2 = 0 - Z

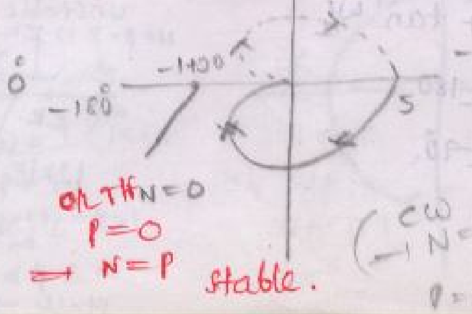
⇒ Z = 2



N=0, OLTF P=0

N=P

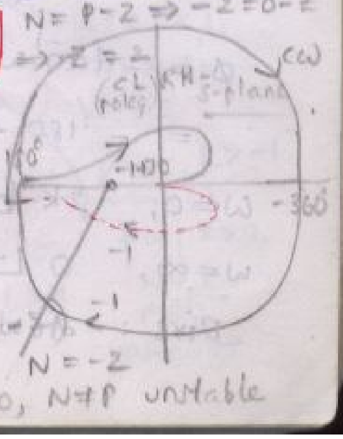
stable.



OLTF N=0

P=0

⇒ N=P stable.



(cw -1 N = -1)

N = -2

P=0, N≠P unstable

Q. $GH = \dfrac{1}{s^3(s+1)}$ 3 half O'leg.

$\longrightarrow P = 0$

$\omega = 0, \quad \infty \angle -270^\circ$

$\omega = \infty, \quad 0 \angle -360^\circ$

Dire : CW

SD → CW

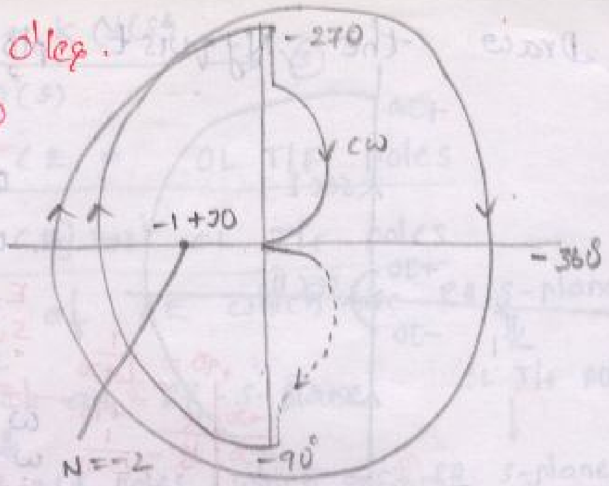
$P = 0$

$N \neq P$

unstable

$N = P - Z$

$-2 = 0 - Z \Rightarrow Z = 2$ CL poles on RH-splane.



Q. $GH = \dfrac{K}{(s+1)(s+2)(s+3)}$

$\omega = 0, \quad \dfrac{K}{6} \angle 0^\circ$

$\omega = \infty, \quad 0 \angle -270^\circ$

Dire : CW

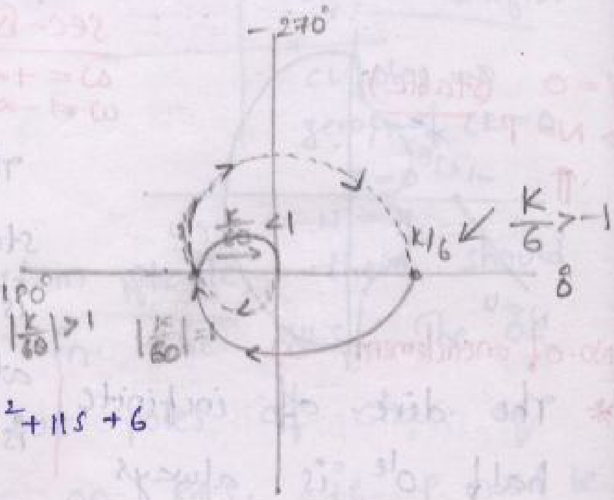
$(s+1)(s+2)(s+3) = s^3 + 6s^2 + 11s + 6$

$s^3 + 11s = 0$

$\omega_{pc} = \sqrt{11}$

$M = \dfrac{K}{\sqrt{(1+\omega^2)(4+\omega^2)(9+\omega^2)}\Big|_{\omega=\sqrt{11}}} = 1$

$= \dfrac{K}{60}$



For stable,

$N = 0$

$P = 0$

$N = P$ – stable

$\dfrac{K}{60} < 1 \Rightarrow K < 60$

and $\dfrac{K}{6} > -1 \Rightarrow K > -6$

$-6 < K < 60$

Q. Find the range of K-value for $GH = \dfrac{K(s+2)}{(s+1)(s-1)}$ for system stability. $\longrightarrow P = 1$

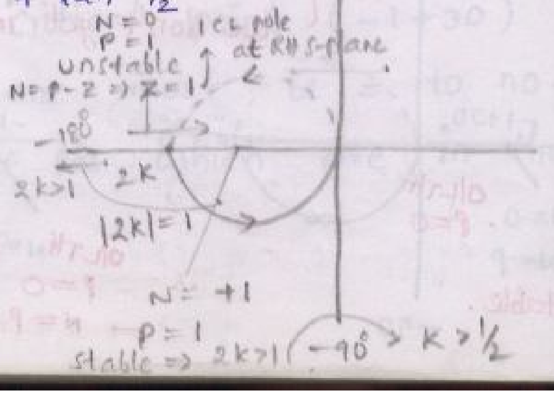
$\phi = -\tan^{-1}\omega - (180 - \tan^{-1}\omega) + \tan^{-1}\omega/2$

$= -(180 + \tan^{-1}\omega/2$

$\omega = 0, \quad 2K \angle -180$

$\omega = \infty, \quad 0 \angle -90^\circ$

Dire : ACW



N = 0
P = 1
unstable
N = P - Z ⇒ Z = 1
1 CL pole at RH s-plane

N = +1
P = 1
stable ⇒ 2K > 1 ⇒ K > 1/2

Q. $G(s).H(s) = \frac{k(s+3)}{s(s-1)} \rightarrow P = 1$

$\phi = -90-(180-\tan^{-1}\omega)+\tan^{-1}\omega/_3$

$= -270 + \tan^{-1}\omega + \tan^{-1}\omega/_3$

$\omega=0, \quad \infty \angle{-270^\circ}$

$\omega=\infty, \quad 0 \angle{-90^\circ}$

Dire : ACW

→ No. of terms less than 2.

$-180 = -270 + \tan^{-1}\omega + \tan^{-1}\omega/_3$

$90 = \tan^{-1}\left[\frac{\omega+\omega/3}{1-\omega^2/3}\right]$

$\Rightarrow \infty = \frac{\omega+\omega/3}{1-\omega^2/3} \Rightarrow \omega = \sqrt{3}$.

$M = \frac{k\sqrt{\omega^2+9}}{\omega\sqrt{1+\omega^2}}\Big|_{\omega=\sqrt{3}}$

$M = k$

for $k>1$,

$N = +1 \quad (+2-1)$

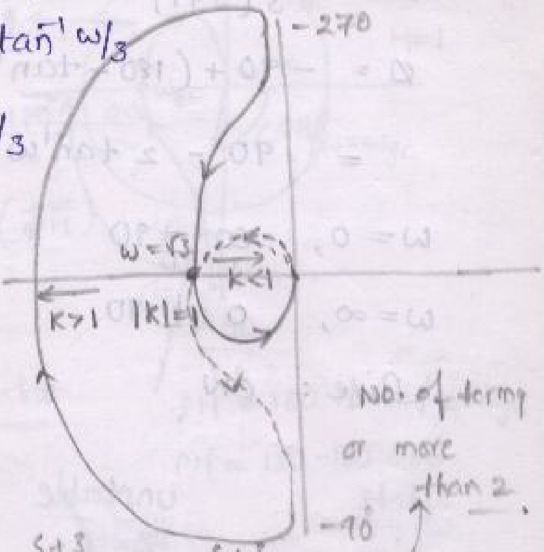
$P = 1$

stable

for $k<1$

$N = -1$

$P = 1$



No. of terms or more than 2.

$\frac{s+3}{s(s-1)} = \frac{s+3}{-s(1-s)}$

$= +s(s+3)(1+s) = 0$

s^3+4s^2+3s

$s \rightarrow j\omega, \quad -j\omega^3+3j\omega = 0$

$\Rightarrow \omega = \sqrt{3}$

$N = P - Z \Rightarrow -1 = 1 - Z \Rightarrow Z = 2$

2 CL p' on RH s-plane

↑ unstable.

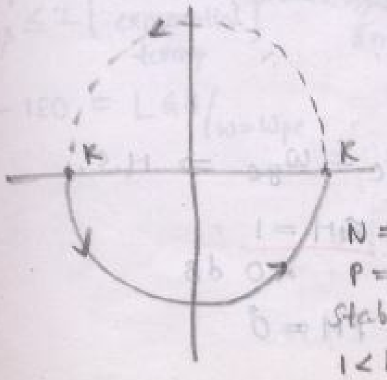
Q. $GH = \frac{k(s+5)}{s-5} \rightarrow P=1$

$\phi = -180 + 2\tan^{-1}\omega/_5$

$\omega=0, \quad k \angle{-180}$

$\omega=\infty, \quad k \angle{0^\circ}$

Direction: ACW



N = 1

P = 1

stable k>1

$1 < k \leq \infty$

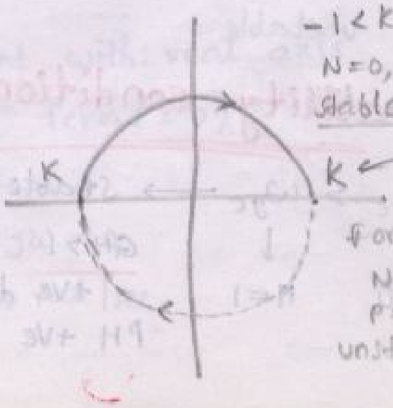
Q. $GH = \frac{k(s-2)}{s+2} \rightarrow P=0$

$\phi = +180 - 2\tan^{-1}\omega/_2$

$\omega=0, \quad k \angle{180^\circ}$

$\omega=\infty, \quad k \angle{0^\circ}$

Direction: CW



$-1<k<1$,

N=0, P=0

stable

← k>−1

for, k>0,

N≠P

P≠0

unstable

Q. $GH = \frac{s-1}{s(s+1)} \longrightarrow P=0$

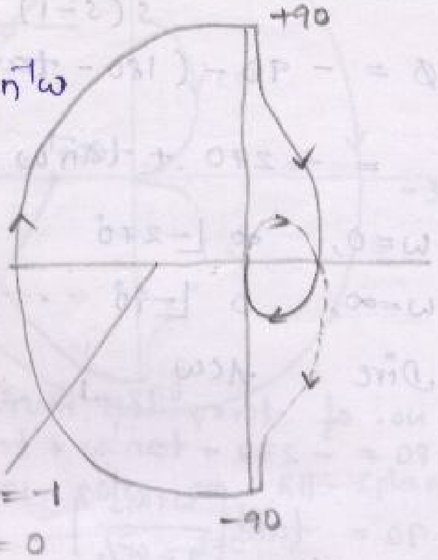
$\phi = -90 + (180 - \tan^{-1}\omega) - \tan^{-1}\omega$

$= 90 - 2\tan^{-1}\omega$

$\omega = 0, \quad \infty \angle +90$

$\omega = \infty, \quad 0 \angle -90$

Dire : CW



N = -1

P = 0

unstable ← Z = 1

Q. $GH = \frac{1}{s(s-1)} \longrightarrow P=1$

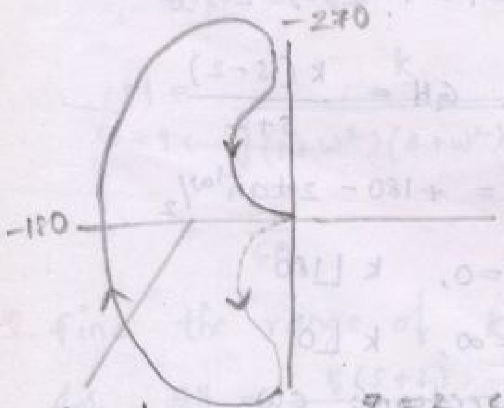
$\phi = -90 - (180 - \tan^{-1}\omega)$

$= -270 + \tan^{-1}\omega$

$\omega = 0, \quad \infty \angle -270$

$\omega = \infty, \quad 0 \angle -180$

Dire : ACW



N = -1

P = 1 unstable

Z = 2 CL RH

Q. $GH = \frac{1}{s(-s+1)} \longrightarrow P=1$

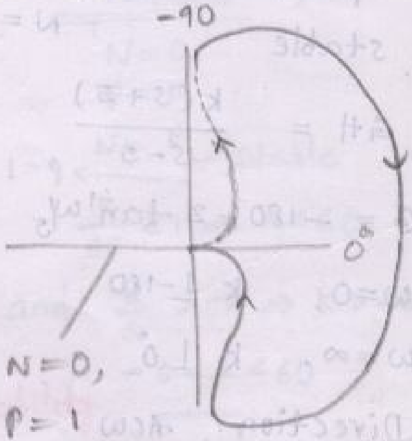
$\phi = -90 - (-\tan^{-1}\omega)$

$= -90 + \tan^{-1}\omega$

$\omega = 0, \quad \infty \angle -90$

$\omega = \infty, \quad 0 \angle 0$

Dire : ACW



N = 0,

P = 1

unstable, Z = 1 CL RH.

⇒ Stability conditions :

$\omega_{pc} > \omega_{gc} \longrightarrow$ stable

$\omega_{pc} \downarrow -180$

$\omega_{gc} \downarrow M = 1$

GH > 1

+ve dB

PM +ve

$\omega_{pc} = \omega_{gc} \Rightarrow$ M.S.

GH = 1

= 0 dB

PM = 0°

$\omega_{pc} < \omega_{gc} \Rightarrow$ unstable

Q..

GM <1

-ve dB

PM -ve

$\omega_{pc} > \omega_{gc}$

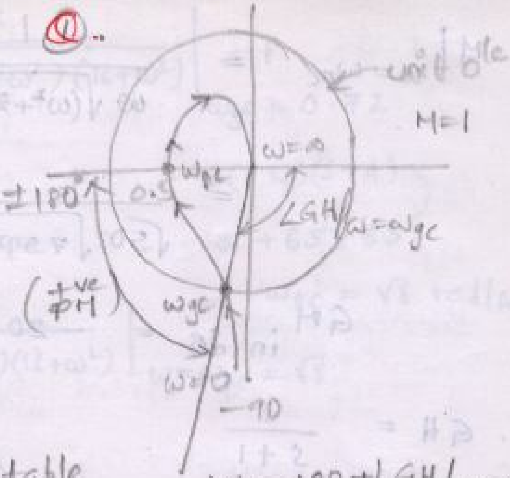
stable

$GM = \frac{1}{M/_{\omega=\omega_{pc}}}$

$= \frac{1}{<1/_{\omega=\omega_{pc}}} > 1$

stable

$= \frac{1}{0.5} = 2$

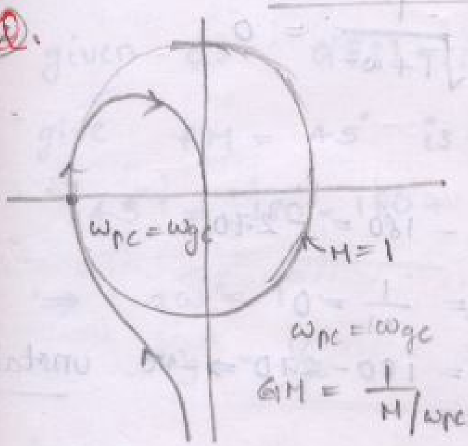


$PM = 180 + \angle GH/_{\omega=\omega_{gc}}$

$PM = 180 - 100 = 80°$

stable

Q.



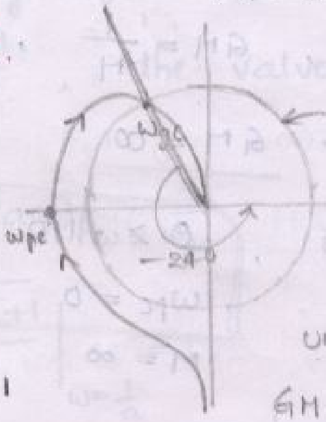
$\omega_{pc} = \omega_{gc}$

$GM = \frac{1}{M/_{\omega_{pc}}} = \frac{1}{1} = 1$

$PM = 180 - 180 = 0°$

M.S.

Q.



$PM = 180 - 240$

$= -60$

-ve.

$\omega_{pc} < \omega_{gc}$

unstable.

$GM = \frac{1}{M/_{\omega=\omega_{pc}}}$

$= \frac{1}{10} = 0.1 < 1$

whenever plot intersect ±180°, with a mag. of <1 (GM>1) then the system is stable, if M=1, then m.s, if mag. (M) >1 (GM<1) unstable.

Q. find the gain margin for $GH = \frac{1}{s(s+5)(s+10)}$

ω_{pc}

no. of terms ≤ 2 [T/f consists exponential terms]

$-180 = \angle GH/_{\omega=\omega_{pc}}$

≥ 2

Intersection point with real axis

(imaginary terms = 0)

$s^3 + 15s^2 + 50s$

$-j\omega^3 + 50j\omega = 0$

$\omega_{pc} = \sqrt{50}$ rad/sec

$$M|_{\omega=\omega_{pc}} = \frac{1}{\omega\sqrt{(\omega^2+25)(100+\omega^2)}}$$

$$= \frac{1}{\sqrt{50}\sqrt{75\times 150}} \Rightarrow GM = \frac{1}{M|_{\omega=\omega_{pc}}} = 750$$

$$GM_{\text{in dB}} = -20\log\frac{1}{\sqrt{50\times 75\times 150}} = \underline{20\log 750\ (+ve)}$$

Q. $GH = \frac{1}{s+1}$

$-180 = -\tan^{-1}\omega$

$\underline{\omega_{pc} = \infty}$, $GM = \frac{1}{M}$, $M = \frac{1}{\sqrt{1+\omega^2}} = 0$

$\Rightarrow \underline{GM = \infty}$

Q. $GH = \frac{1}{s^3}$, $\boxed{\begin{matrix}0 > -180 \\ \omega_{pc} = 0\end{matrix}}$, $\underline{-180 = -270}$

$M = \infty$

$GM = \frac{1}{M} = 0$

$\underline{PM = 180-270 \Rightarrow -ve}$ $\underline{\text{unstable}}$

Q. $GH = \frac{1}{s(s+1)}$

$\omega_{gc} \longrightarrow$ using magnitude condition

$|GH|_{\omega=\omega_{gc}} = 1$ $\left|\frac{1}{\omega\sqrt{1+\omega^2}}\right|_{\omega=\omega_{gc}} = 1$

$PM = 180 + \angle GH|_{\omega=\omega_{gc}}$ $\Rightarrow \omega = 0.78$ rad/sec.

$PM = 180-90-\tan^{-1}\omega|_{\omega=\omega_{gc}=0.78}$

$= 52^\circ$.

Q. The OL T/F of a system is $GH = \frac{k}{s(s+2)(s+4)}$.
Determine the value of k, (i) so that $PM = 60^\circ$,
(ii). so that $GM = +20$ dB

$PM = 60 = 180-90-\tan^{-1}\omega/2 - \tan^{-1}\omega/4$

$\Rightarrow 30 = \tan^{-1}\left[\frac{\omega/2+\omega/4}{1-\omega^2/8}\right]$

$\Rightarrow \omega = \omega_{gc} = 0.72$ rad/sec

Magnitude condi. $\left| \frac{k}{\omega\sqrt{(4+\omega^2)(16+\omega^2)}} \right|_{\omega_{gc}=0.72} = 1$

$\Rightarrow \quad k = 6.2$

$s(s+2)(s+4) \rightarrow s^3+6s^2+8s$

$GM = -20 \log M/_{\omega=\omega_{pc}}$

$\Rightarrow \omega_{pc} = \sqrt{8}$ rad/sec

$$20 = -20 \log \left. \frac{k}{\omega\sqrt{(4+\omega^2)(16+\omega^2)}} \right|_{\omega=\omega_{pc}=\sqrt{8}}$$

$\Rightarrow \quad k = 4.8.$

Q. The OL T/F of a unity f/b control system is given as $G(s) = \frac{as+1}{s^2}$, the value of 'a' to give PM = 45° is —

$$45 = 180 - 180 + \tan^{-1}(a\omega)/_{\omega=\omega_{gc}}$$

$\Rightarrow a\omega = 1$

$\omega_{gc} = 1/a$

$$\left. \frac{\sqrt{(a\omega)^2+1}}{\omega^2} \right|_{\omega=\frac{1}{a}} = 1$$

$$\Rightarrow a^2 = \frac{1}{\sqrt{2}}$$

Q. In the GH plane, the Nyquist plot of T/F $GH = \frac{\pi e^{-0.25s}}{s}$ passes through the -ve real axis, at a point — ?

1). (-0.25, J0)

2). (-0.5, J0)

3). (-1, J0)

4). (-2, J0)

$$-180 = -90 - 0.25\omega \times \frac{180}{\pi} \Big|_{\omega=\omega_{pc}}$$

$\Rightarrow \omega_{pc} = 2\pi$ rad/sec

exp, m=1

$M = \frac{\pi}{\omega} \Big|_{\omega_{pc}=2\pi} = 0.5 \quad (-0.5, J0)$

$e^{j\theta} = \cos\theta + j\sin\theta$

exponential terms never effect magnitude

$= \sqrt{\cos^2\theta + \sin^2\theta}$

but effects phase angle

* whenever the T/F not gives the mag. of '1' at any freq. then consider $\omega_{gc} = 0$.

State Space Analysis :

15 - 06 - 07 .

State gives the future behaviour of the system based on past history and present i/p of the system. * The initial (past) state of system is described by state variable.

~~Limitations~~

→ NO. of state variables:- If electrical n/w given, the no. of state variables = sum of the inductors & conductors

if a differential eq. given, the no. of state var:s = order of the differential eq.

Limitations of T/F Analysis :-

(1). The T/F analysis is valid only for LTI systems, where as SSA is valid for dynamic [linear, non- linear, time varient, time invarient] systems.

(2). The T/F analysis cannot give any idea about controllability and observability.

(3). T/F Analysis is more suitable for SISO systems. where as SSA suitable for MIMO.

Standard form of D. state model :-

state eq. (or) dynamic eq.

$$\underset{\substack{\downarrow\\ \text{Differential}\\ \text{state vector}}}{\dot{x}} = \underset{\substack{\downarrow\\ \text{state}\\ \text{Matrix}}}{A}x + \underset{\substack{\downarrow\\ \text{i/p}\\ \text{matric}}}{B}U$$

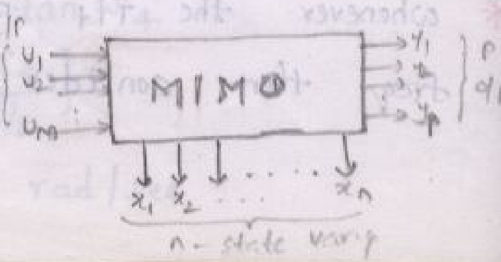
(o/p eq.)

$$\underset{\substack{\downarrow\\ \text{o/p}\\ \text{vector}}}{y} = \underset{\substack{\downarrow\\ \text{o/p}\\ \text{Matrix}}}{C}\overset{\substack{\text{state vector}\\ \uparrow}}{x} + \underset{\substack{\downarrow\\ \text{Transmission Matrix}}}{D}\overset{\substack{\text{i/p vector}\\ \uparrow}}{U}$$

NOTE : D is always zero, if the circuit not present the active elements.

Order of Matrices :-

Consider the MIMO system,



state vector $= \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}_{n\times 1}$ o/p vector $= \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_p \end{bmatrix}_{p\times 1}$ i/p vector $= \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_m \end{bmatrix}_{m\times 1}$

$$* \quad \underset{n\times 1}{\dot{x}} = \overset{n\times n}{A}\underset{n\times 1}{x} + \overset{n\times m}{B}\underset{m\times 1}{U} \qquad \underset{p\times 1}{y} = \overset{p\times n}{C}\underset{n\times 1}{X} + \overset{p\times m}{D}\underset{m\times 1}{U}$$

Q. find the order of Matrices:-

(1). $\frac{d^2y}{dt^2} + 10\frac{dy}{dt} + 5y = 10\,u(t)$

$n = 2$. i/p $\overset{(m)}{=} 1$, o/p $= 1$ (P)

$$\underset{2\times 1}{\dot{x}} = \overset{2\times 1}{\underset{2\times 2}{A}X} + \overset{1\times 1}{\underset{2\times 1}{B}U} \quad ; \quad \underset{1\times 1}{y} = \underset{1\times 2}{C}\overset{2\times 1}{X} + \underset{1\times 1}{D}\overset{1\times 1}{U}$$

Q. obtain the state model by using

$y''' + 2y'' + 3y' + y = u$

Let $n = 3$. (no. of state var.s = no. of differential state variables)

$\hookrightarrow x_1 = y, \quad \dot{x}_1 = \dot{y} = x_2$

$\dot{x}_2 = \ddot{y} = x_3$

$\dot{x}_3 = \dddot{y}$ To get $\dot{x}_3$ relationship with state var.s sub all 4 eq.s in the given system.

$\Rightarrow \dot{x}_3 + 2\dot{x}_2 + 3\dot{x}_1 + x_1 \Rightarrow \dot{x}_3 + 2x_3 + 3x_2 + x_1 = u$

$\Rightarrow \dot{x}_3 = U - x_1 - 3x_2 - 2x_3$

$\dot{x} = AX + BU$ $(n = 3, p = 1, m = 1)$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -3 & -2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}u \quad \& \quad [y] = [1 \ 0 \ 0]\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Q. $y'''' + 10y''' - 6y'' + 7y' + 5y = 10\,u(t)$

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -5 & -7 & +6 & -10 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 10 \end{bmatrix} \text{ and } c = [1 \ 0 \ 0 \ 0]$$

Q. Obtain the state model for given T/F,

$$\frac{y(s)}{U(s)} = \frac{10s+5}{s^3+6s^2+7s+8} \qquad B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix} \qquad y^n = s^n = \dot{x}_n$$

$C \leftarrow$; $A \leftarrow$; $\dot{x}_3$; $\dot{x}_2 = x_3$; $\dot{x}_1 = x_2$; x_1

$$y(s) = 10x_2 + 5x_1$$

$$U(s) = \dot{x}_3 + 6x_3 + 7x_2 + 8x_1 \Rightarrow \dot{x}_3 = U(s) - 8x_1 - 7x_2 - 6x_3$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -8 & -7 & -6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} [U] \;\; \& \;\; [y] = [5 \;\; 10 \;\; 0] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Q. $T/f = \dfrac{s^2+5s+10}{s^4+3s^3+6s^2+5}$

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -5 & 0 & 6 & -3 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} ; \quad C = [10 \;\; 5 \;\; 1 \;\; 0]$$

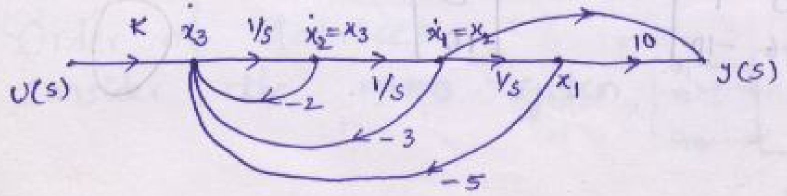
Q. $T/f = \dfrac{7s+6}{(s+1)(s+2)(s+3)} \xrightarrow{\text{directly}} A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -3 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$$= \frac{7s+6}{s^3+6s^2+12s+6} \qquad C = [6 \;\; 7 \;\; 0]$$

Q. $T/f = \dfrac{7s+6}{(s+2)^3\,(s+5)}$

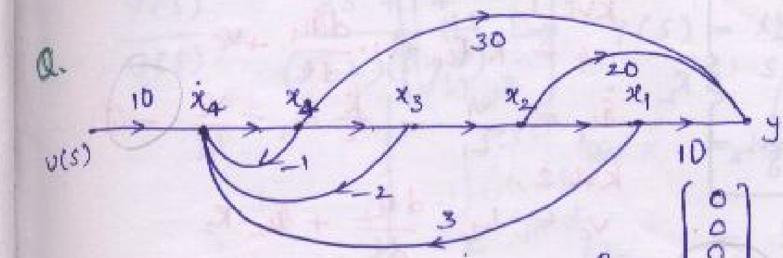
$$A = \begin{bmatrix} -2 & 1 & 0 & 0 \\ 0 & -2 & 1 & 0 \\ 0 & 0 & -2 & 0 \\ 0 & 0 & 0 & -5 \end{bmatrix}$$

Q. Obtain the A, B, C matrices for given signal flow graph.



$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix} = \begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ -5 & -3 & -2\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix} + \begin{bmatrix}0\\ 0\\ k\end{bmatrix}[u] \;;\; [y] = [10 \;\; 7 \;\; 0]\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}$$

Q.



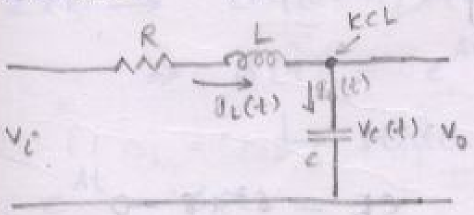
$$A = \begin{bmatrix}0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\\ 3 & 0 & -2 & -1\end{bmatrix}$$

$$B = \begin{bmatrix}0\\ 0\\ 0\\ 10\end{bmatrix} \;;\; C = [10 \;\; 20 \;\; 0 \;\; 30]$$

PROcedure for obtain the state eq for electrical n/w:-

1. select the state var.s as volt. across capacitor and current through inductor. The no. of state var.s = sum of inductors and capacitors.
2. write the independent KCL & KVL, Apply KCL at capacitor junction and KVL through inductor
3. The resultant eq. must consists state var.s differential state var.s, i/p and o/p var.s

⇒ Obtain the state model for the given electrical n/w.



KCL at I_c

$$i_L(t) = i_c(t)$$
$$= c \cdot \frac{dV_c(t)}{dt}$$
$$\dot{V}_c(t) = \frac{I_L(t)}{c} \rightarrow ①$$

KVL through inductor

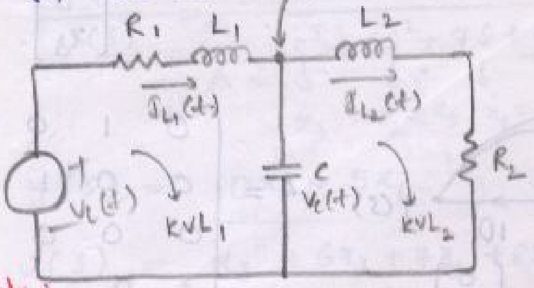
$$V_i(t) - R\,i_L(t) - L\frac{d\,i_L(t)}{dt} - V_c(t) = 0$$

$$\dot{i}_L(t) = \frac{V_i(t)}{L} - \frac{R}{L} I_L(t) - \frac{V_c(t)}{L} \rightarrow ②$$

$$\begin{bmatrix}\dot{V}_c(t)\\ \dot{i}_L(t)\end{bmatrix} = \begin{bmatrix}0 & 1/c\\ -1/L & -R/L\end{bmatrix}\begin{bmatrix}V_c(t)\\ i_L(t)\end{bmatrix} + \begin{bmatrix}0\\ 1/L\end{bmatrix}[V_i(t)]$$

$$[V_0(t)] = [1 \;\; 0]\begin{bmatrix}V_c(t)\\ i_L(t)\end{bmatrix}$$

whenever same ~~no.~~ kind of elements connected in series or ~~kcl~~ parallel then it should be treated as single element.



$$\begin{bmatrix} v_c(t) \\ i_{L_1}(t) \\ i_{L_2}(t) \end{bmatrix}$$

KVL 1,

$$v_1 = R\, i_{L_1} + L_1 \cdot \frac{di_{L_1}}{dt} + v_c$$

$$\dot{i}_{L_1} = \frac{v_1}{L_1} - \frac{R}{L}\, i_{L_1} - \frac{v_c}{L_1} \quad ②$$

KVL 2,

$$v_c = L_2 \cdot \frac{di_{L_2}}{dt} + i_{L_2} \cdot R_2$$

$$\dot{i}_{L_2} = \frac{v_c}{L_2} - \frac{R_2}{L_2} \cdot i_{L_2} \quad ③.$$

KCL,

$$i_{L_1} = i_{L_2} + i_c$$

$$i_{L_1} = i_{L_2} + C \cdot \frac{dv_c}{dt}$$

$$\dot{v}_c = \frac{i_{L_1}}{c} - \frac{i_{L_2}}{c} \quad ①$$

$$\begin{bmatrix} \dot{v}_c \\ \dot{i}_{L_1} \\ \dot{i}_{L_2} \end{bmatrix} = \begin{bmatrix} 0 & 1/C & -1/C \\ -1/L_1 & -R/L_1 & 0 \\ 1/L_2 & -0 & -R_2/L_2 \end{bmatrix} \begin{bmatrix} v_c \\ i_{L_1} \\ i_{L_2} \end{bmatrix} + \begin{bmatrix} 0 \\ 1/L_1 \\ 0 \end{bmatrix} v_1$$

T/f from state Model :-

$$\frac{Y(s)}{U(s)} = C[sI - A]^{-1} B + D$$

$$\frac{Y(s)}{U(s)} = C \cdot \frac{\text{Adj}[sI - A]}{|sI - A|} B + D$$

$$|sI - A| = 0 \xrightarrow{} CE \xrightarrow{\text{Roots of}}$$

$$\xrightarrow{CE} CL \text{ poles} \rightarrow \text{eigen values.}$$

Q. consider the state model that is,

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} +2 & +3 \\ -4 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 3 \\ 5 \end{bmatrix} [u] \; ; \; [y] = [1 \quad 1] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

(i). find the nature of system (ii). obtain stability
(iii). obtain the T/f.

$$s^2 - 4 + 12$$

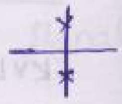
$$s^2 + 8$$

$$T/f = [1 \quad 1] \frac{\begin{bmatrix} s-2 & -3 \\ 4 & s+2 \end{bmatrix}}{s^2+8} \begin{bmatrix} 3 \\ 5 \end{bmatrix}$$

$$\frac{\begin{bmatrix} 3s - 6 - 15 \\ 12 - 5s + 10 \end{bmatrix}}{s^2+8} = \frac{8s+1}{s^2+8}$$

$$CE = s^2 + 8 = 0$$

$$\Rightarrow s = \pm j\sqrt{8}$$

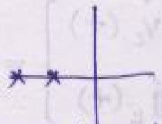


$\xi = 0$ undamped → M.S.

Q. Obtain the T/f,

$$[\dot{x}] = \begin{bmatrix} 0 & 3 \\ -2 & -5 \end{bmatrix} [x] + \begin{bmatrix} 1 \\ 1 \end{bmatrix} [u] \; ; \quad Y = [2 \quad 1][x]$$

$$T/f = [2 \quad 1] \frac{\begin{bmatrix} s+5 & +3 \\ -2 & s \end{bmatrix}}{s^2+5s+6} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \frac{3s+14}{(s+2)(s+3)}$$



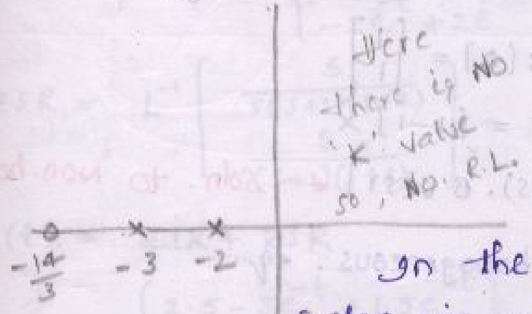
over damped $\xi > 1$
stable

Q. find the unit step response for the above state model and also draw the R.L diagram.

$$\frac{Y(s)}{U(s)} = \frac{3s+14}{(s+2)(s+3)} \Rightarrow Y(s) = \frac{3s+14}{s(s+2)(s+3)}$$

$$= \frac{14}{6s} - \frac{4}{s+2} + \frac{5}{3}\cdot\frac{1}{(s+3)}$$

$$= \frac{14}{6} - 4e^{-2t} + \frac{5}{3}e^{-3t}$$



In the above system, there is NO system gain parameter, hence RL diagram is nothing but loc. of p&z's.

Solution to the state eq :-

$\dot{x} = Ax + BU$ ← Non homogeneous state eq.

(1). L.T. :-

ZIR → due to initial condi.

ZSR → due to i/p

$$x(t) = \boxed{L^{-1}[[sI-A]^{-1}x(0)]} + \boxed{L^{-1}[[sI-A]^{-1}BU(s)]}$$

(2). Classical Method :-

$$x(t) = e^{At}x(0) + \int_0^t e^{A(t-\tau)}.BU(\tau).d\tau$$

ZIR : Natural (or) free force, system impulse.

ZSR : forced response,

$$ZIR \rightarrow L^{-1}[[sI-A]^{-1}x(0)] = e^{At}x(0)$$

$$\Rightarrow \phi(t) = e^{At} = L^{-1}[[sI-A]^{-1}]$$

↳ state transition Matrix.

$$e^{At} = \phi(t)$$

$$e^{A(t-\tau)} = \phi(t-\tau)$$

$$L^{-1}[sI-A]^{-1} = \phi(t) \Rightarrow [sI-A]^{-1} = \phi(s)$$

$$ZSR \rightarrow L^{-1}[\phi(s).BU(s)] = \int_0^t \phi(t-\tau).BU(\tau).d\tau$$

Properties of S.T.M :-

$$STM \ \phi(t) = e^{At} = L^{-1}[[sI-A]^{-1}]$$

1. $\phi(0) = I$ [Identity Matrix]
2. $\phi^k(t) = (e^{At})^k = e^{A(kt)} = \phi(kt)$.

3. $\phi^{-1}(t) = \phi(-t)$

4. $\phi(t_1 + t_2) = \phi(t_1) . \phi(t_2)$.

5. $\phi(t_2 - t_1) . \phi(t_1 - t_0) = \phi(t_2 - t_0)$

Q. Obtain the time response for the given system,

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ -2 & 0 \end{bmatrix} x \quad \text{where} \quad x[0] = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$y = [1 \; -1] \, x$$

$x(t) = e^{At} x(0) + L^{-1}[\phi(s) . B\,U(s)]$ ← soln. to Non. homo. eq

$\dot{x} = AX + BU$ → Non- homogeneous . eq .

$\dot{x} = AX$ → Homogeneous eq . , $U = 0$.

Soln. of homogeneous eq : $x(t) = e^{At} x(0)$.

The given system is homogeneous because $U(s) = 0$,

STM $\phi(t) = e^{At} = L^{-1}[[sI - A]^{-1}]$ $\quad x(t) = e^{At} x(0)$

$$= L^{-1}\begin{bmatrix} \frac{s}{s^2+2} & \frac{+1}{s^2+2} \\ \frac{-2}{s^2+2} & \frac{s}{s^2+2} \end{bmatrix} \neq \begin{bmatrix} \cos\sqrt{2}t & 1/\sqrt{2}\,\sin\sqrt{2}t \\ -\sqrt{2}\sin\sqrt{2}t & \cos\sqrt{2}t \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

{The correct STM is, which gives Identity Matrix for t=0}

$\Rightarrow x(t) = e^{At} . x(0) =$

$y(t) = [1 \; -1] \, x(t) = 3/\sqrt{2} \, \sin\sqrt{2}t$.

Q. find the time response for given

$$\dot{x} = \begin{bmatrix} 0 & -1 \\ +2 & s+3 \end{bmatrix} x + \begin{bmatrix} 0 \\ 5 \end{bmatrix} u \, , \quad x(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \; \& \; y(t) = [0 \; 1] \, x(t)$$

$x(t) = \underbrace{e^{At} . x(0)}_{ZIR} + \underbrace{L^{-1}[\phi(s) . B\,U(s)]}_{ZSR}$

$$\phi(t) = e^{At} = L^{-1}\begin{bmatrix} \frac{s+3}{(s+1)(s+2)} & \frac{+1}{(s+1)(s+2)} \\ \frac{-2}{(s+1)(s+2)} & \frac{s}{(s+1)(s+2)} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{2}{s+1} - \frac{1}{s+2} & \frac{1}{s+1} - \frac{1}{s+2} \\ \frac{-2}{s+1} + \frac{2}{s+2} & \frac{-1}{s+1} + \frac{2}{s+2} \end{bmatrix} = \begin{bmatrix} 2e^{-t} - e^{-2t} & e^{-t} - e^{-2t} \\ -2e^{-t} + 2e^{-2t} & -e^{-t} + 2e^{-2t} \end{bmatrix}$$

→ If we required to find STM, substitute $t=0$ in the given options. $\phi(t)$ at $t=0$, must be the identity matrix.

$$x(t) = ZIR = \begin{bmatrix} 2e^{-t} - e^{-2t} \\ -2e^{-t} + 2e^{-2t} \end{bmatrix}$$

$$ZSR = L^{-1}\begin{bmatrix} \frac{5}{s(s+1)(s+2)} \\ \frac{5}{(s+1)(s+2)} \end{bmatrix} = L^{-1}\begin{bmatrix} \frac{2.5}{s} - 5\frac{1}{s+1} + \frac{2.5}{s+2} \\ \frac{5}{s+1} - \frac{5}{s+2} \end{bmatrix} = \begin{bmatrix} 2.5 - 5e^{-t} + 2.5e^{-2t} \\ 5e^{-t} - 5e^{-2t} \end{bmatrix}$$

$$x(t) = ZIR + ZSR$$

$$= \begin{bmatrix} 2.5 - 3e^{-t} + 1.5e^{-2t} \\ 3e^{-t} - 3e^{-2t} \end{bmatrix} \quad \& \quad Y(t) = 3e^{-t} - 3e^{-2t}$$

$y(t) = [0 \; 1]\, x(t)$

controllability :-

A system is said to be controllable if it is possible to transfer the initial state to desired state in a finite time interval by the controlled i/p.

Kalman's test for controllability :-

$$Q_c = [B \quad AB \quad A^2B \; \ldots \; A^{n-1}B]$$

Rank of Q_c = Rank of A

$|Q_c| \neq 0 \rightarrow$ controllable.

Q. check controllability; $T/f = \dfrac{1}{s^3 + 2s^2 + 3s + 4}$

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -4 & -3 & -2 \end{bmatrix} ; \quad B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$n = 3$.

(B, AB, A^2B)

$$Q_c = \begin{array}{c} \begin{matrix} B & AB & A^2B \end{matrix} \\ \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & -2 \\ 1 & -2 & 1 \end{bmatrix} \end{array} ; \quad |\phi_c| \neq 0 \rightarrow \text{controllable.}$$

Q. $$\dot{x} = \begin{bmatrix} 1 & 1 \\ 2 & -1 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$$Q_c = \begin{array}{c} \begin{matrix} B & AB \end{matrix} \\ \begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix} \end{array} \longrightarrow \text{controllable}$$

Q. $\dot{x}_1 = -2x_1 + u$, $\dot{x}_2 = 3x_1 - 5x_2$

$$A = \begin{bmatrix} -2 & 0 \\ 3 & -5 \end{bmatrix},\quad B = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \;:\; Q_c = \begin{bmatrix} 1 & -2 \\ 0 & 3 \end{bmatrix} \rightarrow \text{controllable}$$

Observability :-

A system is said to be observable, if it is possible to determine initial states of the system by observing the o/p's in a finite time interval.

Kalman's Test for Observability :-

$$Q_0 = [\; C^T \quad A^TC^T \quad (A^T)^2C^T \;\ldots\; (A^T)^{n-1}C^T \;]$$

(or) $$Q_0 = \begin{bmatrix} C \\ CA \\ CA^2 \\ \vdots \\ CA^{n-1} \end{bmatrix} ;$$ Rank of Q_0 = Rank of A

$|Q_0| \neq 0$
$\rightarrow$ Observable

Q. Check the controllability & observability,

$$\dot{x} = \begin{bmatrix} 2 & 0 \\ -1 & 1 \end{bmatrix} x + \begin{bmatrix} 1 \\ -1 \end{bmatrix} u \;;\quad y = [1 \quad 1]\,x$$

$$Q_c = \begin{bmatrix} 1 & 2 \\ -1 & -2 \end{bmatrix},\quad |Q_c| = 0 \rightarrow \text{Not controllable}$$

$$Q_0 = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix},\quad |Q_0| = 0 \rightarrow \text{Not observable.}$$

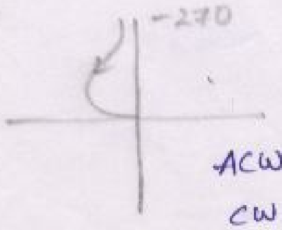
Q. $\dot{x}_1 = -2x_1 + x_2 + u$

$\dot{x}_2 = -x_2 + u$

$y = x_1 + x_2$

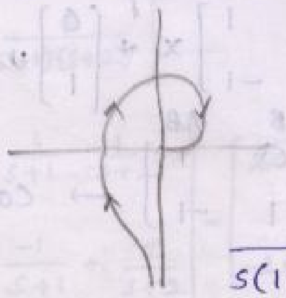
$$Q_c = \begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix} \xrightarrow{|Q_c| = 0} \text{Not controllable}$$

$$Q_0 = \begin{bmatrix} 1 & 1 \\ -2 & 0 \end{bmatrix} \rightarrow |Q_0| \neq 0$$
observable.



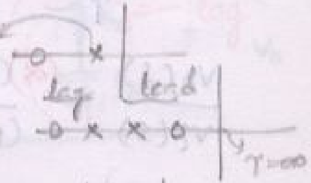
ACW → zero
CW → pole

$$\frac{k(1+sT_1)}{s^3}$$



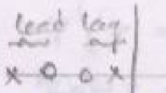
$$\frac{k}{s(1+sT_1)(1+sT_2)(1+sT_3)}$$

Compensator :-

1. Lead → high pass  → +ve angle $\xrightarrow{\text{given by}}$ zero 
2. Lag → Low pass  → −ve angle $\xrightarrow{\text{given by}}$ pole 

3. Lead - Lag → 

→ τ Lead > τ lag
4. Lag - Lead → 

→ τ lag > τ lead 

Each compensator gives the one finite pole and one finite zero.

when a sinusoidal i/p is applied to the n/w it produces a sinu. steady state o/p having a ph. lead w.r.t. i/p then the n/w is called lead compensator. The lead compensator speedup the transient response and increase the margin of system stability and also increases the error const. [ie ss error decreases].

If the ss o/p has the ph. lag then the n/w is called lag compensator. The lag compensator improves the ss behaviour without effecting the transient response. (both ph. lag & lead occurs but in different freq. regions.)

The lag-lead or lead-lag improves the both transient and ss behaviour.

Lead Compensator :-

S_1 : T/F

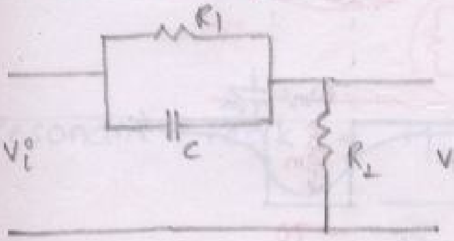
S_2 : T- const form

S_3 : locate P/Z - s-plane

S_4 : B.P & Identify filter.

S_5 : $\omega_m \to \phi_\omega|_{=\omega_m}$

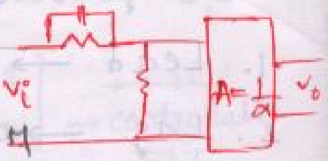
$\Rightarrow M_\omega|_{=\omega_m}$



$$\frac{V_0(S)}{V_i(S)} = \frac{R_2}{R_2 + \frac{R_1}{SCR_1+1}}$$

$$= \frac{R_2(1+SCR_1)}{R_1+R_2+SCR_1R_2} = \frac{R_2(1+SCR_1)}{(R_1+R_2)\left(1+\frac{R_2}{R_1+R_2}\cdot SCR_1\right)}$$

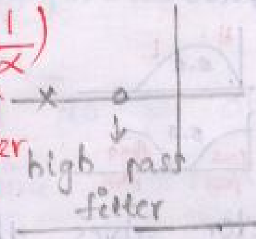
Let α - lead const. $= \frac{R_2}{R_1+R_2} < 1$



τ - lead time const. $= R_1 C$

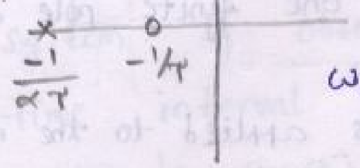
$$\frac{V_0(s)}{V_i(s)} = \frac{(\alpha)(1+\tau s)}{(1+\alpha\tau s)} \cdot \left(\frac{1}{\alpha}\right)$$

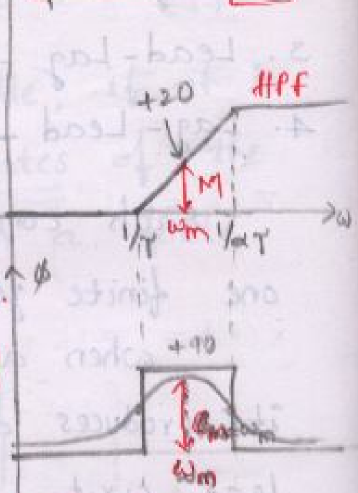
amplifier gain



$S_z = -1/\tau$

$S_p = -1/\alpha\tau$





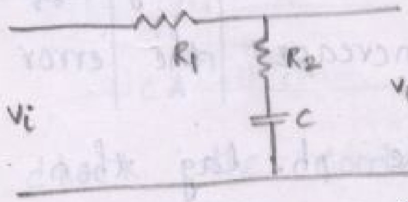
$\omega_m = \frac{1}{\tau\sqrt{\alpha}}$; $M = 10 \log 1/\alpha$.

$\phi_m = \sin^{-1}\left(\frac{1-\alpha}{1+\alpha}\right)$

α - Attenuation factor

b'coz α is < 1. The main dis.adv in lead comp. is signal strength is attenuated. So elimi-nate attenuation we required to connect amplifier with gain of $1/\alpha$ in series to compen.

Lag compensator :-



$$\frac{V_0(s)}{V_i(s)} = \frac{R_2 + 1/sC}{R_1 + R_2 + 1/sC}$$

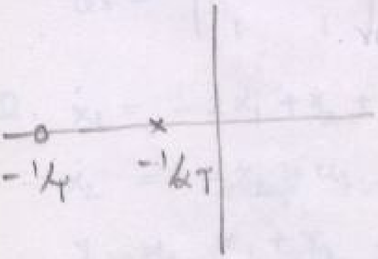
$$= \frac{1 + sCR_2}{1 + \frac{R_1+R_2}{R_2} \cdot sCR_2}$$

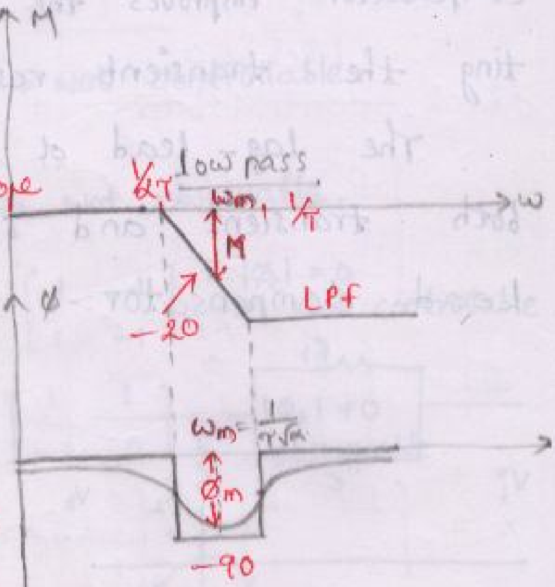
α - lag constant $= \frac{R_1+R_2}{R_2} > 1$

τ - lag time constant $= R_2 C$

$$\frac{V_0(s)}{V_i(s)} = \frac{1+\tau s}{1+\alpha\tau s}$$

Initial slope $= 0$





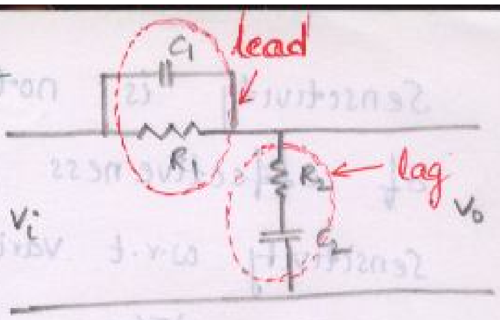
In compensators zero location is fixed, the change is only in poles location.

$M = 10 \log 1/\alpha$

$\omega_m = \frac{1}{\tau\sqrt{\alpha}}$

$\phi_m = \sin^{-1}\left(\frac{\alpha-1}{\alpha+1}\right)$.

Lead-Lag compensator:-

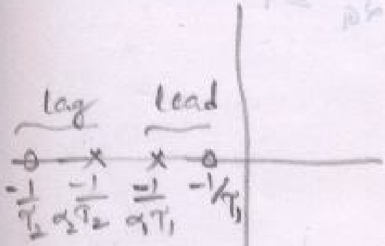


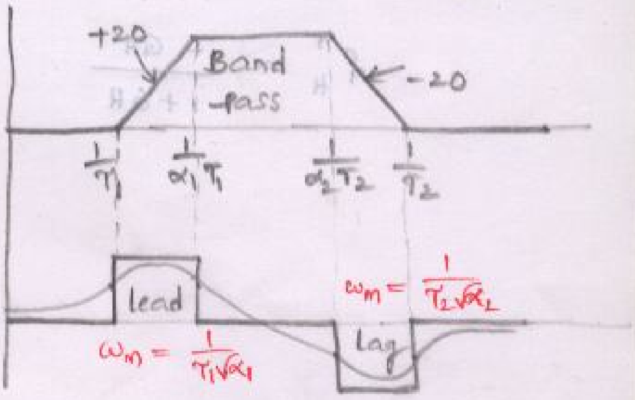
$\tau_{lead} > \tau_{lag}$

$$\frac{V_0(s)}{V_i(s)} = \overset{lead}{\frac{1+T_1 s}{1+\alpha T_1 s}} \cdot \overset{lag}{\frac{1+T_2 s}{1+\alpha T_2 s}}$$

τ_1 – lead $\tau = R_1 C_1$ $\quad$ τ_2 – lag $\tau = R_2 C_2$

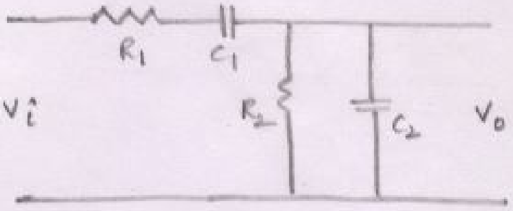
α_1 – lead const. $= \frac{R_2}{R_1+R_2} < 1$, α_2 – lag const. $= \frac{R_1+R_2}{R_2} > 1$

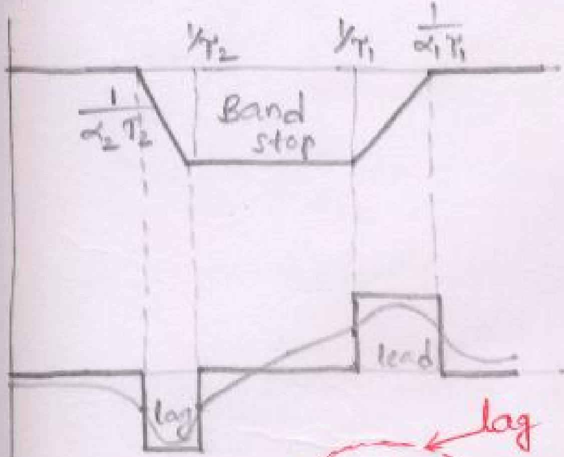


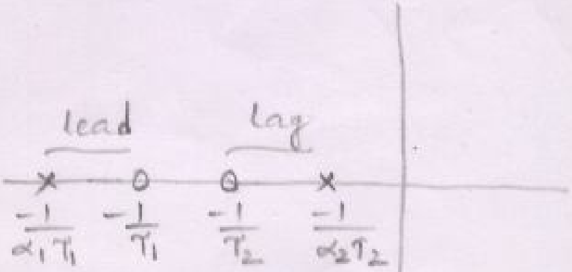


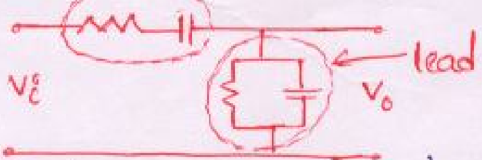
Lag-lead compensator:-

$\tau_{lag} > \tau_{lead}$









Resonant Peak $= M_r = \dfrac{1}{2\xi\sqrt{1-\xi^2}}$

$\omega_r = \omega_n\sqrt{1-2\xi^2}$

B.W. $= \omega_b = \omega_n\sqrt{1-2\xi^2+\sqrt{2-4\xi^2+4\xi^2}}$

$\uparrow$ B.W $\propto \frac{1}{t_r \downarrow}$

Smallest $\xi \Rightarrow$ BW $\uparrow$

Sensitivity is nothing but a measurement of effectiveness of f/b.

Sensitivity w.r.t variations in $G(s) - S_G^T = \frac{\partial T/T}{\partial G/G}$

$$S_H^T = \frac{\partial T/T}{\partial H/H} = \frac{\partial T}{\partial H} \cdot \frac{H}{T}$$

$$= \frac{\partial T}{\partial G} \times \frac{G}{T}$$

⇒ CLCS:

$$S_G^T = \frac{1}{1+GH}$$

$$S_H^T = \frac{-GH}{1+GH}$$

⇒ OLCS: $S_G^T = 1$

CONTROLS

NOTES

(Modified)

GATE 2009

JANARDHANARAO

control Systems (15M)

FRI. 08/08/08

The LTI System is nothing but RLC n/ω. because the RLC components gives the linear transfer char.s and its values are not changes w.r.t time [Time invarient].

* $L[t^n] = \frac{n!}{s^{n+1}}$

* $L[t^n . e^{\pm at}] = \frac{n!}{(s\mp a)^{n+1}}$

* $L[e^{\pm at}] = \frac{1}{s\mp a}$

* $L[\sin bt] = \frac{b}{s^2+b^2}$

* $L[\cos bt] = \frac{s}{s^2+b^2}$

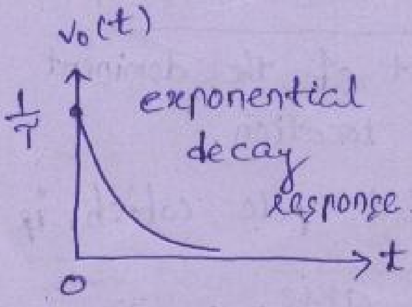
* $L[e^{\pm at} \sin bt] = \frac{b}{(s\mp a)^2+b^2}$

* $L[f(t-\tau)] = e^{-s\tau} f(s)$.

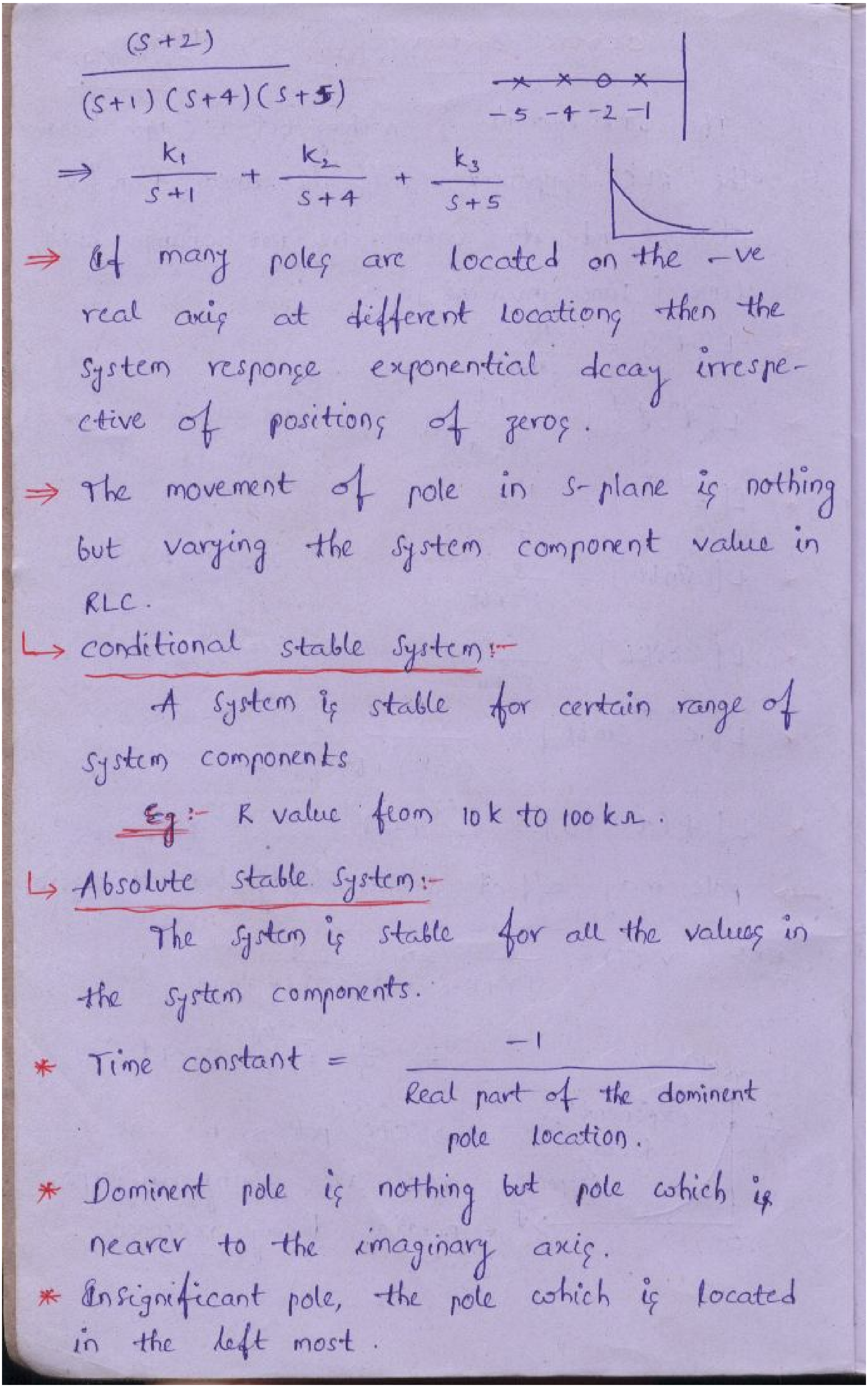
→ pole may effect s.s. stability but not a zero.

Ex:- $V_o(s) = \frac{1}{s\tau+1} \Rightarrow \frac{1}{\tau(s+\frac{1}{\tau})}$

$\Rightarrow \frac{1}{\tau} e^{-t/\tau} = V_o(t)$.



⇒ one pole on the -ve real axis giving an exponential decay response.



$$\frac{(s+2)}{(s+1)(s+4)(s+5)}$$

$$\Rightarrow \frac{k_1}{s+1} + \frac{k_2}{s+4} + \frac{k_3}{s+5}$$

⇒ If many poles are located on the −ve real axis at different locations then the system response exponential decay irrespective of positions of zeros.

⇒ The movement of pole in s-plane is nothing but varying the system component value in RLC.

↳ Conditional stable system :-

A system is stable for certain range of system components

Eg :- R value from 10k to 100kΩ.

↳ Absolute stable system :-

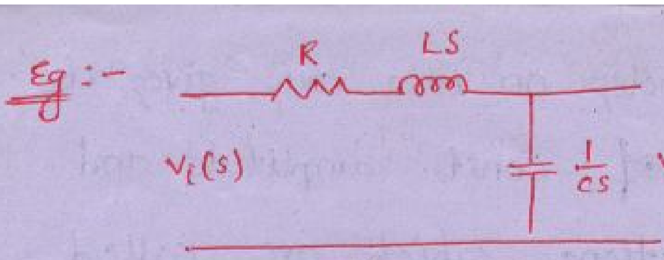
The system is stable for all the values in the system components.

* Time constant $= \dfrac{-1}{\text{Real part of the dominent pole location.}}$

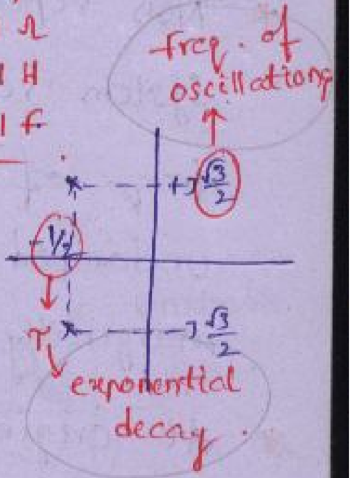
* Dominent pole is nothing but pole which is nearer to the imaginary axis.

* Insignificant pole, the pole which is located in the left most.

Eg :-



Let,
R = 1 Ω
L = 1 H
C = 1 f



$$\frac{V_o(s)}{V_i(s)} = \frac{1/cs}{R+LS+\frac{1}{cs}} = \frac{1}{s^2+s+1}$$

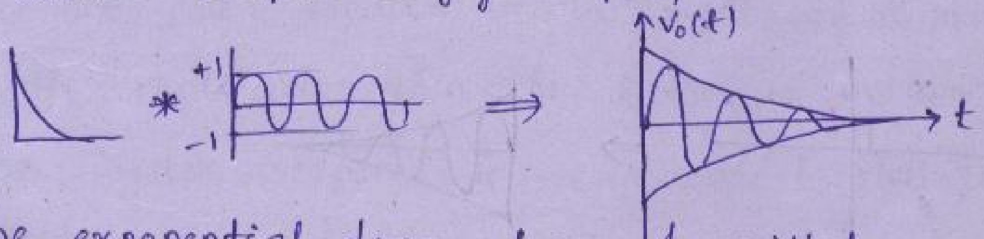
System is stable.

$$\Rightarrow V_o(t) = k \cdot e^{-1/2 t} \cdot \sin\left(\frac{\sqrt{3}}{2}\right)t$$

* In a complex conjugate poles, the real part gives the exponential decay where as ima. part gives the freq. of oscillations. The total system response is called exponential decay freq. of oscillations.

$$\Rightarrow V_o = k \cdot e^{(\text{real part})t} \cdot \sin/\cos(\text{ima. part}) \cdot t$$

↳ whenever complex conjugate poles.



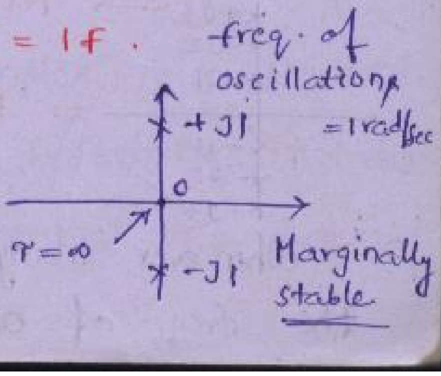
The exponential decay freq. of oscillations are called damped oscillations.

* A System which produces damped oscillations is called under damped system.

case 2) : R = 0 Ω, L = 1 H, C = 1 f.

$$\frac{V_o(s)}{V_i(s)} = \frac{~~SH~~\ 1}{s^2LC + sCR + 1}$$

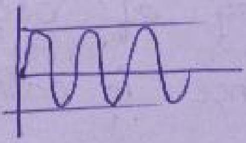
$$= \frac{~~S~~\ 1}{s^2+1}$$

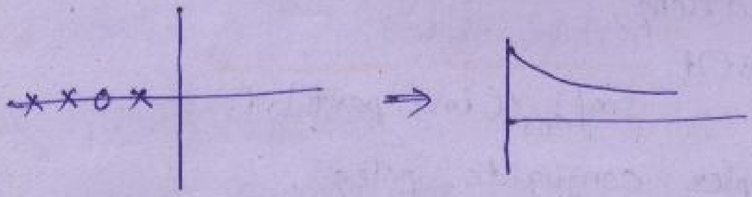


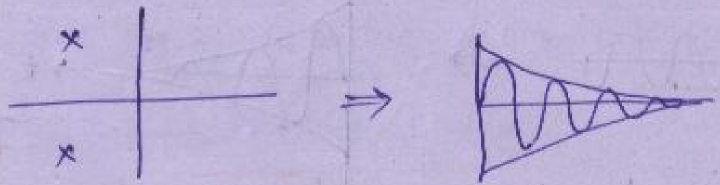
* Non repeated poles on ima. axis gives the system response of const. amplitude and freq. of oscillations, which are called undamped oscillations and the system is marginally stable.

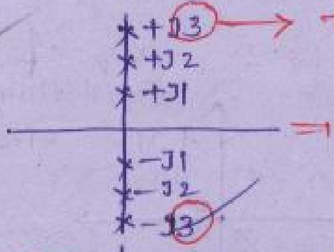
* At origin $\tau = \infty$. , As poles moves towards left hand side the system τ decreases, stability improves and system gives very quick response.

→ $v_0(t) = \sin t$







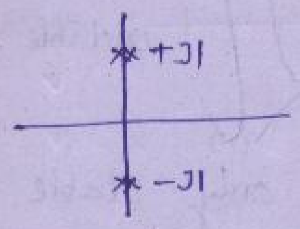


→ freq. of oscillations

⇒ Marginally stable.

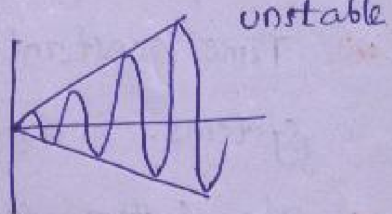
→ whenever many poles lies on ima. axis then the freq. of oscillations are nothing but largest intersection point on the ima. axis.

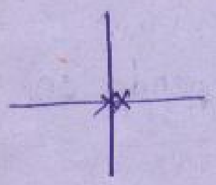
Repeated poles on ima axis



$\Rightarrow \frac{1}{(s^2+1)^2} \Rightarrow t.\sin t$

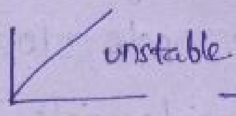
$\Rightarrow$





$\Rightarrow \frac{1}{s^2} \Rightarrow t$

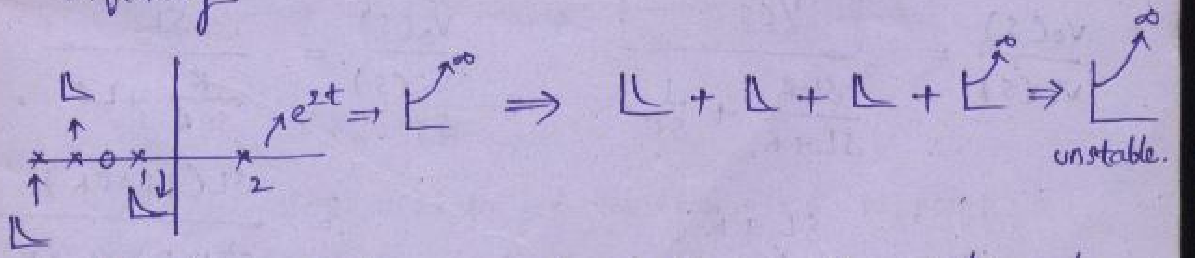
$\Rightarrow$



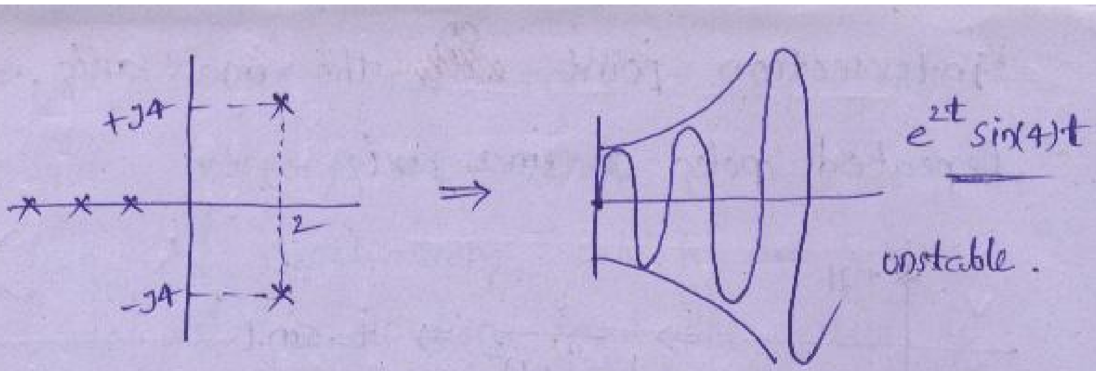
→ ramp.

* Repeated poles on ima. axis ⇒ System is unstable b'coz system response is increasing amplitude oscillations. (&f finite ima. term).

→ Many poles are lies in the left of s-plane but one pole located in the right of s-plane on the real axis then the system is unstable, b'coz system response is exponential rise to infinity

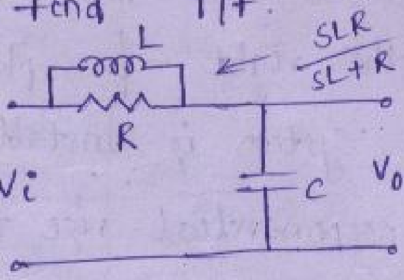


→ Many poles are located in the left of s-plane but one pair of complex poles located in the right of s-plane ⇒ System is unstable b'coz System response is exponential rise freq. of oscillations.



- * Time constant is defined for only stable systems.
- * The system stability / response is depends on the right most side pole location.
- * If right most side pole location in the left hand side ⇒ stable.
- * If it is on ima. axis ⇒ Marginally stable.
- * If it is on right hand side ⇒ unstable.

Q. find T/f.

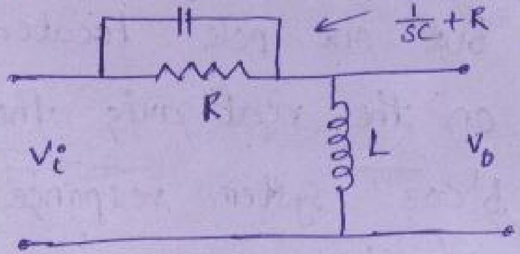


$$\frac{V_o(s)}{V_i(s)} = \frac{1/Cs}{\frac{SLR}{SL+R}+\frac{1}{SC}}$$

$$= \frac{SL+R}{S^2LCR+SL+R}$$

Q.

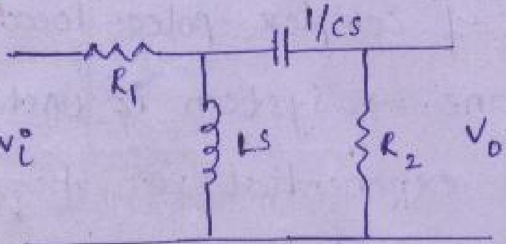
$$\frac{R/SC}{\frac{1}{SC}+R} = \frac{R}{SCR+1}$$



$$\frac{V_o(s)}{V_i(s)} = \frac{SL}{\frac{R}{SCR}+LS}$$

$$= \frac{SL(1+SCR)}{S^2LCR+SL+R}$$

Q.



$$\frac{V_o(s)}{V_i(s)} =$$

$$\frac{V_o(s)}{V_i(s)} = \frac{sL\cdot R_2}{R_1\left[sL+\frac{1}{sC}+R_2\right]+sL\left[\frac{1}{sC}+R_2\right]}$$

find T/f for; $\frac{d^3y}{dt^3} + 6\cdot\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = 10\frac{dx}{dt}$

⇒ x : i/p $\quad s^n = \frac{d^n}{dt^n}$

y : o/p

$$\Rightarrow y(s)\left[s^3+6s^2+3s+2\right] = 10x(s).s$$

$$\Rightarrow \frac{y(s)}{x(s)} = \frac{10.s}{s^3+6s^2+3s+2}$$

Given find

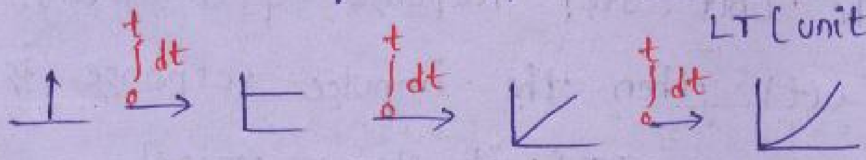
Q1. Signal response ⟶ T/f.

$$T/f = \frac{LT[\text{given signal response}]}{LT[\text{given i/p}]}.$$

Eg:- Given: unit ramp response

then to find $T/f = \frac{LT[\text{unit ramp res.}]}{LT[\text{unit ramp}]}$

Q2.



Q3.

Step1 : find T/f.

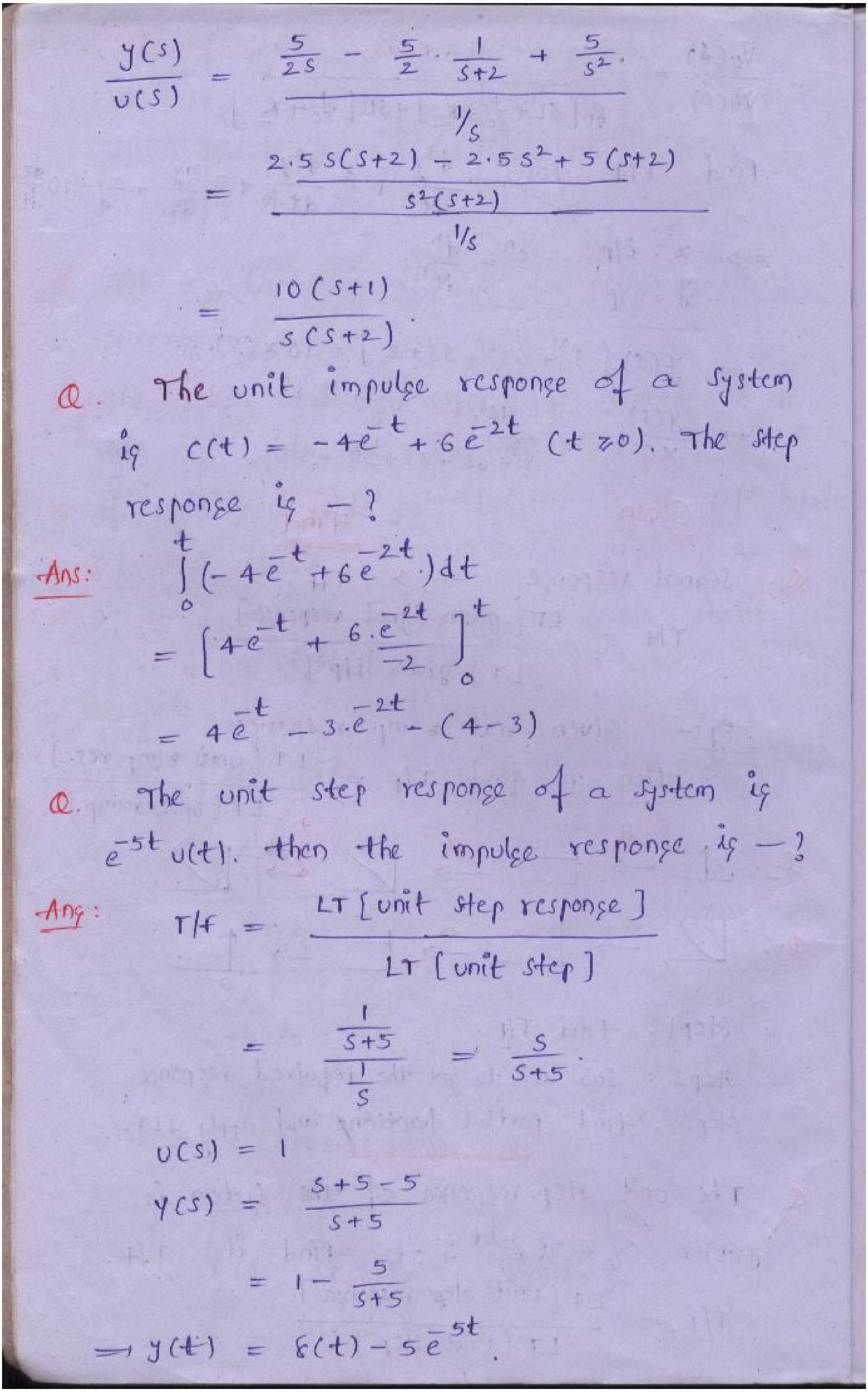
step2 : Sub. u(s) to get the required response.

step3 : find partial fractions and apply ILT.

Q. The unit step response of the system is

$y(t) = 5/2 - 5/2\, e^{-2t} + 5t$, find its T/f.

$$T/f = \frac{LT[\text{unit step response}]}{LT[\text{unit step}]}$$



$$\frac{y(s)}{u(s)} = \frac{\frac{5}{2s} - \frac{5}{2}\cdot\frac{1}{s+2} + \frac{5}{s^2}}{1/s}$$

$$= \frac{\frac{2.5\,s(s+2) - 2.5\,s^2 + 5(s+2)}{s^2(s+2)}}{1/s}$$

$$= \frac{10(s+1)}{s(s+2)}.$$

Q. The unit impulse response of a system is $c(t) = -4e^{-t} + 6e^{-2t}$ $(t \geq 0)$. The step response is — ?

Ans:

$$\int_0^t (-4e^{-t} + 6e^{-2t})\,dt$$

$$= \left[4e^{-t} + 6.\frac{e^{-2t}}{-2}\right]_0^t$$

$$= 4e^{-t} - 3.e^{-2t} - (4-3)$$

Q. The unit step response of a system is $e^{-5t}u(t)$. then the impulse response is — ?

Ans:

$$T/F = \frac{LT\,[\text{unit step response}]}{LT\,[\text{unit step}]}$$

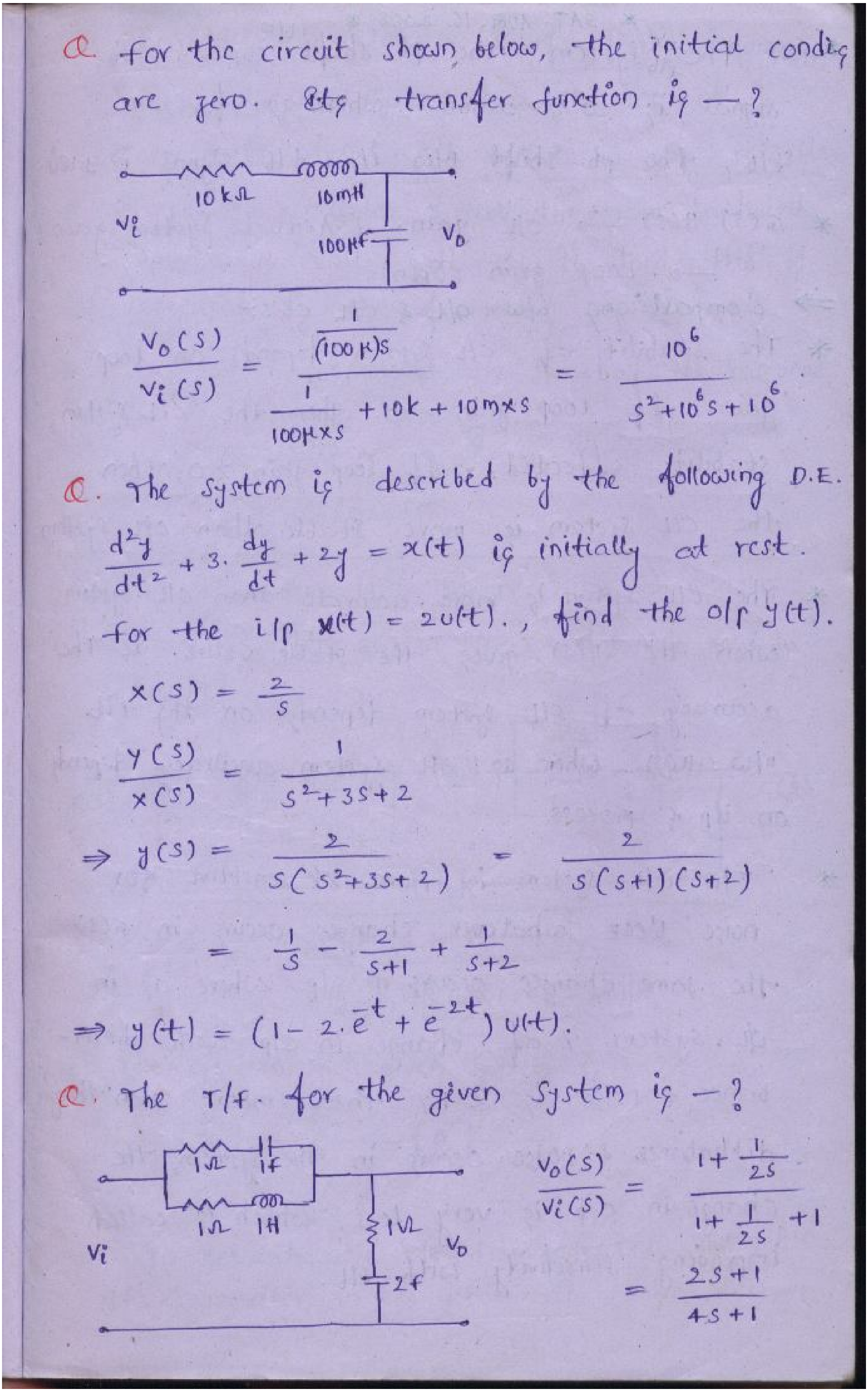
$$= \frac{\frac{1}{s+5}}{\frac{1}{s}} = \frac{s}{s+5}.$$

$U(s) = 1$

$$Y(s) = \frac{s+5-5}{s+5}$$

$$= 1 - \frac{5}{s+5}$$

$$\rightarrow y(t) = \delta(t) - 5e^{-5t}.$$



Q. For the circuit shown below, the initial condi
are zero. Its transfer function is — ?

$$\frac{V_o(S)}{V_i(S)} = \frac{\frac{1}{(100\,\mu)S}}{\frac{1}{100\mu \times S} + 10k + 10m \times S} = \frac{10^6}{S^2+10^6 S+10^6}$$

Q. The system is described by the following D.E.

$\frac{d^2y}{dt^2} + 3 \cdot \frac{dy}{dt} + 2y = x(t)$ is initially at rest.

For the i/p $x(t) = 2u(t)$., find the o/p $y(t)$.

$$X(S) = \frac{2}{S}$$

$$\frac{Y(S)}{X(S)} = \frac{1}{S^2+3S+2}$$

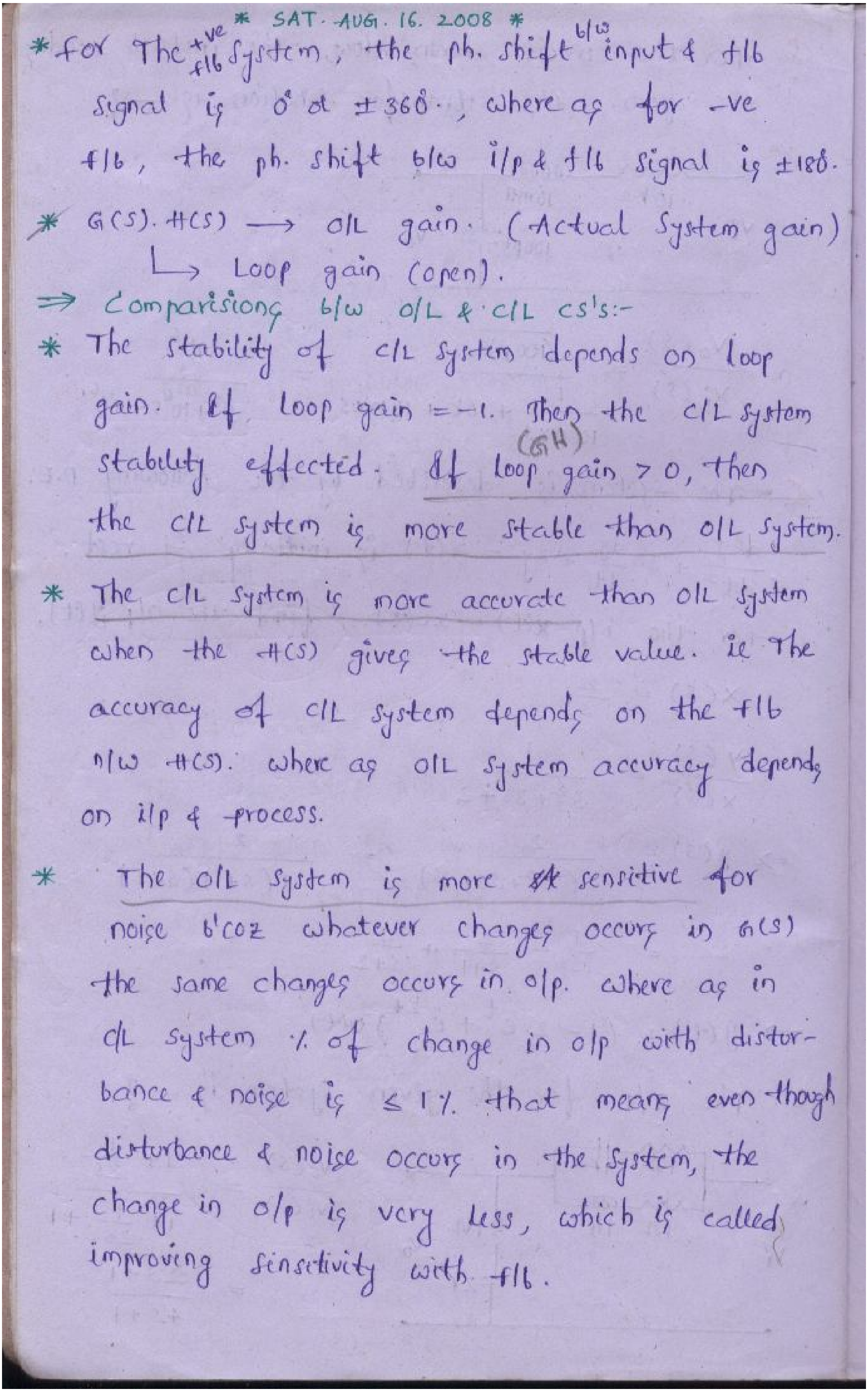
$$\Rightarrow y(S) = \frac{2}{S(S^2+3S+2)} = \frac{2}{S(S+1)(S+2)}$$

$$= \frac{1}{S} - \frac{2}{S+1} + \frac{1}{S+2}$$

$$\Rightarrow y(t) = (1 - 2 \cdot e^{-t} + e^{-2t})\, u(t).$$

Q. The T/F. for the given system is — ?

$$\frac{V_o(S)}{V_i(S)} = \frac{1+\frac{1}{2S}}{1+\frac{1}{2S}+1} = \frac{2S+1}{4S+1}$$



* SAT. AUG. 16. 2008 *

* For The +ve f/b System, the ph. shift b/w input & f/b signal is 0° or ±360°, where as for -ve f/b, the ph. shift b/w i/p & f/b signal is ±180°.

* G(S). H(S) → O/L gain. (Actual System gain)
  └→ Loop gain (open).

⇒ Comparisions b/w O/L & C/L CS's:-

* The stability of C/L system depends on loop gain. If loop gain = -1. Then the C/L system stability effected. If loop (GH) gain > 0, then the C/L system is more stable than O/L system.

* The C/L system is more accurate than O/L system when the H(S) gives the stable value. ie The accuracy of C/L system depends on the f/b n/w H(S). where as O/L system accuracy depends on i/p & process.

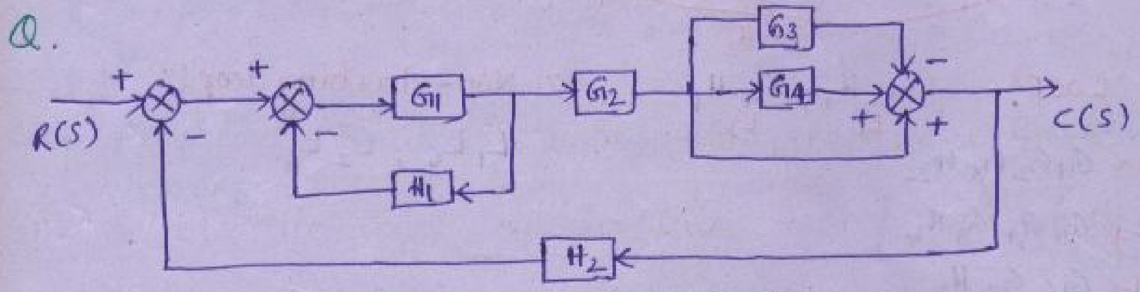
* The O/L system is more ~~sk~~ sensitive for noise b'coz whatever changes occurs in G(S) the same changes occurs in o/p. where as in C/L system % of change in o/p with disturbance & noise is ≤ 1%. that means even though disturbance & noise occurs in the system, the change in o/p is very less, which is called improving sensitivity with f/b.

- * Reliability depends on no. of discrete comp.s the o/L control system has the less no. of components, hence o/L system is more reliable.
- * for any perticular system the gain, bandwidth product is const. With f/b the band width is increased by the factor of $1+G(S).H(S)$.
- * Band width represents the speed of the response, as BW increases the system gives the quick response.

$$\uparrow BW \propto \frac{1}{t_R}\downarrow \propto \overset{\text{response}}{(\text{speed})}$$

- * In o/L system it is not necessary to measure the o/p., where as in c/L cs the o/p must be measured.

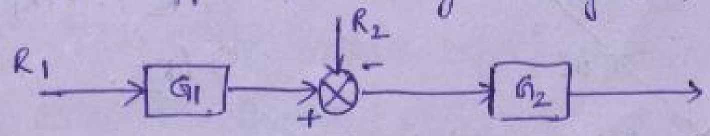
Q.



Sol.

$$\frac{C(S)}{R(S)} = \frac{\boxed{G_1 G_2 (1+G_4-G_3)}}{1+G_1H_1+\boxed{[G_1G_2(1+G_4-G_3)]}H_2}$$

Q. The o/p to the given system is -?



Sol. $G_1 G_2 R_1 - G_2 R_2$

⟹ To get the o/L T/f from c/L T/f, Subtract the numerator in the denominator, when $H(S)=1$.

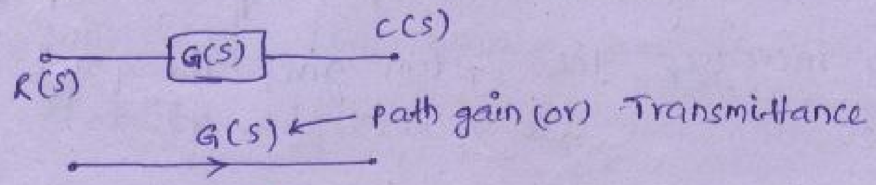
$$= \frac{G}{1+G-G}$$

⟹ To get the C/L T/f from O/L, add the numerator term in the denominator when H(S) = 1.

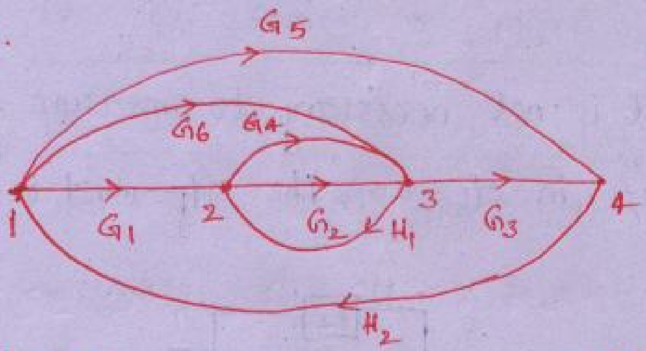
$$= \frac{G}{1+G}.$$

⟹ Signal flow Graphs:

⟹ Set of Linear Algebraic eq.s represents the system.



Q.



forward paths:

$G_1G_2G_3$

G_6G_3

$G_1G_4G_3$

G_5

Loops: $\underset{L_1}{G_2H_1}$, $\underset{L_2}{G_4H_1}$

L_3 $G_1G_2G_3H_2$

L_4 $G_1G_4G_3H_2$

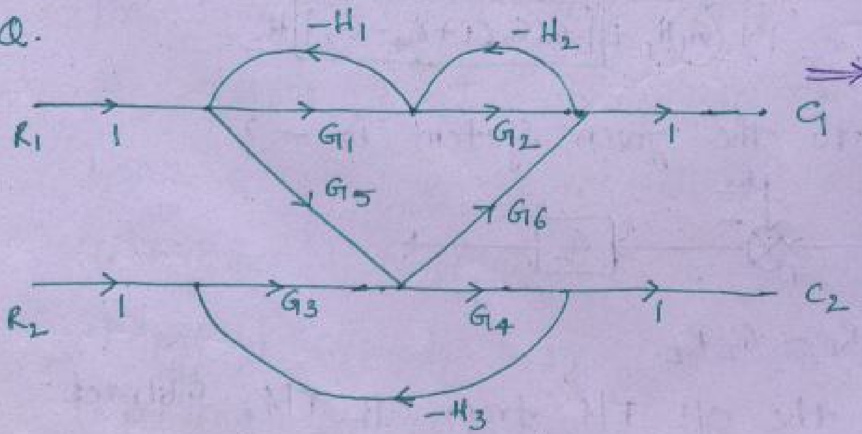
L_5 $G_6G_3H_2$

L_6 G_5H_2

2 Non-touching loops:

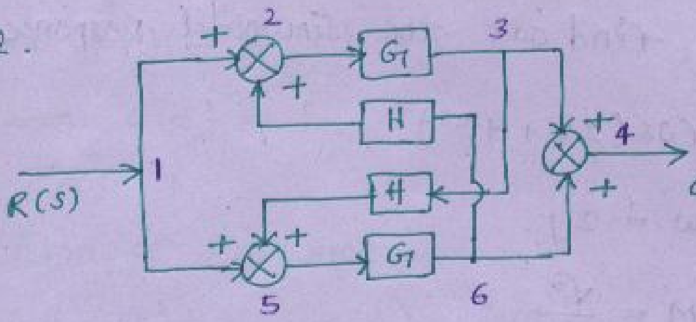
L_1L_6, L_2L_6

Q.



⟹ find C_1/R_1, C_1/R_2, C_2/R_1 and C_2/R_2.

Q.



forward paths:

$P_1 = 1234 \longrightarrow G$

$P_2 = 123564 \longrightarrow G^2H$

$P_3 = 1564 \longrightarrow G$

$P_4 = 156234 \longrightarrow G^2H$

Loops:

$L_1 = 23562$

$$T/f = \frac{G+G^2H+G+G^2H}{1-G^2H^2}$$

$$= \frac{2G}{1-GH}.$$

→ Procedure to draw signal-flow graph for Electrical Network :-

(1). Select the nodes as a series branch var.s in the same order.

(2). Each component in an electrical n/w gives one f. path & one -ve f/b path except the last element where we takes o/p.

Last element gives only f. path.

(3). Take the ratio of impedance for series branch elements as a path gain and take the same impedance for shunt branch elements.

Time Domain Analysis ⇒

Q. Identify $C_{ss}(t)$ & $C_{tr}(t)$. in the following Response.

$$c(t) = \underbrace{5 + 2\sin 3t}_{\uparrow \; C_{ss}(t)} + \underbrace{e^{-10t} + e^{-10t}\cdot t + e^{-5t}\sin 3t}_{\uparrow \; C_{tr}(s)}$$

Q. $\frac{C(S)}{R(S)} = \frac{S+1}{S+2}$, find out the sinusoidal Response for $r(t) = 10\cos(2t + 45^\circ)$.

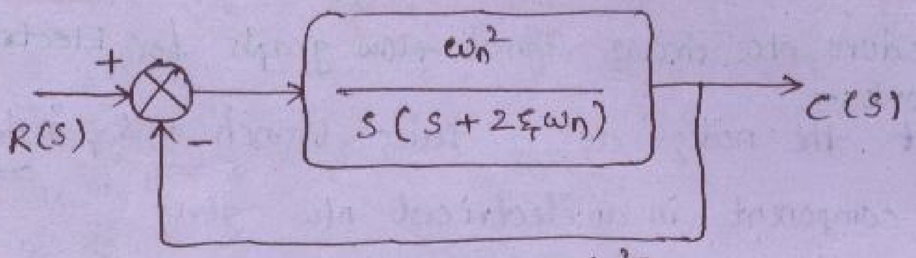
$\omega = 2, \quad S = j\omega = 2j.$

$\Rightarrow \frac{2j+1}{2j+2} \Rightarrow M = \frac{\sqrt{5}}{\sqrt{8}}$

$\phi = \frac{\tan^{-1}(2/1)}{\tan^{-1}(2/2)} = 18.43^\circ.$

$\therefore \; c(t) = 10 \times \sqrt{\frac{5}{8}} \cdot \cos(2t + 45^\circ + 18.43^\circ)$

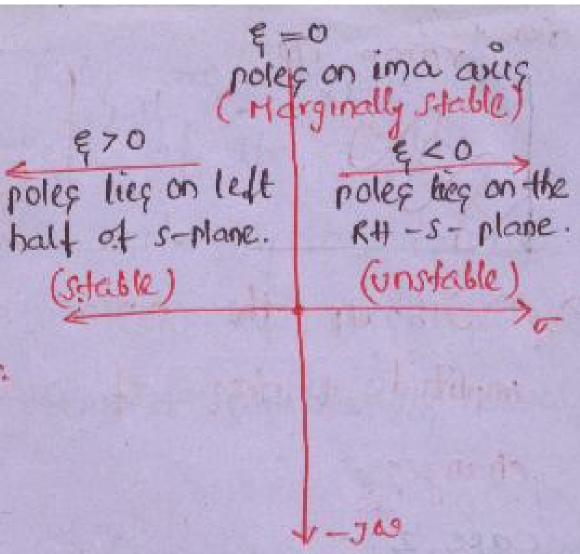
SECOND ORDER SYSTEMS:



$$\frac{C(S)}{R(S)} = \frac{\omega_n^2}{S^2 + 2\xi\omega_n S + \omega_n^2}$$

* The II order system response completely depends on ξ.

* II order system is stable for all +ve of ξ
$0 < \xi \le \infty$.
B'coz for +ve values of ξ, the poles lies on the left half of s-plane.



IMPULSE RESPONSE:

$r(t) = \delta(t)$

$\Rightarrow R(s) = 1.$

case. 1:

Impulse Response :-

$\xi = 0$:- $r(t) = \delta(t)$

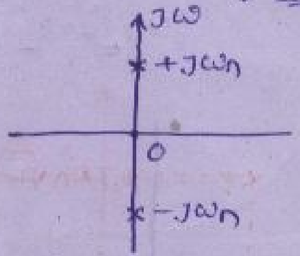
$R(s) = 1.$

$$C(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

$$\Rightarrow C(s) = \frac{\omega_n^2}{s^2 + \omega_n^2}$$

$\Rightarrow c(t) = \omega_n \cdot \sin \omega_n t.$

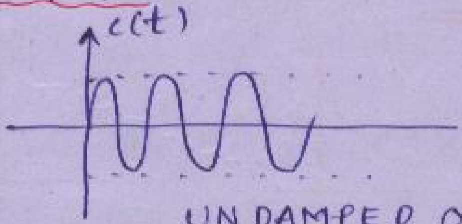
[Non-repeated poles on imaginary axis].
marginally stable



* freq of oscillations = ω_n rad/sec

* $T = -\frac{1}{0}$
$= \infty.$

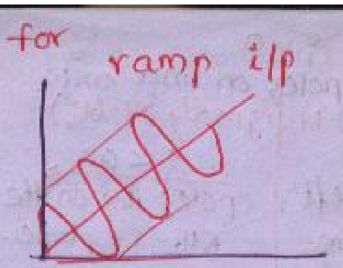
Response:

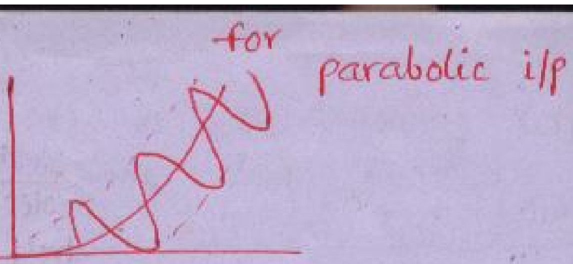


UNDAMPED OSCILLATIONS. $= \omega_n$ rad/sec.

when $\xi = 0$, the poles on the ima axis which are not repeated, the System is M. stable and the system response is const amplitude and freq. of oscillations, which are called undamped oscillations. The System which gives undamped oscillations is called undamped system.

when $\xi = 0$, irrespective of all i/p's, Response is undamp.





for $\xi=0$, with i/p the nature of the system (const. amplitude & freq. of oscillations around i/p not changes.

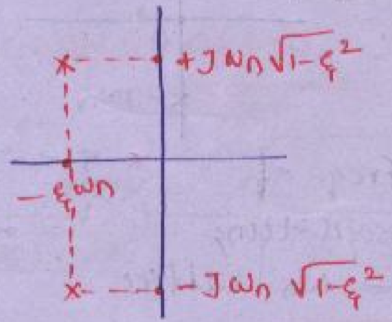
Case 2:-

$0<\xi<1$:-

$$s_1, s_2 = \frac{-2\xi\omega_n \pm \sqrt{4\xi^2\omega_n^2 - 4\omega_n^2}}{2}$$

$$= -\xi\omega_n \pm \omega_n\sqrt{\xi^2-1}, \quad \xi>1$$

$$= -\xi\omega_n \pm j\omega_n\sqrt{1-\xi^2}, \quad \xi<1$$



$\tau = \frac{1}{\xi\omega_n}$

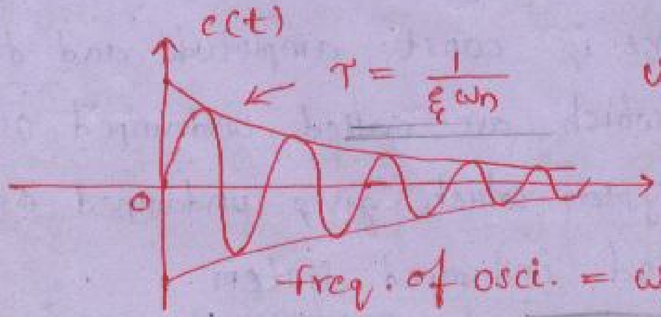
freq. of oscillation $= \omega_n\sqrt{1-\xi^2}$ (ω_d)

stable.

Response:

$$c(s) = \frac{\omega_n^2}{(s+\xi\omega_n - j\omega_n\sqrt{1-\xi^2})(s+\xi\omega_n + j\omega_n\sqrt{1-\xi^2})}$$

$$\Rightarrow c(t) = K.e^{-(\xi\omega_n)t}.\sin(\omega_n\sqrt{1-\xi^2})t$$



under damped system

when $0<\xi<1$, the poles are complex conju. which are at the Left half of the s-plane. The system is stable. The system response is exponential dacay freq of oscillations which are called damped oscillations ie ω_d

$\omega_d = \omega_n\sqrt{1-\xi^2}$.

* A system which produce damped oscillations are called under damped system.

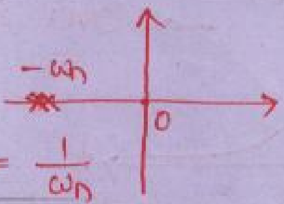
case 3:-

$\xi = 1$:-

$$C(s) = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2} = \frac{\omega_n^2}{s^2+2\omega_n s+\omega_n^2}$$

$$= \frac{\omega_n^2}{(s+\omega_n)^2}$$

$\Rightarrow c(t) = \omega_n^2 . t . e^{-\omega_n t}$

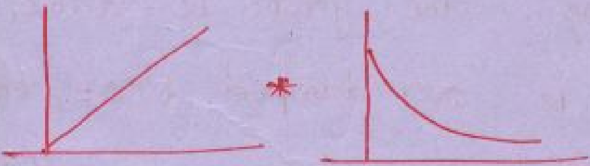


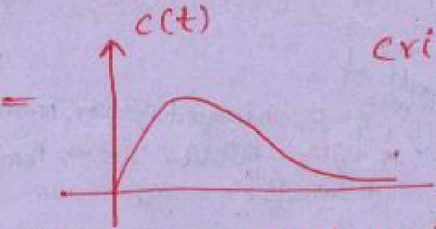
$\tau = \frac{1}{\omega_n}$

freq. of osci. = 0

stable

Response:





critical damped system

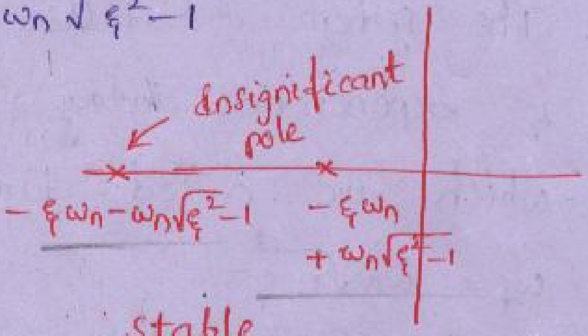
when $\xi = 1$, the pole on the -ve real axis at the same location, the system is stable, the system response is critical damped. b'coz it generates critically or hardly one damped oscillation.

Case 4 :-

$\xi > 1$:-

$$s_1, s_2 = -\xi\omega_n \pm \omega_n\sqrt{\xi^2-1}$$

$$\tau = \frac{1}{\xi\omega_n - \omega_n\sqrt{\xi^2-1}}$$



freq. of oscillations = 0. rad./sec.

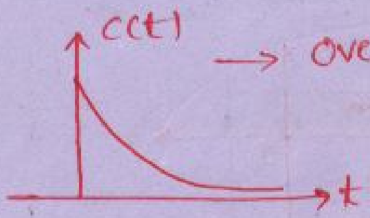
stable

Response:

$$C(s) = \frac{\omega_n^2}{(s + \xi\omega_n - \omega_n\sqrt{\xi^2-1})(s + \xi\omega_n + \omega_n\sqrt{\xi^2-1})}$$

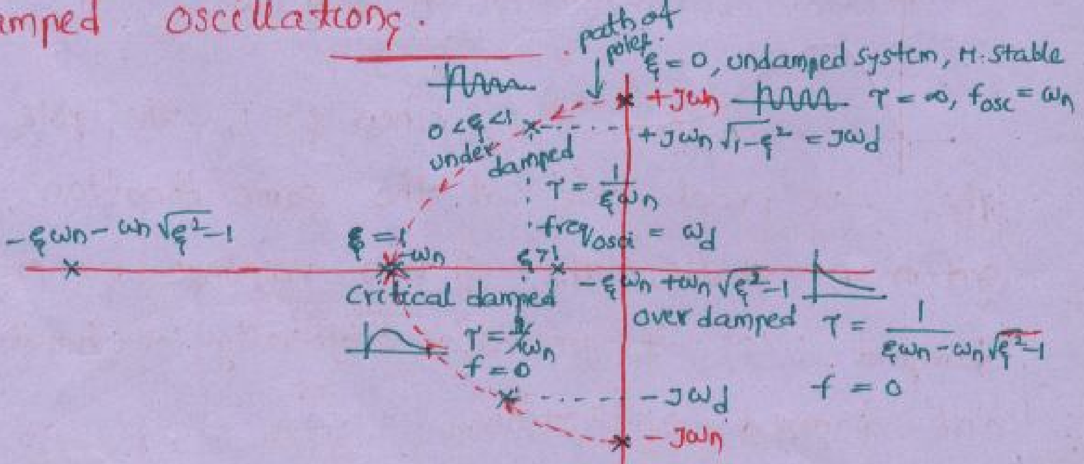
insignificant pole

$$\rightarrow c(t) = K \cdot e^{-(\xi\omega_n - \omega_n\sqrt{\xi^2-1})t}$$



→ over damped.
↳ means over comes

when $\xi > 1$, the poles on the -ve real axis at different locations, the system is stable, the system response is over damped b'coz the system response elemenates or over comes the damped oscillations.



⇒ when ξ increases from 0 to 1, the poles moves towards L.H.S and near to the real axis. Hence the system time constant & freq. of oscillations are decreased. when $\xi \geq 1$ and increases ^one pole moves on -ve real axis towards ima. axis the freq. of oscillations becomes zero b'coz the poles lies on the real axis only. and hence system τ increases, & f_{OO} are reduced to zero. when $\xi > 1$ and increases the system τ increases b'coz one poles moves towards the origin on the real axis.

order of Time constants :

$\tau_{undamped}$ >	$\tau_{overdamped}$ >	$\tau_{underdamped}$ >	$\tau_{critical\ damped}$
(∞)	$\left(\frac{1}{\xi\omega_n - \omega_n\sqrt{\xi^2-1}}\right)$	$\left(\frac{1}{\xi\omega_n}\right)$	$\left(\frac{1}{\omega_n}\right)$
(Marginal stable)	(stable)	(stable)	(stable)
	Largest τ	Medium τ	lowest τ

Large Time const. ⟶ slow response (or) sluggish

Unit Step Response :-

$r(t) = 1 \cdot u(t)$

$R(s) = \frac{1}{s}$

$$C(s) = \frac{\omega_n^2}{s \cdot (s^2 + 2\xi\omega_n s + \omega_n^2)}$$

when $\xi \geq 0$, & $\xi < 1$ $(0 \leq \xi < 1)$ the unit step response of the system

$\cos\theta = \xi$

$\sin\theta = \cos\omega t$

$$c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin\left\{(\omega_n\sqrt{1-\xi^2})t + \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi}\right\}$$

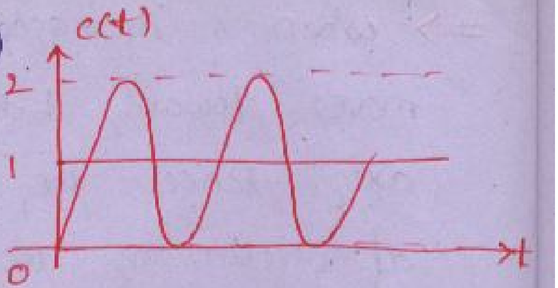
or $\cos^{-1}\xi$

Case . 1 :-

$\xi = 0$: $c(t) = 1 - \sin(\omega_n t + \pi/2)$

$\Rightarrow c(t) = 1 - \cos(\omega_n t)$

Undamped, Marginally stable.



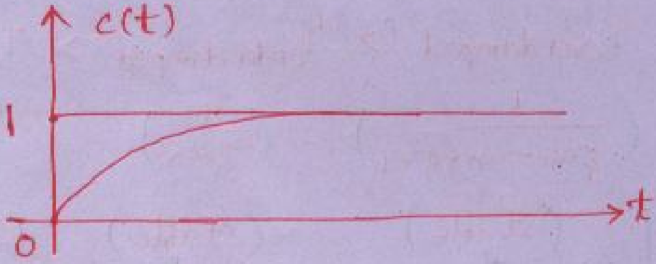
Case. 3

$\xi = 1$:-

$$C(s) = \frac{\omega_n^2}{s(s+\omega_n)^2}$$

$$= \frac{1}{s} - \frac{\omega_n}{(s+\omega_n)^2} - \frac{1}{(s+\omega_n)}$$

$$\Rightarrow c(t) = (1 - \omega_n \cdot t \cdot e^{-\omega_n t} - e^{-\omega_n t}).u(t)$$



Case. 4

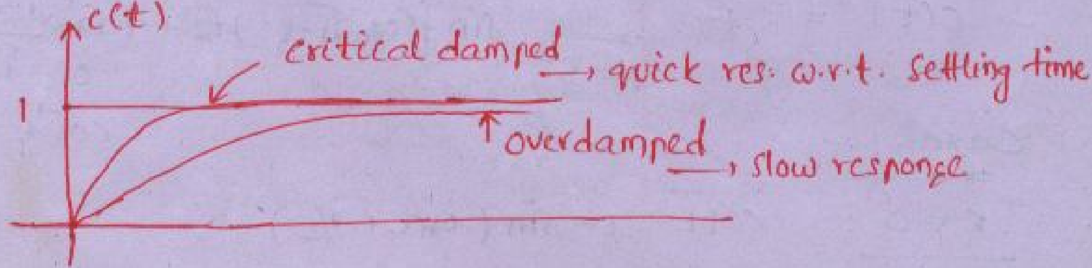
$\xi > 1$:-

Insignificant pole

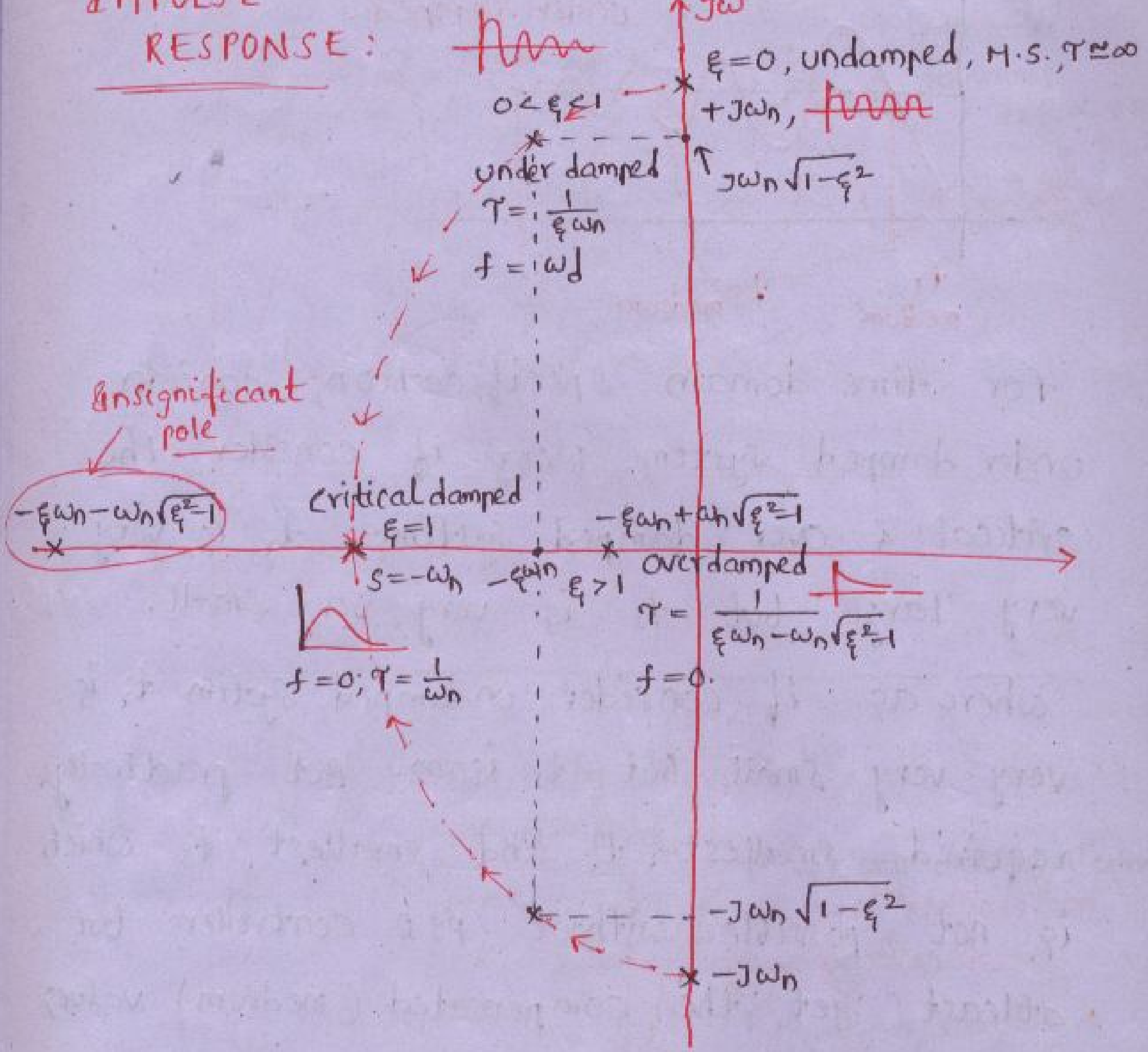
$$C(s) = \frac{\omega_n^2}{s(s+\xi\omega_n - \omega_n\sqrt{\xi^2-1})(\underline{s+\xi\omega_n + \omega_n\sqrt{\xi^2-1}})}$$

$$= \frac{1}{s} - \frac{K}{s+\xi\omega_n - \omega_n\sqrt{\xi^2-1}}$$

$$\Rightarrow c(t) = (1 - K \cdot e^{-(\xi\omega_n - \omega_n\sqrt{\xi^2-1})t}.)\,u(t)$$

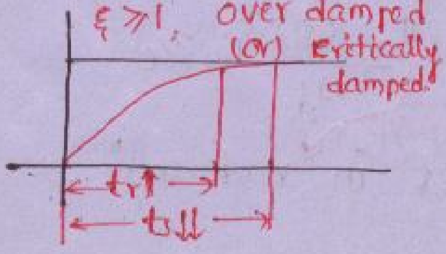


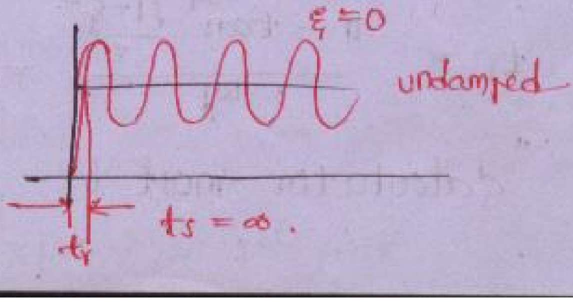
IMPULSE RESPONSE:

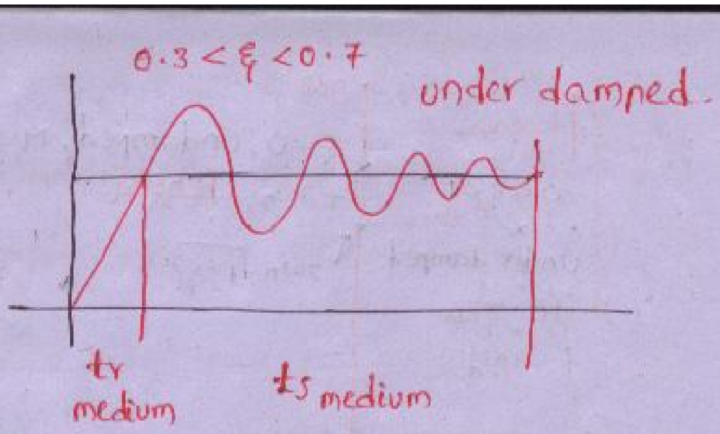


TIME DOMAIN SPECIFICATIONS:

Test signals	Tr.res.	SS.res.	Stability	
Impulse	✓	×	✓	→ practically not exist
Step	✓	✓	✓	→ BOUNDED widely used
Ramp	✓	✓	×	UNBOUNDED
parabolic	✓	✓	×	







for time domain specifications consider under damped system. b'coz if consider the critical & over damped systems, t_r is very very large but t_s is very very small.

where as if consider undamped system t_r is very very small but t_s is ∞. But practically required smallest t_r and smallest t_s which is not possible without P&D controllers but atleast get the compensated (medium) values of t_r & t_s are possible when selected ξ 0.3 to 0.7.

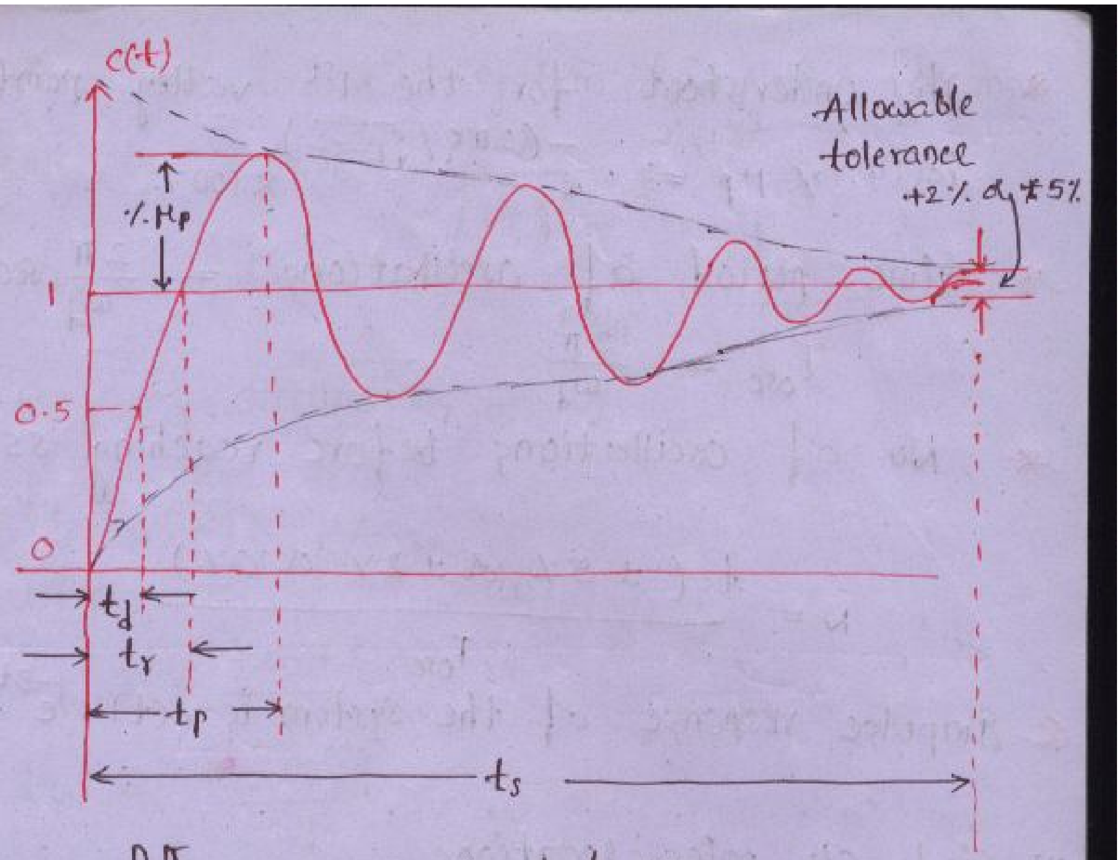
* when $\xi > 0$ & $\xi < 1$ the unit step res. of the system is

$$c(t) = 1 - \frac{e^{-\xi \omega_n t}}{\sqrt{1-\xi^2}} \cdot \sin\left[(\omega_n\sqrt{1-\xi^2})t + \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi}\right]$$

* $t_d = \dfrac{1+0.7\xi}{\omega_n}$ sec

* $t_r = \dfrac{\pi - \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi}}{\omega_d} = \dfrac{\pi - \cos^{-1}\xi}{\omega_d}$ sec

"calculator must be set in radians".



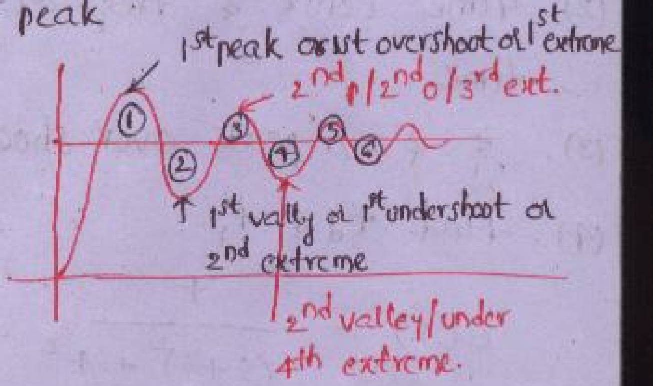
* $t_p = \frac{n\pi}{\omega_d}$; n = 1 by default

1st peak

$= \frac{\pi}{\omega_d}$, 1st peak

$= \frac{3\pi}{\omega_d}$, 2nd peak

$= \frac{2\pi}{\omega_d}$, 1st valley



* $t_s = 3\tau = \frac{3}{\xi\omega_n} \longrightarrow \pm 5\%$

$t_s = 4\tau = \frac{4}{\xi\omega_n} \longrightarrow \pm 2\%$

$t_s = 5\tau = \frac{5}{\xi\omega_n} \longrightarrow 0\%$

* $\% M_p = \frac{c(t_p) - c(\infty)}{c(\infty)} \times 100\%$

$= [c(t_p) - 1] \times 100\%$

$= e^{-(n\pi\xi/\sqrt{1-\xi^2})} \times 100\%$

$= e^{-(\pi\xi/\sqrt{1-\xi^2})} \times 100\%$, 1st peak.

* The undershoot for the 1st valley point is $\% M_p = e^{-(2\pi\xi/\sqrt{1-\xi^2})} \times 100\%$

* Time period of oscillations $= \frac{2\pi}{\omega_d}$ sec.

$$T_{osc} = \frac{2\pi}{\omega_d}$$

* No. of oscillations before reaching ss is

$$N = \frac{t_s(\pm 5\% \text{ or } \pm 2\% \text{ or } 0\%)}{T_{osc}}$$

Q. Impulse response of the system is $c(t) = e^{-3t} \cdot \sin 4t$

(1). Find C/L poles locations

(2). Time const. & freq. of oscillations ie ω_d, peak time

(3). ξ & % peak over shoot.

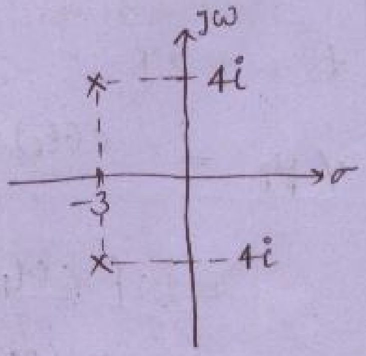
(4). Find t_d & t_r.

$$C(s) = \frac{4}{(s+3)^2 + 4^2}$$

$$= \frac{4}{s^2+6s+25}$$

$\rightarrow \omega_n = 5; \quad 2\xi(5) = 6$

$\rightarrow \xi = 3/5$.

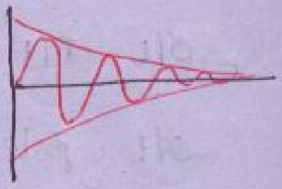


C/L poles:

$$s^2+6s+25 = 0$$

$$\Rightarrow s = -3 \pm 4i$$

for under damped system, impulse response will be $c(t) = \frac{\omega_n}{\sqrt{1-\xi^2}} \cdot e^{-\xi\omega_n t} \sin \omega_d t$



Time constant $\tau = \frac{1}{\xi\omega_n}$.

freq. of oscillations $= \omega_d$

$$t_p = \frac{\pi}{\omega_d}$$

$$\% M_p = e^{-\pi\xi/\sqrt{1-\xi^2}} \times 100\%$$

$$t_d = \frac{1+0.7\xi}{\omega_n}$$

$$t_r = \frac{\pi - \cos^{-1}\xi}{\omega_d}$$

ROOT LOCUS:

Relationship b/w o/L T/F poles & zeros with c/L T/F poles:

$$o/L\ T/F = G(s).H(s) = k.\frac{N(s)}{D(s)}.$$

o/L poles $D(s) = 0$ —①

o/L zeros $N(s) = 0$ —②

A c/L system stabelety is given by char. eq.

$$1 + G(s).H(s) = 0.$$

$$1 + k:\frac{N(s)}{D(s)} = 0$$

→ char. eq. ⇒ c/L T/F poles:

$$D(s) + k\,N(s) = 0.$$

The c/L poles are nothing but sum of o/L poles and o/L zero's with the fun. of system gain k.

Case (1):

$k = 0;$

$$k = \left| -\frac{D(s)}{N(s)} \right| = 0.$$

c/L poles $D(s) = 0.$

when $k = 0$, o/L poles = c/L poles.

case (2):

$k = \infty;$ c/L pole = $N(s) = 0$

when $k = \infty$, c/L poles = oL zero's.

The RL diagram start at o/L pole, where $k = 0$ and end at o/L zeros where $k = \infty$.

Q. find where RL diagram starts & ends.

for $G(s).H(s) = \dfrac{K(s+1)}{s(s+3)(s+5)}$

Starts at OL pole $(K=0) \Rightarrow 0, -3, -5$.

Ends at OL zeros $(K=\infty) \Rightarrow -1, \underbrace{\infty, \infty}_{\text{Along Asympto.}}$

Q. Check whether the following points lies on RL or not ?

$$G(s).H(s) = \frac{k}{s(s+3)(s+5)}$$

(i). $s = -2$ (ii). $s = -4$.

$$\angle GH\Big|_{s=-2} = \frac{\angle k}{\angle s \ \angle(s+3) \ \angle(s+5)}$$

$$= \frac{\angle k}{\angle -2 . \angle +1 . \angle +3}$$

$$= \frac{0^\circ}{\pm 180^\circ . 0^\circ . 0^\circ} = \pm 180^\circ .$$

∴ Satisfies Angle condition, so $s=-2$ lies on RL.

(ii). $\angle GH\Big|_{s=-4} = \dfrac{\angle k}{\angle -4 . \angle -1 . \angle 1}$

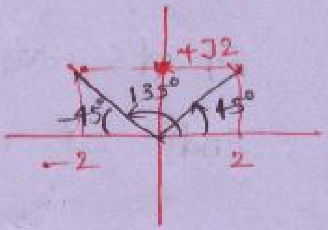
$$= \frac{0^\circ}{\pm 180 . \pm 180^\circ . 0^\circ}$$

$$= \mp 360^\circ \longrightarrow \text{Not Satisfy}.$$

So $s=-4$ is not lie on RL.

Q. $G(s).H(s) = \frac{k}{s(s+4)}$.

(i). $s = -2 + J2$ find system gain at given point.



$$\angle GH = \frac{\angle k}{\angle(-2+J2).\angle(2+J2)}$$

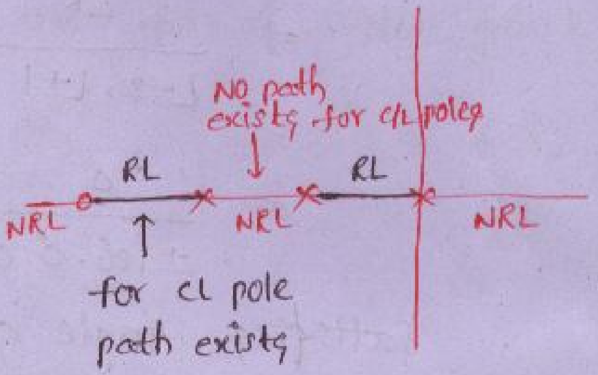
$$= \frac{0}{135^\circ . 45^\circ} = -180^\circ$$, satisfy. AC.

$$MC := \left|\frac{k}{s(s+4)}\right| = 1$$

$$\Longrightarrow \left|\frac{k}{(-2+J2)(2+J2)}\right| = 1$$

$$\longrightarrow \underline{\underline{k = 8}} \qquad \frac{k}{\sqrt{8}.\sqrt{8}} = 1.$$

③. Real axis loci :-



Q. Identify which are on RL.

$$GH = \frac{k(s^2+2s+2)}{s^2(s+2)(s+4)(s+6)}$$

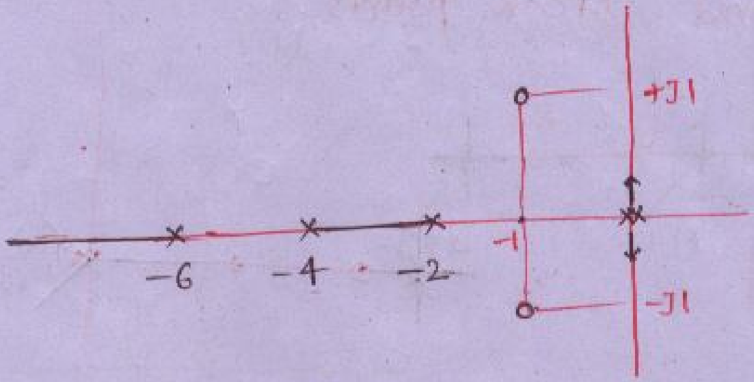
(1). $s \overset{\checkmark}{=} 0$ (2). $s \overset{\times}{=} -1$, (3). $s \overset{\checkmark}{=} -2$

(4). $s \overset{\checkmark}{=} -3$ (5). $s \overset{\checkmark}{=} -4$, (6). $s \overset{\checkmark}{=} -6$

(7). $s \overset{\checkmark}{=} -\infty$ (8). $s = \overset{\checkmark}{-1+J1}$ (9). $s = \overset{\checkmark}{-1-J1}$

(10): $s \overset{\times}{=} -5$

$$GH = \frac{K(s^2+2s+2)}{s^2(s+2)(s+4)(s+6)} \quad s = -1 \pm j$$



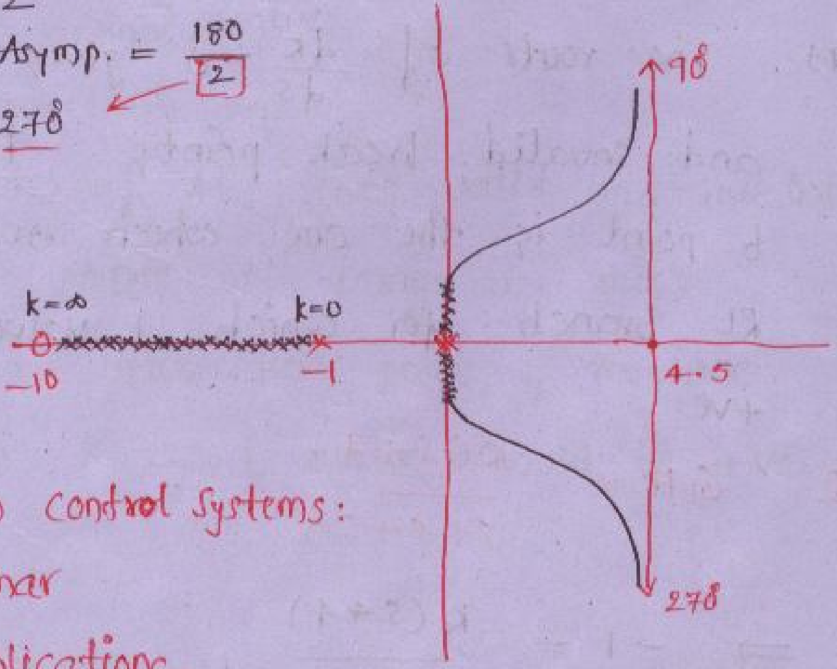
At the position of all poles & zeros, there must be a root locus branch b'coz RL branch start at o/L poles and ends at o/L zeros.

Q. Draw RL $GH = \frac{K(s+10)}{s^2(s+1)}$

$$\sigma = \frac{-1-(-10)}{2} = 4.5.$$

Angle of Asymp. $= \frac{180}{\boxed{2}}$

$= 90°, 270°$



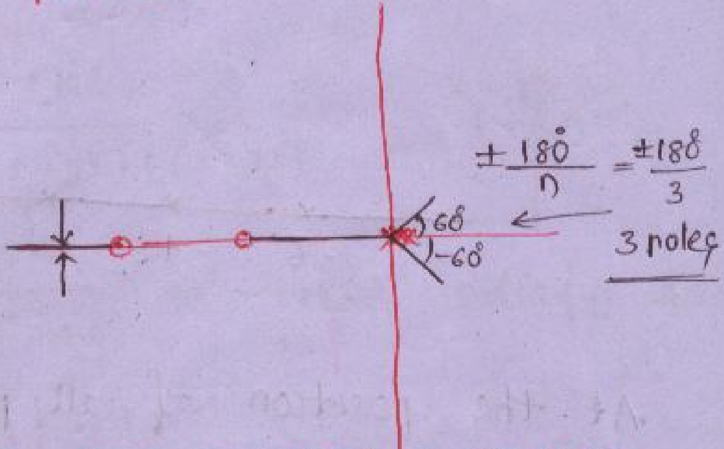
* Problems in Control Systems:

By, Ashok kumar

Sigma publications.

Q $GH = \frac{K(s+2)(s+4)}{s^3}$.

find Break points.



PROCEDURE FOR DETERMINING B. POINTS:-

(1). $G(s)H(s)$ replace by -1

(2). Re arrange above eq. in the form of $K = f(s)$.

(3). defferentiate k w.r.t s and make it equal to zero.

(4). The roots of $\frac{dk}{ds} = 0$ gives the valid and invalid break points. The valid b. point is the one which must be on RL branch for which k value should be +ve.

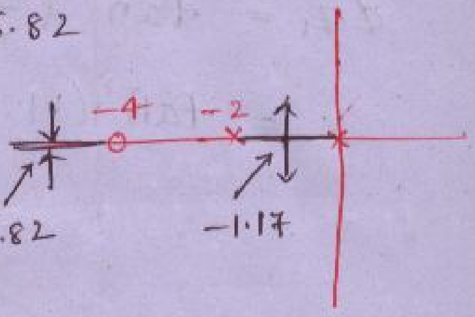
Q $GH = \frac{K(s+4)}{s(s+2)}$

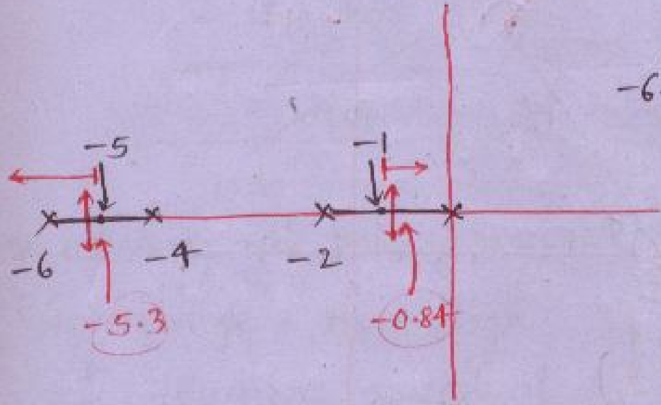
$$\Rightarrow \quad -1 = \frac{K(s+4)}{s(s+2)}$$

$$\Rightarrow \quad K = \frac{-s^2-2s}{s+4}$$

$$\Rightarrow \frac{dk}{ds} = \frac{(-2s-2)(s+4)+s^2+2s}{(s+4)^2} = 0$$

$$= -1.17, -6.82$$





PROCEDURE TO FINDOUT INTERSECTION POINT ON IMAGINARY AXIS:

(1). form Char. eq.

(2). write Routh tabular form

(3). find $k_{marginal}$ value.

(4). form Auxiliary eq.

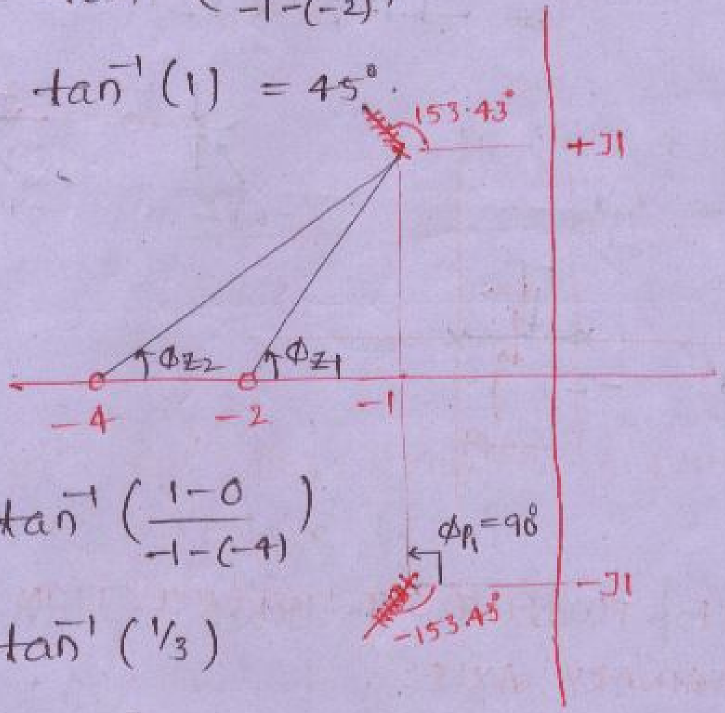
The roots of AE, gives valid and invalid intersection points with imaginary axis.

The valid intersection point is the one for which $k_{marginal}$ value should be +ve.

$$G(s)H(s) = \frac{K(s+2)(s+4)}{(s^2+2s+2)}$$

$$\phi z_1 = \tan^{-1}\left(\frac{1-0}{-1-(-2)}\right)$$

$$= \tan^{-1}(1) = 45^\circ.$$



$$\phi_{Z_2} = \tan^{-1}\left(\frac{1-0}{-1-(-4)}\right)$$

$$= \tan^{-1}(1/3)$$

$$= 18.43^\circ$$

$$\therefore \phi_{\pm} = \phi_{P_1} - (\phi_{Z_1} + \phi_{Z_2})$$

$$= 90 - (45 + 18.43)$$

$$= 26.57^\circ.$$

$$\phi_d = 180^\circ - \phi$$

$$= 153.43^\circ.$$

$$G(s).H(s) = \frac{K(s^2+2s+2)}{(s+2)(s+4)}$$

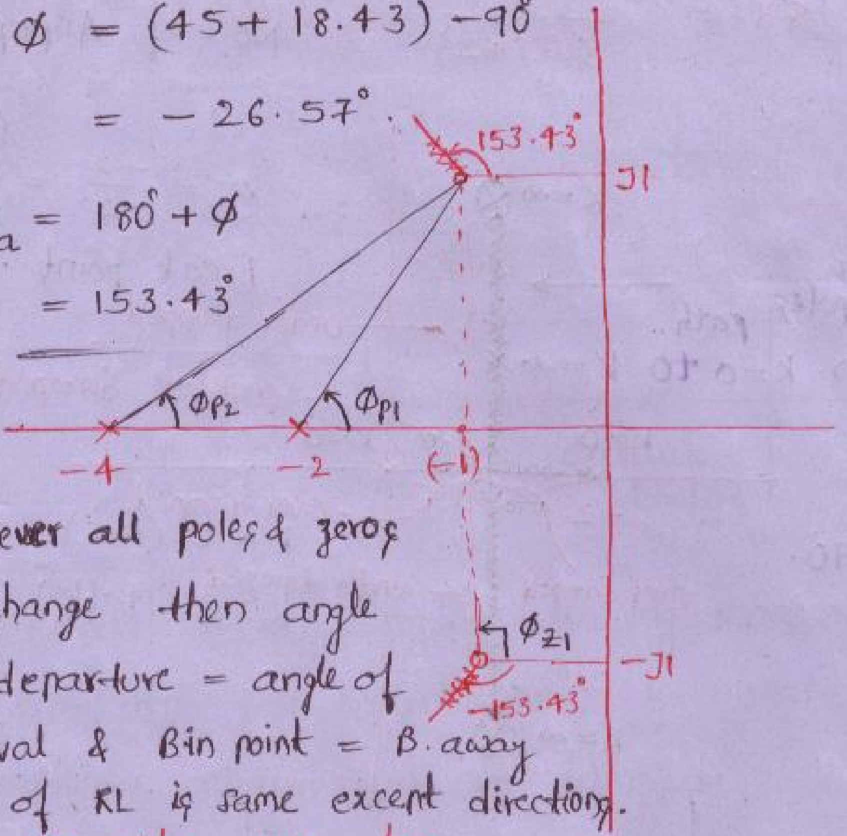
$$\phi_{P_1} = 45^\circ$$

$$\phi_{P_2} = 18.43^\circ.$$

$$\phi_{Z_1} = 90^\circ.$$

$$\therefore \phi = (\phi_{P_1} + \phi_{P_2}) - \phi_{Z_1}$$

$$\Rightarrow \phi = (45 + 18.43) - 90^\circ$$
$$= -26.57^\circ.$$
$$\therefore \phi_a = 180^\circ + \phi$$
$$= 153.43^\circ$$



* whenever all poles & zeros interchange then angle of departure = angle of arraival & Bin point = B. away shape of RL is same except direction.

Q. Draw the RL for,

① $GH = \frac{k}{s(s+2)}$ ② $\frac{k}{s(s^2+2s+2)}$

③ $\frac{ks}{s^2+4}$ ④ $\frac{k(s+2)(s+4)}{(s^2+2s+2)}$

⑤ $\frac{k(s^2+2s+2)}{(s+2)(s+4)}$ ⑥ $\frac{k}{s}, \frac{k}{s^2}, \frac{k}{s^3}, \frac{k}{s^4}$

⑦ $\frac{k}{s(s+1)^2(s+2)}$ ⑧ $\frac{k(s+1)^2}{s(s+2)}$

⑨ $\frac{k}{s(s+k_1)(s^2+2s+2)}$ ⑩ $\frac{k(s+1)}{s^2(s+k_1)}$

$k_1 > 2, \; k_1 < 2, \; k_1 = 2$

① . $GH = \frac{K}{s(s+2)}$

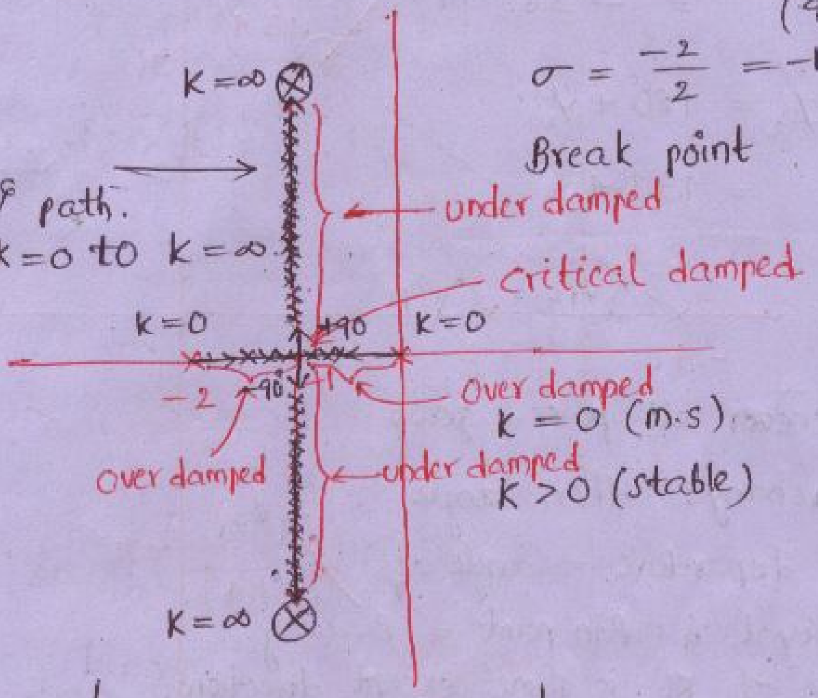
No. of Asymp. = P−Z
= 2
(90°, 270°)

$\sigma = \frac{-2}{2} = -1.$

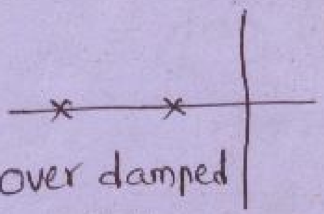
Break point

C/L poles path.
from k=0 to k=∞.

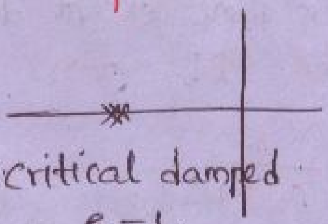
$\frac{\pm 180^\circ}{2}$
$= \pm 90^\circ.$



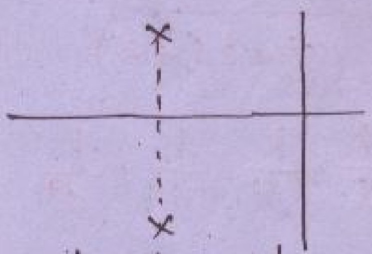
K = 0 (m.s).
K > 0 (stable)



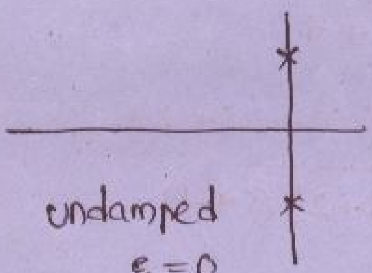
over damped
ξ>1



critical damped
ξ=1.



under damped
0<ξ<1



undamped
ξ=0

find K value at for Break point ? by using magnitude condition.

$$\left|\frac{K}{s(s+2)}\right|_{s=-1} = 1$$

$\Rightarrow K = 1.$

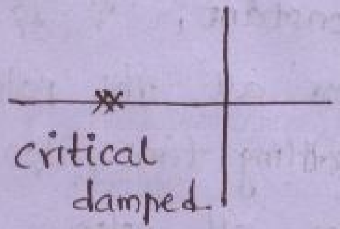
0<K<1, overdamped
K=1, critical
K>1, under damped

for k = 1,

$$GH = \frac{1}{s(s+2)}$$

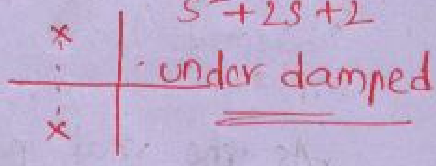
$$\rightarrow \text{C/L T/F } \frac{C(s)}{R(s)} = \frac{1}{s^2+2s+1}$$

$$= \frac{1}{(s+1)^2}$$



CHECKING for k = 2,

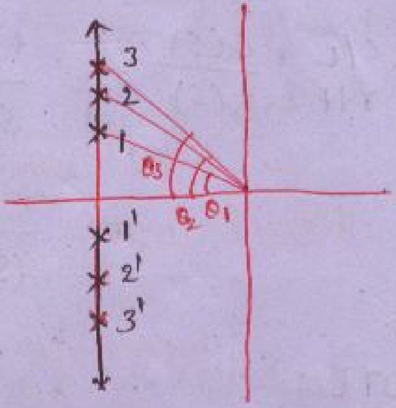
$$\text{C/L T/F} = \frac{2}{s^2+2s+2}$$



NOTE :

- * whenever a RL diagram having more than or equal to two real axis RL branches, then the system is over damped.
- * whenever RL diagram having b. points, the should have the critical damped nature.
- * whenever RL diagram having break away or break in then the system should have the under damped nature.
- * whenever the system having angle of departure or angle of arraival or angle of asymptotes direction towards imaginary axis, the system should have undamped nature.

Q. find the variations in time domain specifications to the given poles path in the s-plane.



As the real part is constant, σ is constant for all the poles hence settling time also same for all poles.

As damped freq. of oscillations increases, time specifications t_r, t_d, t_p must be decreases.

As the inclination of poles increases the damping factor 'ξ' decreases. ($\xi = \cos\theta$)

As damping factor ξ decreases, the % Mp increases ∴ System becomes less stable.

As $\xi \downarrow\downarrow \longrightarrow \% M_p \uparrow\uparrow$.

The optimum value of % Mp is 5% to 40% w.r.t t_s

If it is > 40% ⟶ unstable (less stable)

< 5% ⟶ response becomes slow

w.r.t. t_r. ~~t_s~~

Q. find % Mp for a given ck t/f.

$$\frac{G(s)}{R(s)} = \frac{25}{s^2+25}$$

$\xi = 0 \longrightarrow$ undamped.

$$\% M_p = e^{-\pi\xi/\sqrt{1-\xi^2}} \times 100\%$$

$$= 100\%$$

Q. for $C(s) = \frac{100}{s^2+20s+100}$

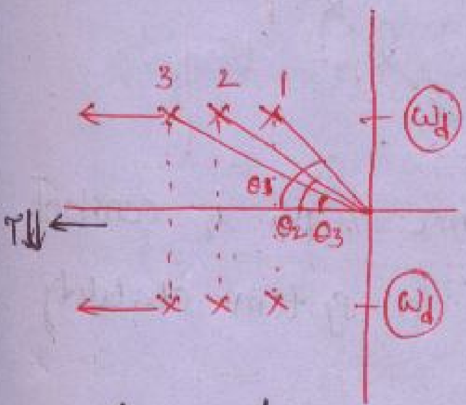
$\omega_n = 10, \quad \xi = 1. \longrightarrow$ critical damped

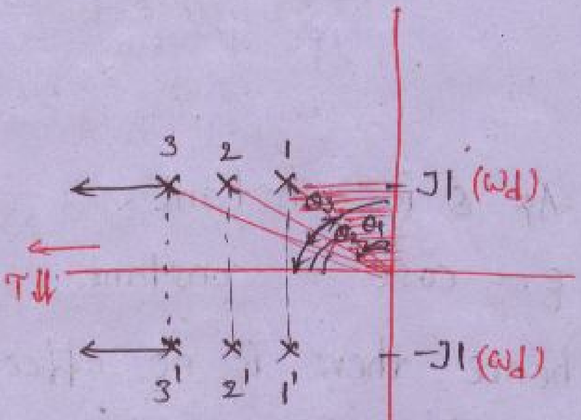
$\% M_p = e^{-\pi\xi/0} \times 100 \%$

$= 0 \%$

As ξ increases, % of M_p decreases. and system becomes more stable.

when $\xi \geq 1$, and increases the % M_p is 0%.





As poles moves towards ∞,

τ decreases & also ts decreases.

As ima. part is constant, the peak time is const. but there exists a slight variations in delay & rise time.

ω_d — const.

$t_p = \frac{\pi}{\omega_d} \rightarrow$ const.

But $t_d = \frac{1+0.7\xi}{\omega_n}$

As inclination of the poles decreases then ξ increases. $\underline{\xi = \cos\theta}$

Hence % Mp decreases → system becomes more stable.

θ → ξ

ξ → % Mp

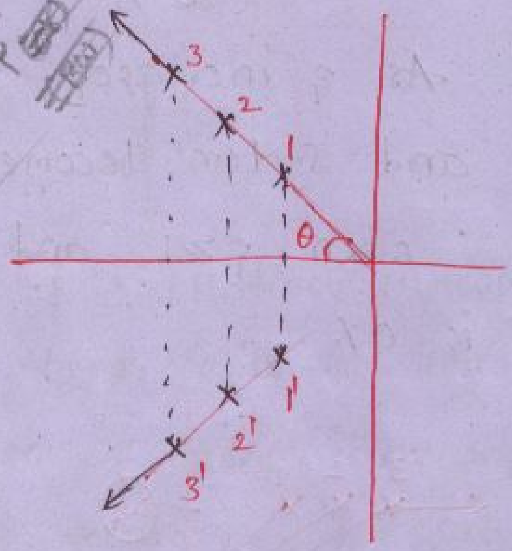
stability → % Mp td

ωd → tr, tp

tr → BW

τ → ts

speed of res. → τ



As θ is constant.

$\xi = \cos\theta$ = constant. hence % Mp is constant.

hence there is no effect on system stability.

ω_d increases,

t_d, t_r, t_p ~~increases~~.

As poles moves to left, τ ↓ ts ↓.

⟹ ts depends only on τ

τ depends on real part of poles.

tp depends only on ω_d

ω_d depends on ima. part

% Mp depends on ξ

ξ depends inclination of poles. (θ).

Q. find time domain specifications for unity f/b system.

$$G(s) = \frac{25}{s(s+4)}.$$

$$\frac{C(s)}{R(s)} = \frac{25}{s^2+4s+25}$$

$\omega_n = 5.$

$\xi = 0.4$

$\omega_d = \omega_n\sqrt{1-\xi^2} = 4.5 \text{ rad/sec}$

$$t_d = \frac{1+0.7\xi}{\omega_n}$$

$= 0.256 \text{ sec}$

$$t_r = \frac{\pi - \cos^{-1}\xi}{\omega_d}$$

$= 0.44 \text{ sec}$

$t_p =$

$= 0.698 \text{ sec}$

$\% M_p =$

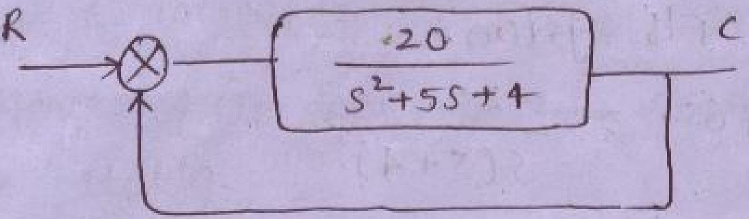
$= 25.8\%$

$t_s =$

$= 2 \text{ secs } (\pm 2\%)$

$$c(t) = 1 - \frac{e^{-4t}}{\sqrt{1-0.16}} \sin(4.5t + \cos^{-1}0.4)$$

Q. find



$$\frac{C}{R} = \frac{20}{s^2+5s+24}$$

$$= \underset{\uparrow}{\frac{20}{24}} \cdot \left[\frac{24}{s^2+5s+24} \cdot \right]$$

Affect the steady state value but not any time domain specifications.

$\omega_n = 4.89$ rad/sec

$\xi = 0.511$

$\omega_d = 4.2$ rad/sec

$t_d = 0.277$ sec

$t_r = 0.501$ sec

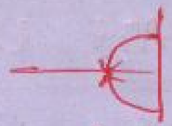
$t_p = 0.745$ sec

$t_s = 1.6$ sec $(\pm 2\%)$

$\% M_p = 15.5\%$

$$c(t) = \frac{20}{24}\left(1 - \frac{e^{-2.5t}}{\sqrt{1-0.511^2}} \sin(4.2t + \cos^{-1} 0.511)\right)$$

As $\xi \uparrow\uparrow$



→ $\tau \downarrow\downarrow \rightarrow t_s \downarrow\downarrow$

→ $\omega_d \downarrow\downarrow \rightarrow t_d, t_r, t_p \uparrow\uparrow$

→ $\% M_p \downarrow\downarrow$

As ξ increases poles moves towards left and near to real axis, hence

(1). damped freq. of oscillations $\omega_d \downarrow\downarrow$, decreases hence time specifications t_d, t_r, t_p increases.

(2). As poles moves towards left, $\tau \downarrow$ and $t_s \downarrow$.

(3). As $\xi \uparrow$, the $\% M_p$ decreases it shows that the system becomes more stable.

Q. find time domain specifications,

$$\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 8y = 8x$$

where $y \rightarrow$ o/p
$x \rightarrow$ i/p.

$$\frac{Y(s)}{X(s)} = \frac{8}{s^2+4s+8}$$

STEADY STATE ERROR

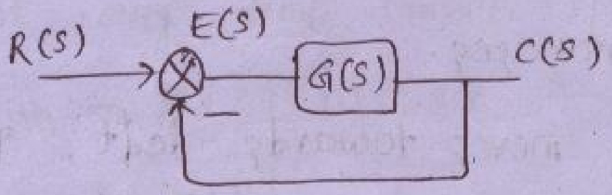
Error is nothing but deviation of the o/p from the reference i/p.

Error at t → ∞ is called ss error.

$$e_{ss} = \underset{t\to\infty}{\text{lt}} \; e(t)$$

$$= \underset{s\to 0}{\text{lt}} \; s \cdot E(s)$$

$$= \underset{s\to 0}{\text{lt}} \; s \cdot \frac{R(s)}{1+G(s)}$$



$$\frac{E(s)}{R(s)} = \frac{1}{1+G(s)}$$

$$\Rightarrow E(s) = \frac{R(s)}{1+G(s)}$$

ss errors depends on two factors,

1. type of i/p (R(s))
2. Type of system (G(s))

* ss errors are valid for unity F/b systems only

* ss errors calculated to c/l stable systems only

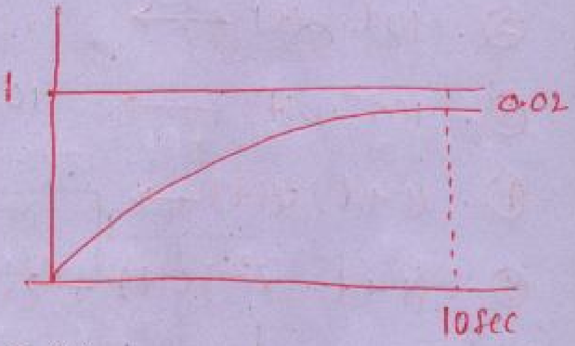
* ss. errors are calculated to c/L system by using o/L T/F.

we required to calc. ss error to only 3 cases.

(1). Type 0 – step i/p
(2). Type 1 – ramp i/p
(3). Type 2 – parabolic i/p.

Q. The unit step response of the system is shown in fig.

(1). find τ.
(2). find t_d, t_r.
(3). find t_p, $\% M_p$



Tolerance is $\pm 2\% = 0.02$

$t_s = 4\tau$

$\rightarrow 10 = 4\tau$

$\Rightarrow \tau = 2.5$ sec.

$c(t) = k.(1-e^{-t/\tau})$

At $t = t_d$, $c(t) = 0.5$.

$\Rightarrow 0.5 = 1.(1-e^{-t_d/2.5})$

$\Rightarrow t_d = 1.78$ sec.

$t_r \longrightarrow 10\%$ to 90%

$t_r = 2.2\tau$ —— (?)

$\rightarrow t_r = 5.5$ sec.

The given response not consists the peak, hence no peak time & peak over shoot.

find steady state error for

$$G(s) = \frac{s+1}{s^2(s+5)(s+10)}$$

①. $10\,u(t)$. $\longrightarrow$

②. $10t \cdot u(t) \longrightarrow$

③. $10t^2 \cdot u(t) \longrightarrow \quad 10\times 2 \cdot \frac{t^2}{2} = 20 \cdot \frac{t^2}{2} u(t)$

$$\downarrow \frac{A}{K} = \frac{20}{1/50} = 1000$$

④. $(1+t)\,u(t) \longrightarrow$

⑤. $(1+t+t^2)\,u(t) \longrightarrow$

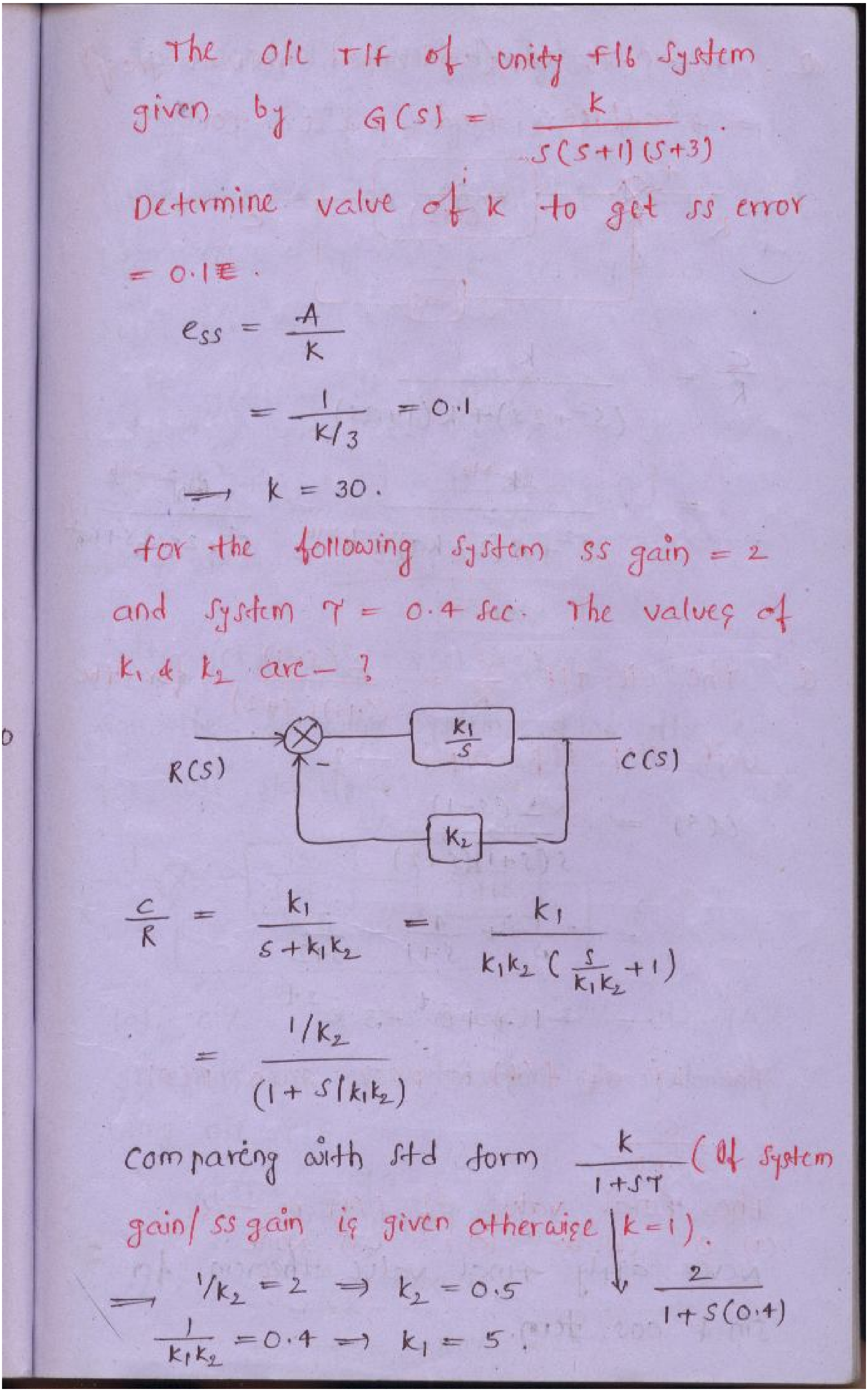
$$\downarrow \quad 2 \cdot \frac{t^2}{2} \qquad e_{ss} = 0+0+\frac{2}{1/50} = 100$$

Q. $$G(s) = \frac{1}{s^2(s+2)(s+10)}$$

char. eq $\Rightarrow s^4 + 15s^3 + 50s^2 + 1 = 0$

In char eq, s term is missing and so the system is unstable.

SS error is valid only for c/L stable system.



The o/l T/F of unity f/b system given by $G(s) = \frac{k}{s(s+1)(s+3)}$.

Determine value of k to get ss error = 0.1E.

$$e_{ss} = \frac{A}{K}$$

$$= \frac{1}{K/3} = 0.1$$

$$\Longrightarrow k = 30.$$

for the following system ss gain = 2 and system τ = 0.4 sec. The values of k_1 & k_2 are— ?

$$\frac{C}{R} = \frac{k_1}{s + k_1k_2} = \frac{k_1}{k_1k_2\left(\frac{s}{k_1k_2} + 1\right)}$$

$$= \frac{1/k_2}{(1 + s/k_1k_2)}$$

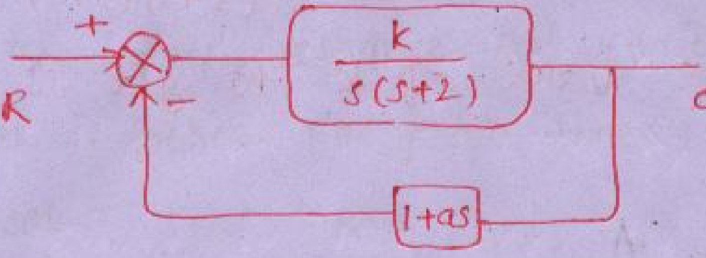
Comparing with std form $\frac{k}{1+s\tau}$ (If system gain/ ss gain is given otherwise $k=1$).

$$\downarrow \quad \frac{2}{1+s(0.4)}$$

$$\Longrightarrow \quad 1/k_2 = 2 \Rightarrow k_2 = 0.5$$

$$\frac{1}{k_1k_2} = 0.4 \Rightarrow k_1 = 5.$$

Q. $\xi = 0.7$, ω_n (undamped natural freq) = 4 rad/sec. find k & a for



$$\frac{C}{R} = \frac{k}{(s^2+2s)+k(1+as)}$$

$$= \frac{\cancel{k}\rightarrow 16}{s^2+s(2+ka)+\cancel{k}\rightarrow 16} = \frac{\cancel{\omega_n^2}\rightarrow 16}{s^2+2\xi\omega_n s+\cancel{\omega_n^2}\,16}$$

Q. The c/c T/F $\frac{C}{R} = \frac{2(s-1)}{(s+1)(s+2)}$ for the unit step i/p, o/p is —?

$$C(s) = \frac{2(s-1)}{s(s+1)(s+2)}$$

$$= -\frac{1}{s} + \frac{4}{s+1} - \frac{3}{s+2}$$

$$= -1 + 4e^{-t} - 3.e^{-2t}$$

The LT of f(t) is F(s). Given F(s)

$$= \frac{\omega}{s^2+\omega^2}$$

The final value of f(t) is —?

(a). 0 (b). ∞ (c). 1. (d). None

Never apply final value theorem for sin & cos term.

Q. The control described by,

$$\frac{d^2y}{dt^2} + 10\cdot\frac{dy}{dt} + 5y = 12(1-e^{-2t}).$$ The

response of the system at t → ∞ is ?

(a). $y = 2$ (b). $y = 6$ (c). $y = 2.5$

(d). $y = -2$

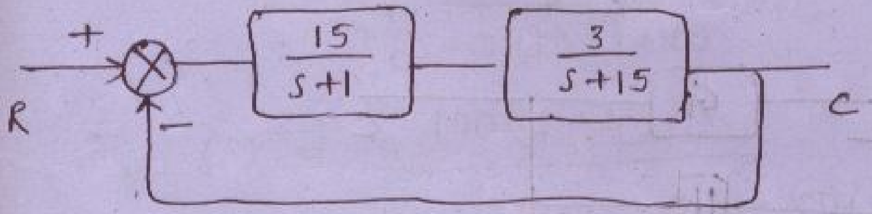
$$\underset{t\to\infty}{lt}\ y(t) = \underset{s\to 0}{lt}\ s.Y(s).$$

$$Y(s)\left[s^2+10s+5\right] = 12\left[\frac{1}{s} - \frac{1}{s+2}\right]$$

$$\Rightarrow Y(s) = \frac{24}{s(s+2)(s^2+10s+5)}$$

$$\underset{t\to\infty}{lt}\ y(t) = \underset{s\to 0}{lt}\ s.Y(s) = 2.4.$$

Q. For the following system, find the e_{ss} for unit step i/p –?



(a). 6%. (b). 25%. (c). 33%. (d). 75%.

SS errors are evaluated for c/l system by using o/l T/F.

$$G(s) = \frac{45}{(s+1)(s+15)}$$

$$\%\ e_{ss} = \frac{A}{1+k} \times 100\%$$

$$= \frac{1}{1+45/15} \times 100\% = 25\%.$$

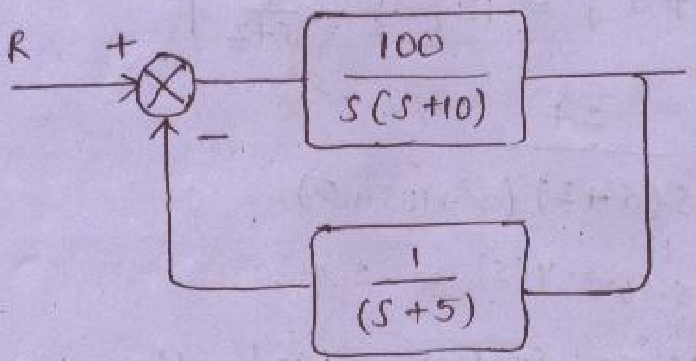
Q. The o/L T/F of unity f/b system is G(s). $e_{ss} = 0$ for

(a). step i/p, Type-0 (b).. Ramp i/p - Type-0

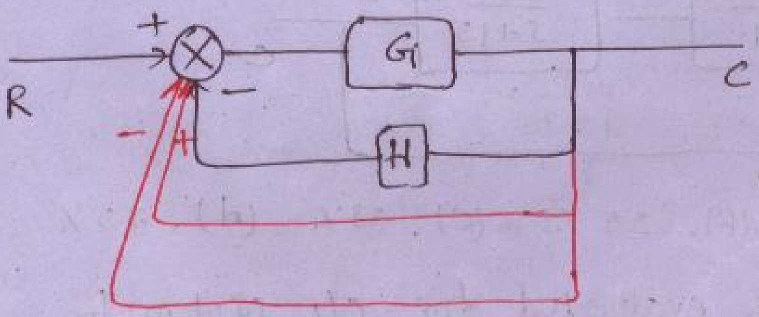
(c). step i/p, Type-1 (d). Ramp i/p - Type-1

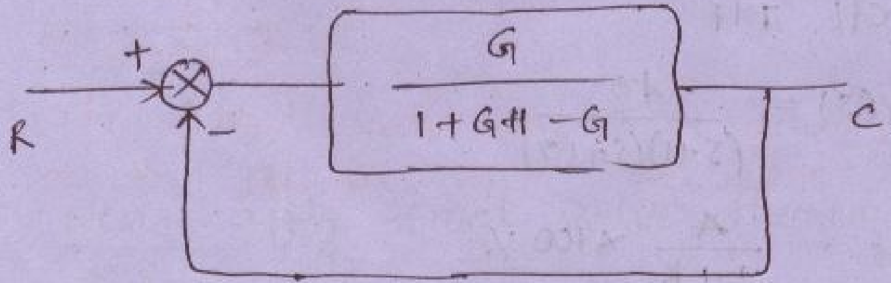
S.S. errors to Non-unity f/b system

Q. find ss errors to the given Non-unity f/b system for the unit step i/p — ?



Given non-unity f/b system should be converted into unity f/b system.





$$G_{NUF}(s) = \frac{G}{1+GH-G}$$

Equilavent o/L T/F
for non-unity f/b
system

Step 1: find c/L T/F : $\frac{C}{R} = \frac{G}{1+GH}$

Step 2: find $G_{NUF}(s)$ by subtracting numerator in the denominator.

$$G_{NUF}(s) = \frac{G}{1+GH-G}$$

Step 3: Compute e_{ss} for type of i/p.

$$\frac{C(s)}{R(s)} = \frac{100}{s(s+10) + \frac{100}{s+5}}$$

$$= \frac{100(s+5)}{s(s+10)(s+5)+100}$$

$$G_{NUF}(s) = \frac{100(s+5)}{s(s+10)(s+5)+100-100(s+5)}$$

$$= \frac{100(s+5)}{s^3+15s^2-50s-400}$$

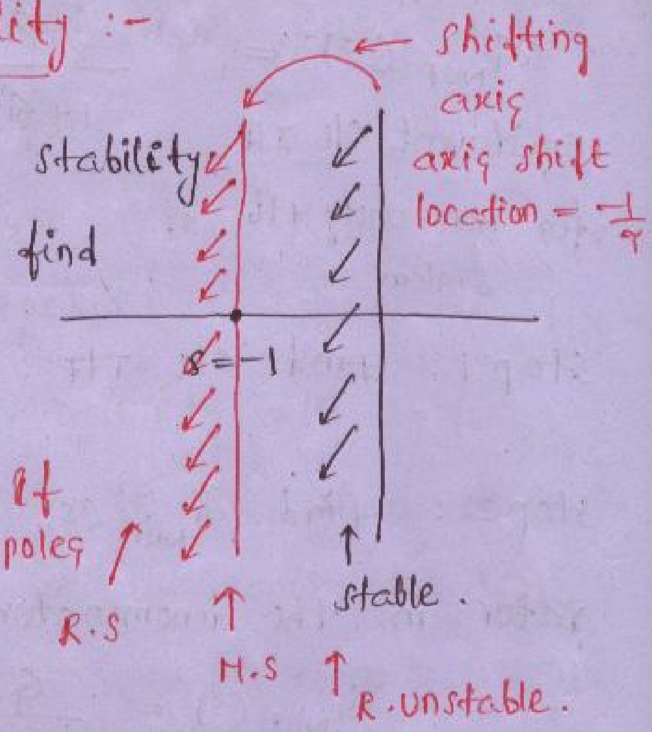
$$e_{ss} = \frac{A}{1+K}$$

$$= \frac{1}{1+\frac{500}{-400}}$$

$$= -4.$$

Relative Stability :-

By using Relative stability concept, we can find System time constant and settling time and time required to reach steady state.



RH CRITERIA :

$as^2 + bs + c = 0$

If $a, b, c > 0 \longrightarrow$ stable

if $a, b, c < 0 \longrightarrow$ stable.

if $b = 0,\ a, c > 0 \longrightarrow$ M.S.

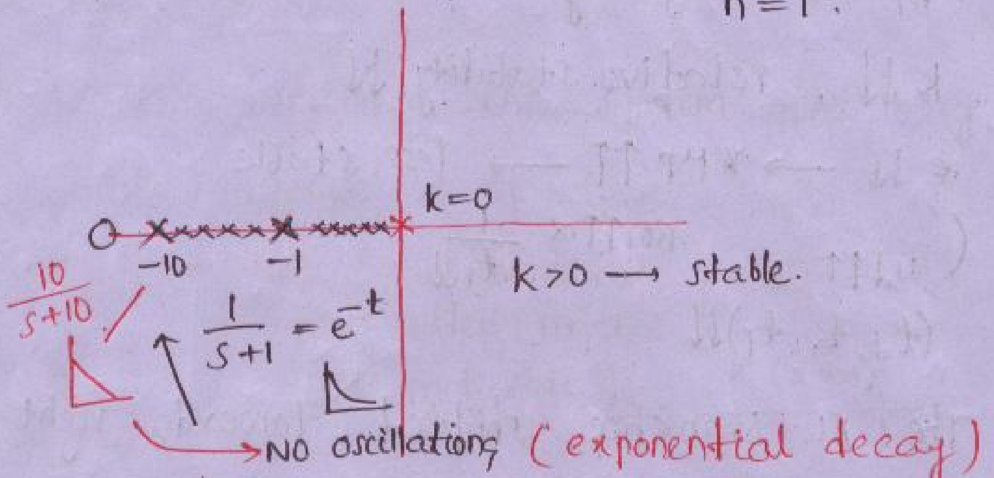
* Addition of poles & zeros should be in the left of s-plane only.

* If zeros are added in the left most side then steady state performance is improved.

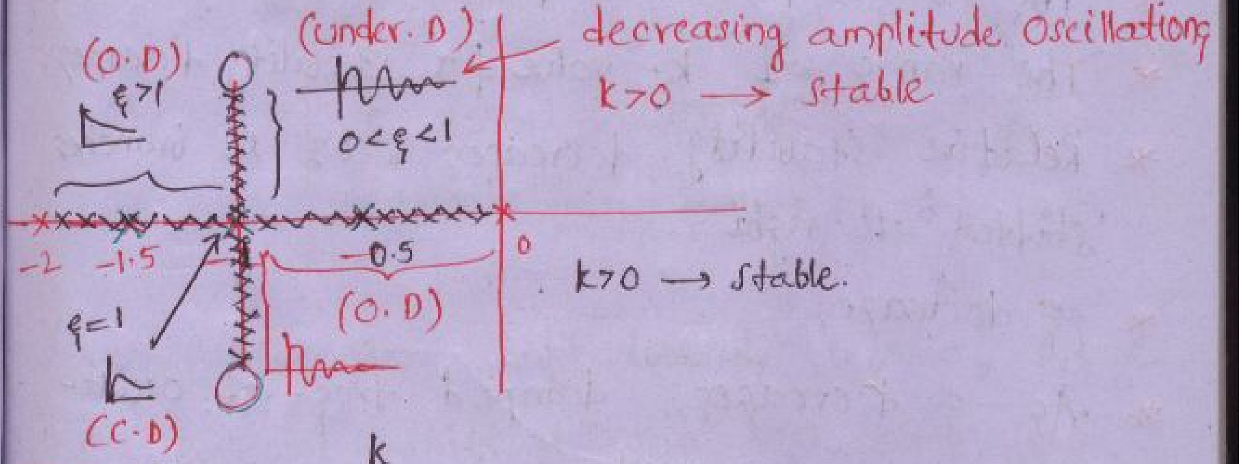
Addition of poles :

$\Rightarrow GH = \frac{k}{s}$

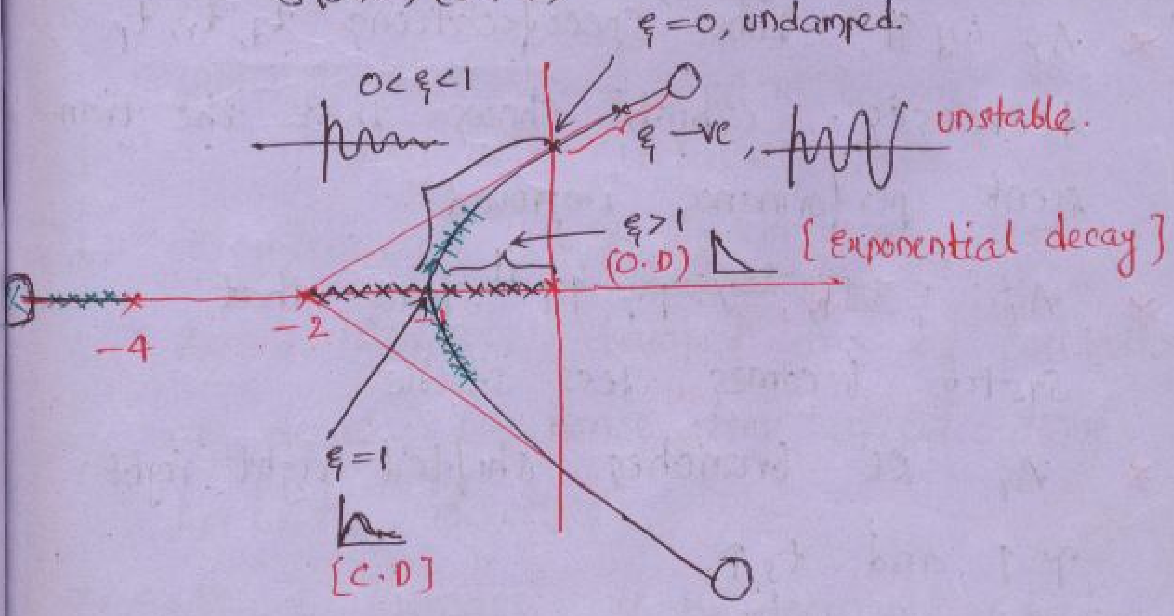
$n = 1.$



$\Rightarrow GH = \frac{k}{s(s+2)}$



$\Rightarrow GH = \frac{k}{s(s+2)(s+4)}$



RL → RH - S·plane

BP → Imaginary

K ↓↓ , relative stability ↓↓

$\xi \downarrow\downarrow \longrightarrow \% M_p \uparrow\uparrow \longrightarrow$ less stable

($\omega_d \uparrow\uparrow$ $\quad BW \uparrow\uparrow \propto \frac{1}{t_r \downarrow\downarrow}$

$(t_d, t_r, t_p) \downarrow\downarrow$

- The RL branches shifted towards right of s-plane.
- The break point shifted towards ima. axis
- The range of k-value for stability decreases.
- Relative stability decreases b'coz RL branches shifted to right.
- ξ decreases
- As ξ decreases, damped freq. of oscillations are increased
- As $\omega_d \uparrow\uparrow$, Time specifications t_d, t_r, t_p decreases. which shows that the transcent performance improved.
- As $\xi \downarrow\downarrow$, $\% M_p \uparrow\uparrow$ it shows that System becomes less stable.
- As RL branches shifted right right $\tau \uparrow$ and $t_s \uparrow$.
- More Oscillatory

* The BW increases b'coz t_r decreases.

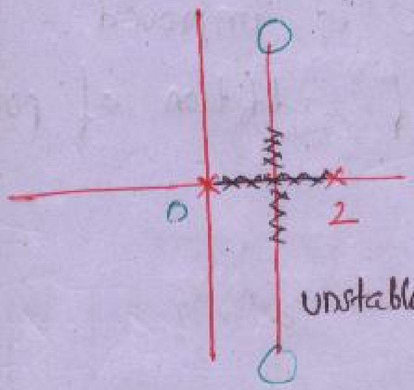
$BW \propto \frac{1}{t_r}$.

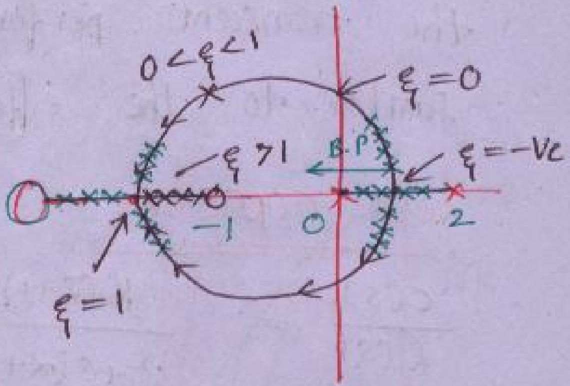
* As BW increases the system gives very quick response w.r.t t_r.

Effect of Addition of Zero's:-

~~[zero added in the~~ left most side].

$$\Rightarrow \frac{K}{s(s-2)} \longrightarrow \frac{K(s+1)}{s(s-2)}$$





1. RL branches shift towards left of s-plane.
2. B. point shift towards ~~ima. axis~~ zero.
3. The range of k value for system stability increases.
4. RS increases.
5. ξ increases
6. As ξ increases, damped freq. of (ω_d) oscillations are decreased hence as $\omega_d \downarrow$, time specifications t_r, t_p, t_d increases.
7. As ξ increases, % M_p decreases which shows that system becomes more stable.

* System becomes more relatively stable.

* System becomes less oscillatory.

* As RL branches turn towards left time const. decreases and also $t_s \downarrow$.

* The BW is decreases b'coz $t_r \uparrow$.

* As BW decreases, system response becomes slow w.r.t t_r.

NOTE :

If zero added near to imaginary axis the transient performance is improved similar to the effect of addition of poles.

ROOT CONTOUR :

$$\frac{C(s)}{R(s)} = \frac{K(s+1)}{s^2+s(\alpha+3)+2} \qquad "\alpha" = ?$$

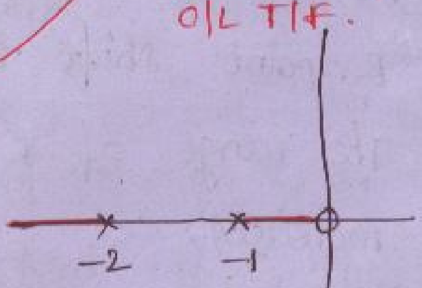
CE : $s^2+s(\alpha+3)+2 = 0$ ← converting into O/L T/F.

$$GH = \frac{\alpha s}{s^2+3s+2}$$

At pole $\alpha = 0$

At zero $\alpha = \infty$

$\uparrow$ system gain.



Draw the RL diagram by considering 'α' as System Gain for the given c/L T/F.

ROOT CONTOUR :-

If T/F of char. eq consists more than one unknown parameter, by varying all parameters from 0 to ∞ drawing a RL diagram is nothing but RC.

Draw RC for the following char.eq

$s^2+as+k=0$.

case 1

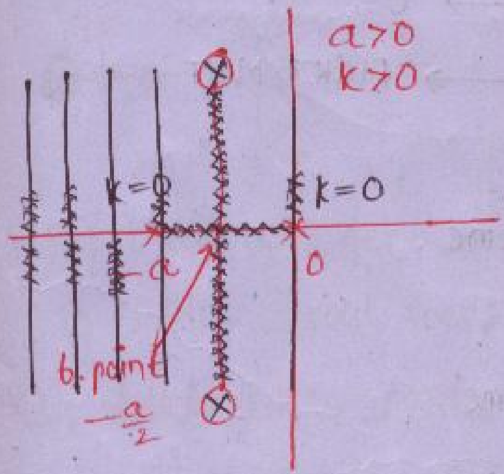
consider,

k as system gain

& a – const.

$$GH(s) = \frac{k}{s^2+as}$$

$$= \frac{k}{s(s+a)}$$



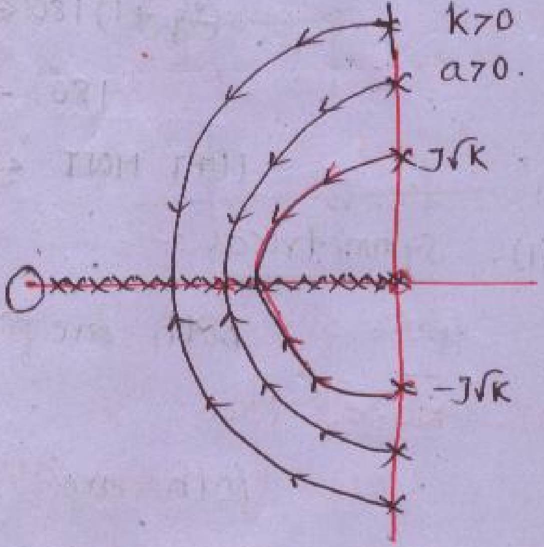
case 2

consider,

'a' as system gain

& k – const.

$$GH(s) = \frac{as}{s^2+k}$$

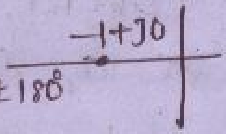


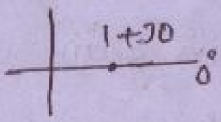
180° RULES **DIRECT ROOT LOCUS** 0° RULES **INVERSE ROOT LOCUS**

(1). $K \rightarrow 0$ to ∞ $k \rightarrow 0$ to $-\infty$

−ve f/b, $\xrightarrow{CE}$ $1+GH=0$ +ve f/b $\xrightarrow{CE}$ $1-GH=0$

$\angle GH = \angle -1+j0$





$\angle GH = \angle 1+j0$

$= \pm(2q+1)180 \pm 180°$ $= \pm 2q(180°)$

= odd multiples of ±180° = Even multiples of ±180°.

$DRL \xleftrightarrow[\text{Replaced}]{} IRL$

$ODD \longleftrightarrow Even$

$(2q+1)180° \longleftrightarrow 2q(180°)$

$180° \longleftrightarrow 0°$.

LEFT MOST ⟷ RIGHT MOST

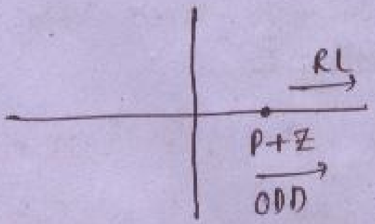
(1). Symmetrical :-

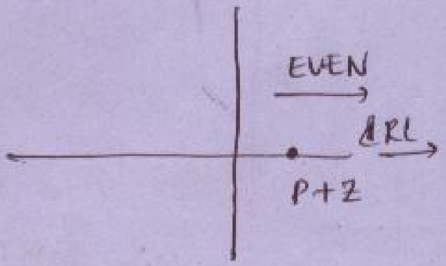
Both are same.

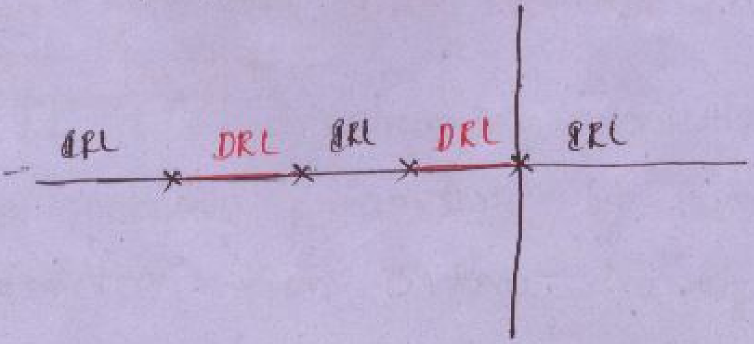
(2). No. of loci:

Both are same.

(3). Real axis loci:







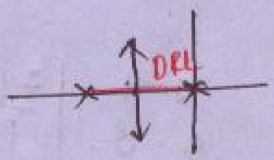
NOTE :

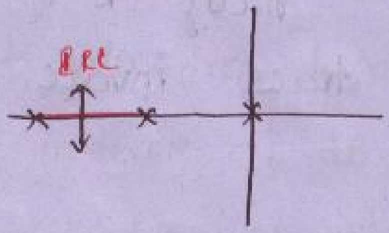
wherever direct RL not exists there must be a IRL.

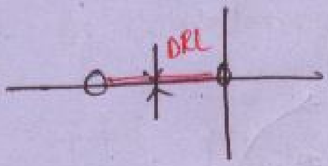
(4). Asymptotes & Centroid :

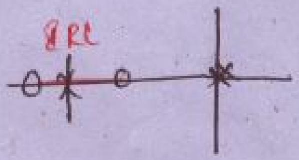
for both same.

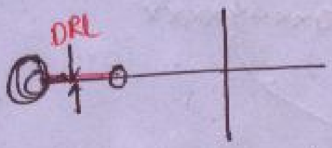
(5). Break point :

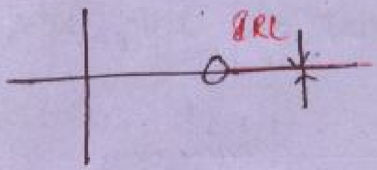












procedure for finding B. points is same in both root locus.

(6). Intersection point with imaginary axis :

procedure is same.

for valid intersection point with ima. axis

$K_{marginal}$ is +ve $K_{marginal}$ is −ve.

(7). Angle of departure :

$\phi_d = 180° - \phi$ $\phi_d = 0° - \phi$

Angle of arrival :

$\phi_a = 180 + \phi$ $\phi_a = 0° + \phi$

Draw the RL for — $G(S) = \frac{k \cdot e^{-S}}{s(s+1)}$

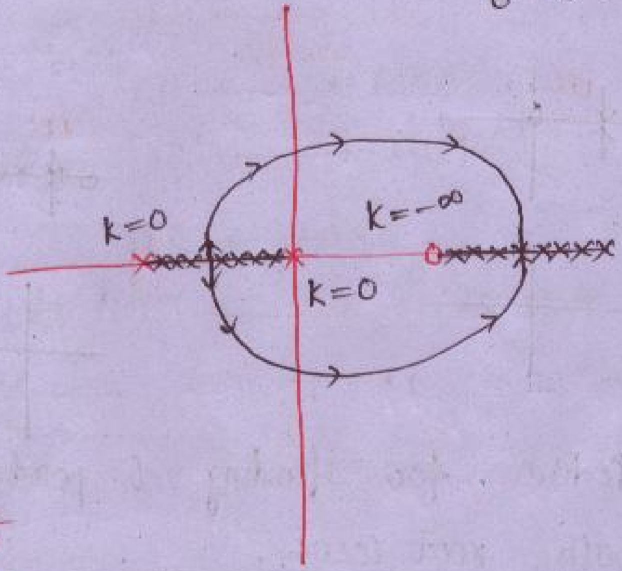
$$G(S) = \frac{K(1-S)}{S(S+1)}$$

$$= \frac{-K(S-1)}{S(S+1)}$$

B'coz K is -ve so we required to draw inverse RL.

N=1
θ=8.

$$\frac{2q(180)}{P-Z}$$



BODE PLOT

To draw freq response of O/L T/F.

To find c/L system stability by using ph. margin, K_{margin}, $K_{cross\ over\ freq}$ & ph. COF.

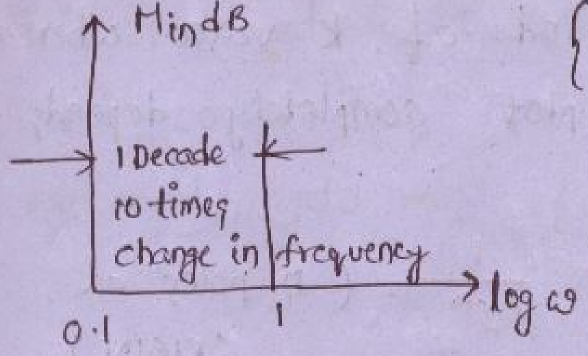
To find relative stability.

The largest gain margin & ph. margin gives more relative stability.

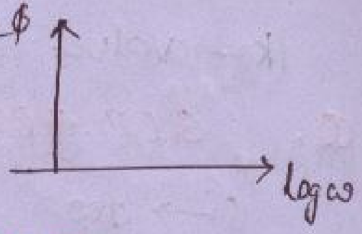
Bode plot consists two plots

(1). Magnitude Vs phase → Magnitude plot

(2). phase plot.



{ Octave → 2 times change in freq. }

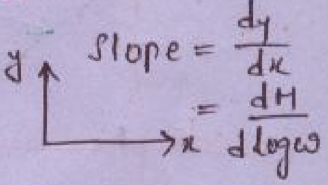


PROCEDURE TO DRAW BODE PLOT :

1. s replaced $j\omega$ to convert into freq domain.
2. find magnitude and write in terms of dB.

   by considering $M_{in\,dB} = 20 \log |G(j\omega)H(j\omega)|$
3. find phase angle $\angle\phi = \tan^{-1}\left(\frac{\text{ima. part}}{\text{real part}}\right)$
4. Draw the magnitude & phase plot by varying freq from min. to max. value.

Q. Draw bode plot for $GH = k$.



$\text{Slope} = \frac{dy}{dx} = \frac{dH}{d\log\omega}$

$s \rightarrow j\omega$

$G(j\omega)\cdot H(j\omega) = k.$

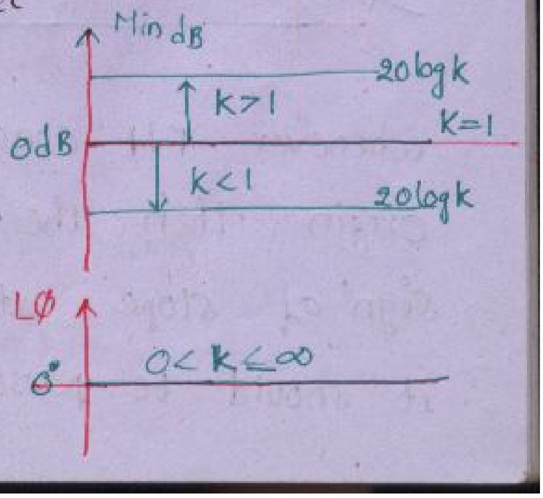
$M = k$

$M_{in\,dB} = 20 \log k.$
- $k = 1, \Rightarrow M = 0$
- $k > 1$ $(k = 10)$ $\Rightarrow M = \underline{+20}\text{dB}$
- $k < 1$ $(k = 0.1)$ $\Rightarrow M = -20\text{dB}.$

$S = \frac{dM}{d\log\omega} = 0\,\text{dB}|_{dec}$

$\angle\phi = \angle k = 0^\circ.$

shift depends on k value
(20log k).



ph. plot is ind. of k value whereas shift in mag. plot completely depends on k - value.

Q. $G(s).H(s) = \frac{1}{s^n}$ (n poles at ORIGIN).

$s \rightarrow j\omega$

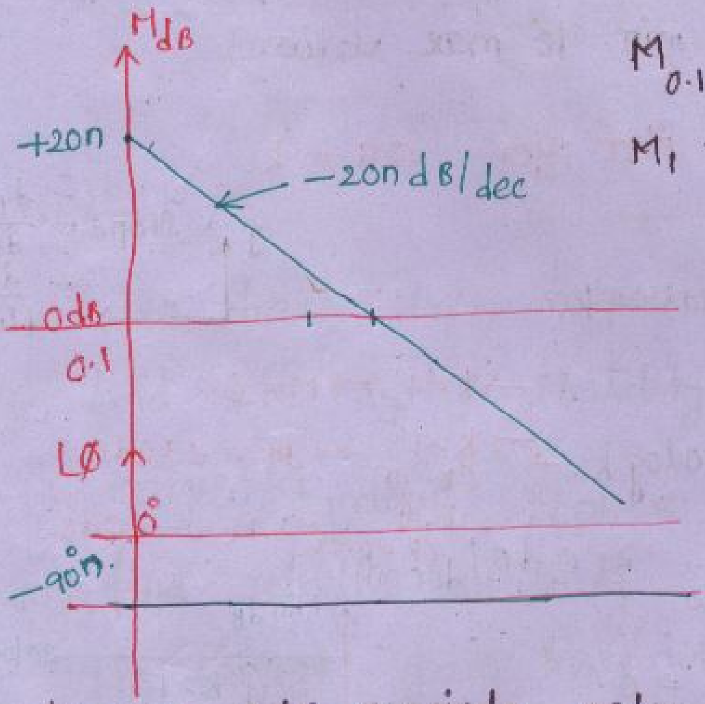
$G(j\omega).H(j\omega) = \frac{1}{(j\omega)^n}$

$M = \frac{1}{\omega^n}$

$M_{dB} = -20n \log \omega$

$S = \frac{dM}{d \log \omega} = -20n \, dB/dec$

$\angle\phi = \frac{\angle 1}{\angle j\omega \cdots n \text{ time}} = -90^\circ . n$



$M_{0.1} = -\text{slope}$

$M_1 = 0.$

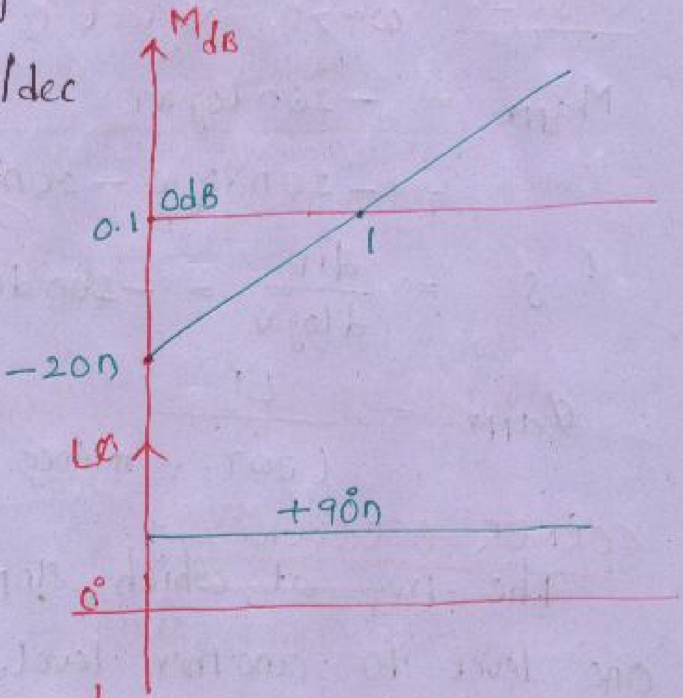
whenever T/F consists poles & zeros at origin then the plot start with opposite sign of slope at a freq. of 0.1 and it should be passes through 0dB line, intersect at $\omega = 1$ and extendended upto first corner freq if existed, otherwise extended upto ∞. (when $k=1$ only).

Q. $G(S).H(S) = S^n$. (n ZEROS ~~poles~~ at ORIGIN).

$$M_{dB} = 20n \log\omega$$

$$S = 20n \text{ dB/dec}$$

$$\angle\phi = +90^\circ . n$$



Q. $G(S).H(S) = \dfrac{1}{(1+ST)^n}$

$S \to j\omega$

$$G(j\omega).H(j\omega) = \frac{1}{(1+j\omega T)^n}.$$

$$M = \left[\frac{1}{\sqrt{1+(\omega T)^2}}\right]^n.$$

$$M_{dB} = -20.n \log\sqrt{1+(\omega T)^2}$$

Actual

$$\phi_{actual} = \frac{\angle 1}{\angle(1+j\omega T)\ldots n \text{ times}}$$

$$= -n. \tan^{-1}(\omega T).$$

Asymptotic or Appr. analysis :-

Case1 : $\omega\tau < 1, \xleftarrow{\omega < 1/\tau}$ Neglect ($\omega\tau$).

$M_{Appr.} = 0$ dB/dec , $S = 0$ dB/dec.

$\phi_{appr.} = 0.$

Case2 : $\omega\tau > 1. \xleftarrow{\omega > \frac{1}{\tau}}$ Neglect 1,

$$M_{appr.} = -20n \log \omega\tau$$

$$= -20n \log\omega - \cancelto{0}{20n \log\tau}$$

$$S = \frac{dM}{d\log\omega} = -20n \text{ dB/dec.}$$

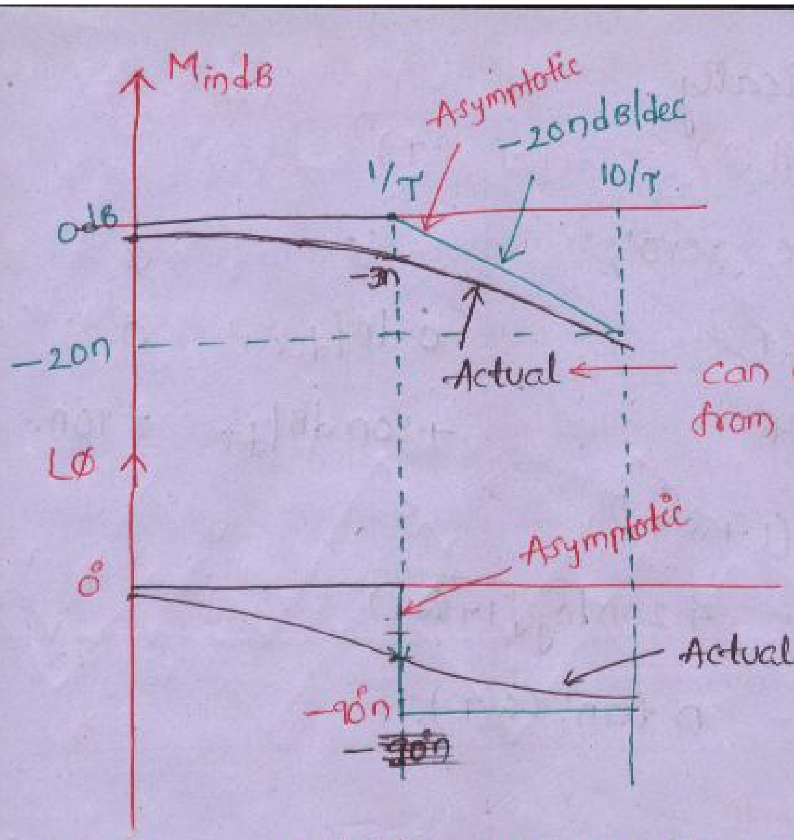
$$\phi_{appr.} = \frac{\angle 1}{(j\omega\tau \ldots n\text{times}} = -90n.$$

CORNER FREQUENCY :-

The freq. at which slopes changes from one level to another level. is called corner freq.

The corner freq.s are nothing but finite poles & finite zeros location in the form of magnitude.

n finite poles	S	ϕ
< CF	0 dB/dec	0
> CF	−20n dB/dec	−90°n.



can be calculated from T/F.

Error = Actual − Asymptotic value

$E_{at\ \omega=\frac{1}{\tau}}$:

$$M_{act\ \omega=1/\tau} = -20n \log \sqrt{1+(\omega\tau)^2}$$
$$= -20n \log \sqrt{2}$$
$$= -3n\,dB$$
$$M_{asy} = 0\,dB.$$
$$E = -3n\,dB.$$

Error in phase plot :

$$\phi_{\omega=1/\tau} = -n\tan^{-1}(\omega\tau)$$
$$= -45°n.$$

NOTE:

Error is max. at corner freq. either above or below corner freq.s the error is decreases

Symmetrically.

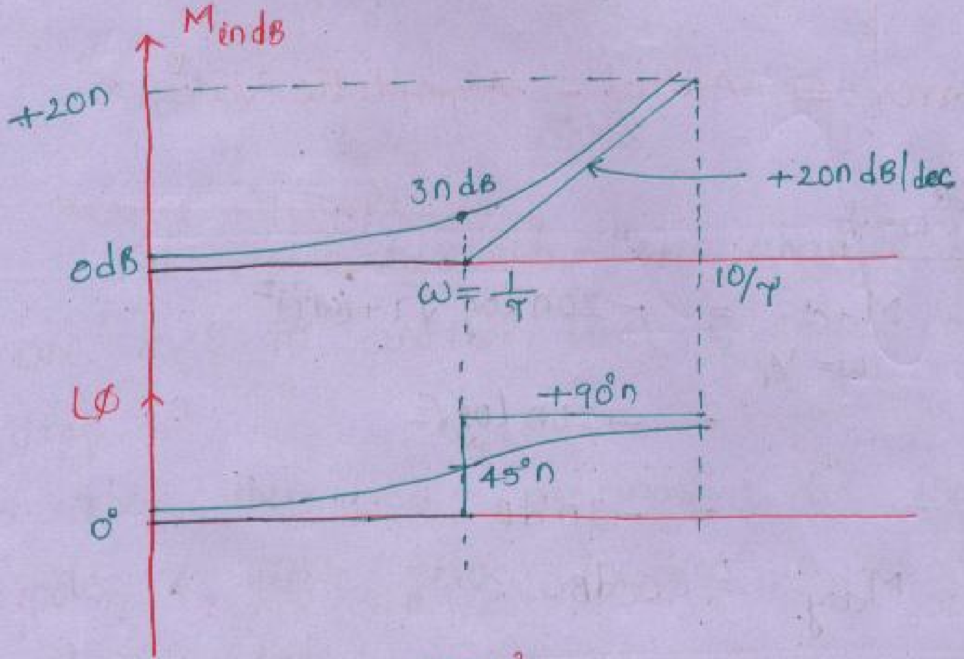
Q. $G(S).H(S) = (1+sT)^n$.

n finite zero's	s	ϕ
$< cf$	0 dB/dec	0
$> cf$	+20n dB/dec	$+90^\circ n$.

$$GH = (1+sT)^n.$$

$$M_{act} = +20n\log\sqrt{1+(\omega T)^2}$$

$$\phi_{act} = n\tan^{-1}(\omega T).$$

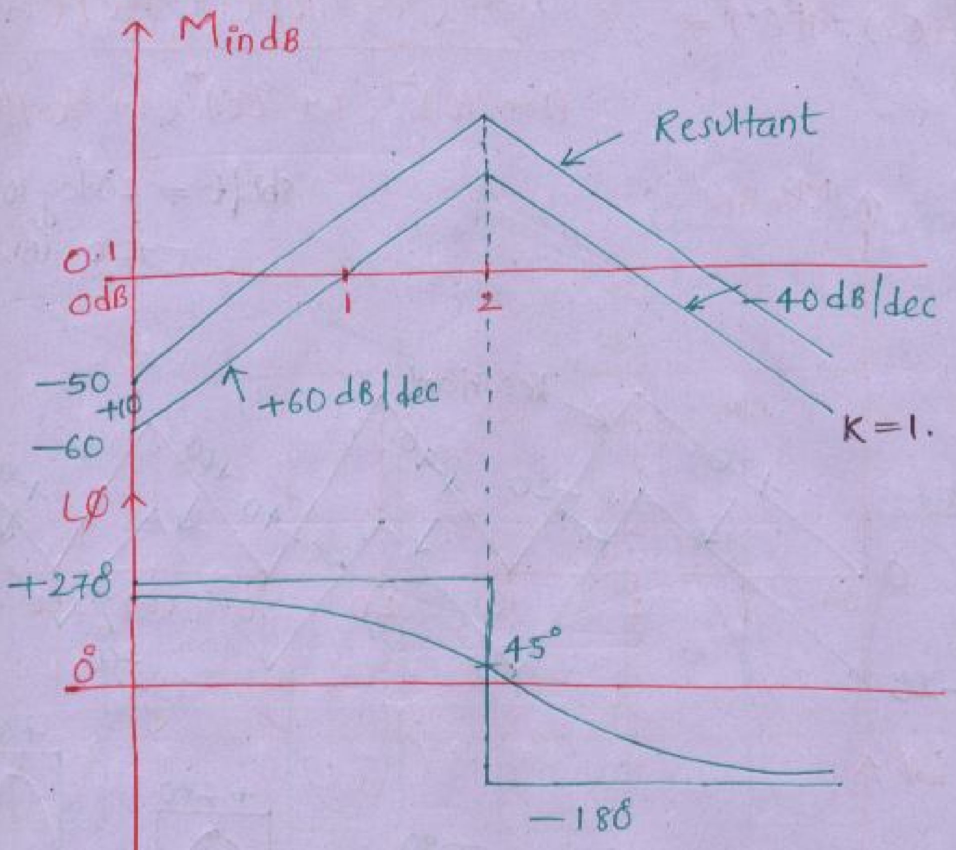


Q. $G(S).H(S) = \dfrac{100.s^3}{(s+2)^5}$

$$\xrightarrow[\text{form}]{\text{Time const.}} \frac{100\cdot s^3}{2^5(1+s/2)^5}$$

$$= \frac{3.125\,s^3}{(1+s/\boxed{2})^5}$$

↑ cf.



$$\underset{\underset{(20\log k)}{\uparrow}}{\text{shift}} = 20\log 3.125$$

$$= +10\text{ dB}.$$

$$\angle\phi\big|_{\omega=2} = 270^\circ - 5\tan^{-1}\left(\frac{\omega}{2}\right)$$

$$= 270^\circ - 5\times 45^\circ$$

$$= 45^\circ.$$

$+20 \rightarrow +90^\circ$

$+60 \rightarrow +90^\circ\times 3$
(3×20)

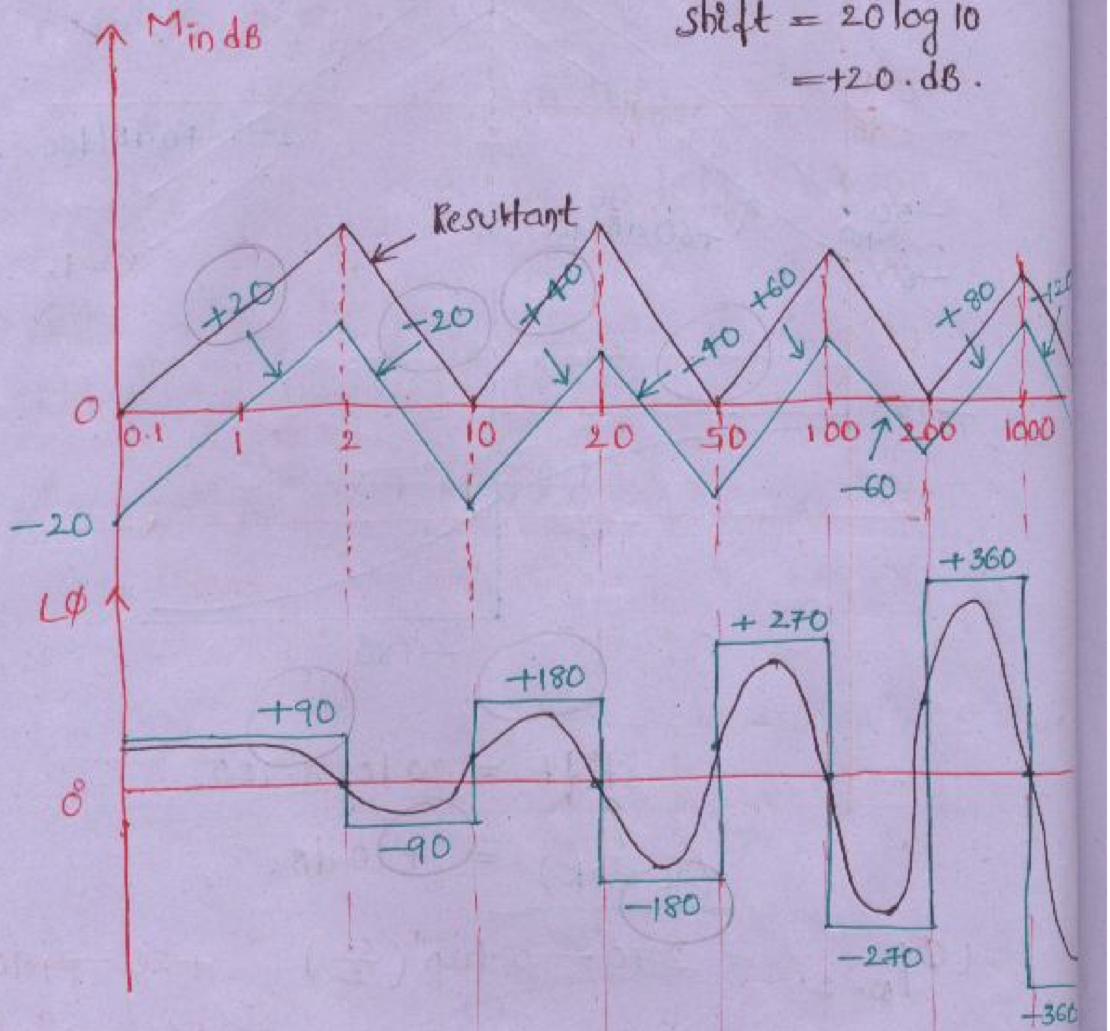
$-80 \rightarrow -360^\circ$

* Initial slope is given by poles & zeros located at origin.

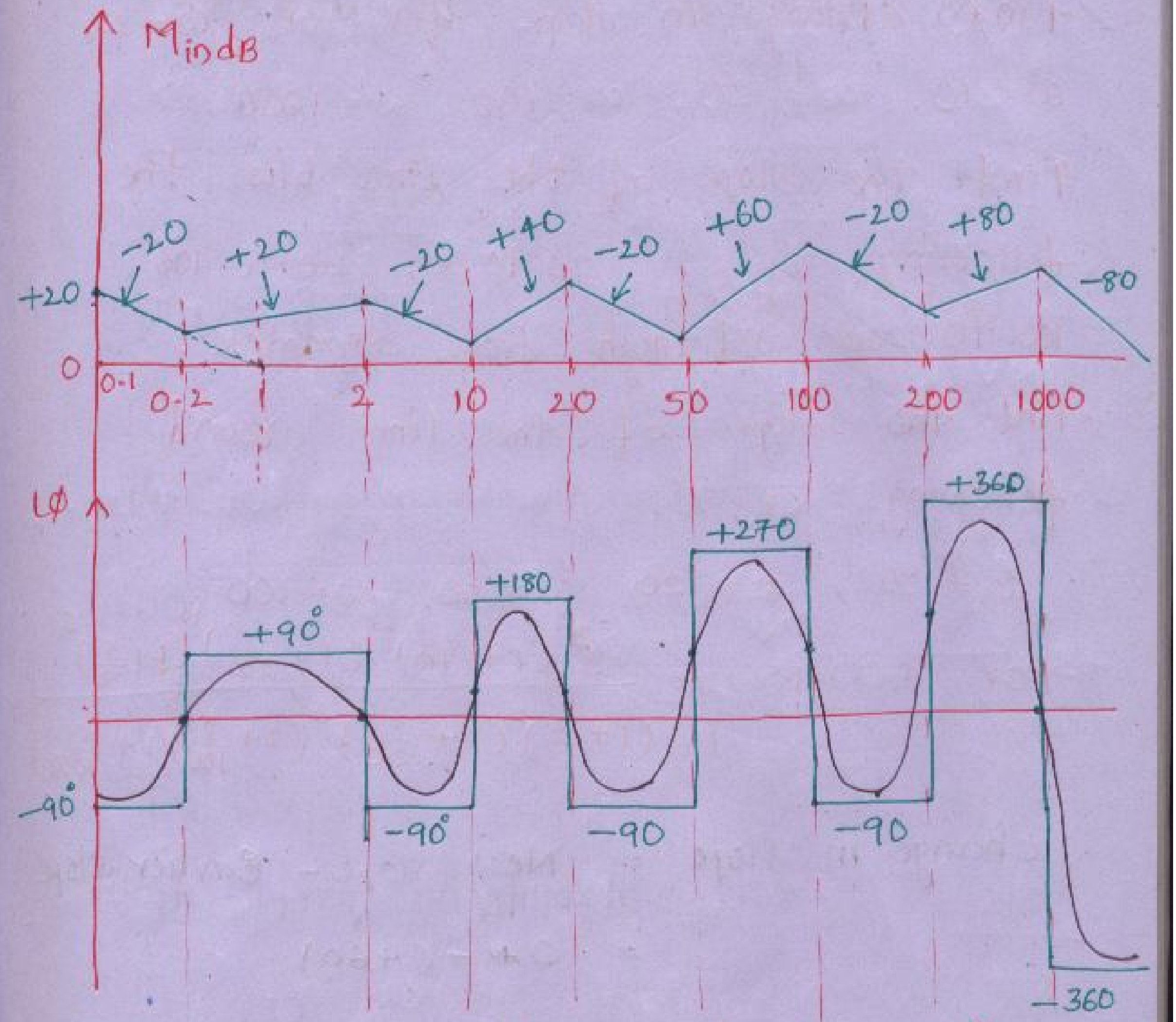
$$G(s).H(s) = \frac{10.s\left(1+s/10\right)^{3}\left(1+s/50\right)^{5}\left(1+s/200\right)}{\left(1+s/2\right)^{2}\left(1+s/20\right)^{4}\left(1+s/100\right)^{6}\left(1+\frac{s}{1000}\right)}$$

shift = 20 log 10

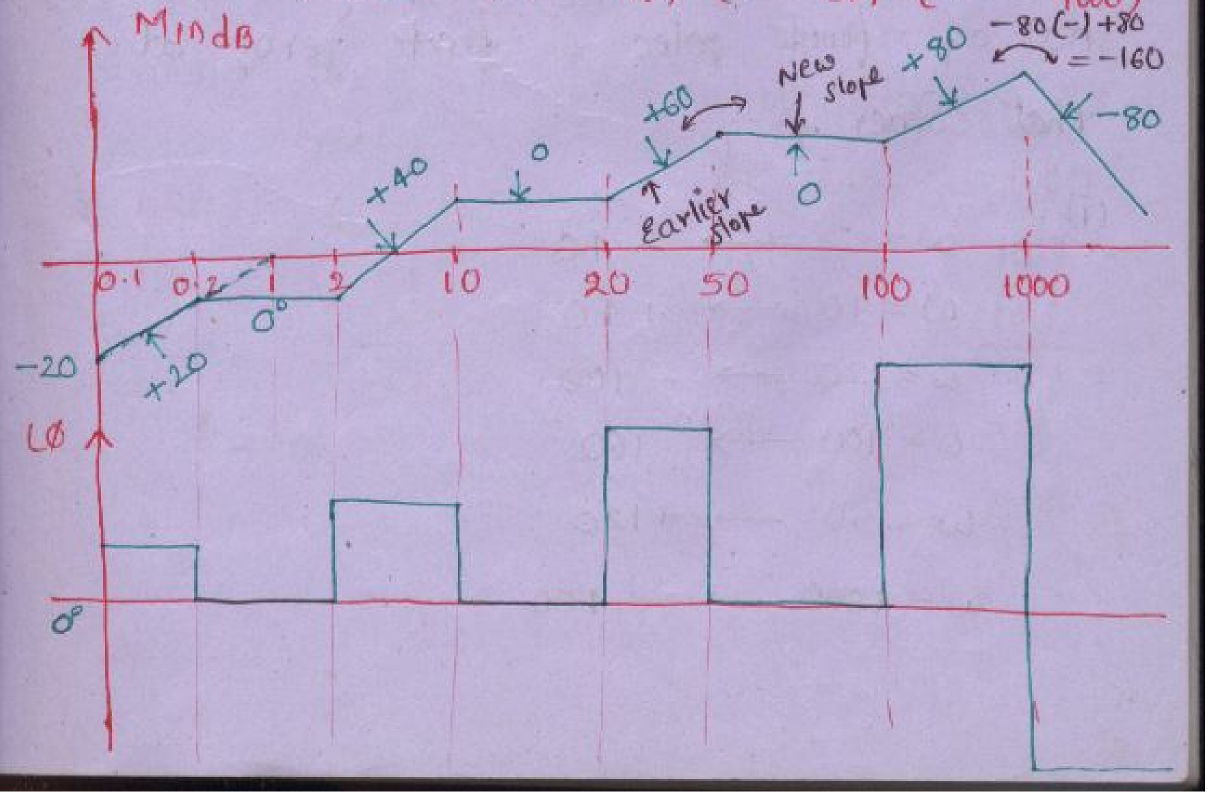
= +20.dB.



$$G(s)H(s) = \frac{\left(1+s/0.2\right)^{2}\left(1+s/10\right)^{3}\left(1+s/50\right)^{4}\left(1+s/200\right)^{5}}{s.\left(1+s/2\right)^{2}\left(1+s/20\right)^{3}\left(1+s/100\right)^{4}\left(1+s/1000\right)^{8}}$$



$$G(s)H(s) = \frac{s(1+s/2)^2(1+s/20)^3(1+s/100)^4}{(s/0.2+1)(1+s/10)^2(1+s/50)^3(1+s/1000)^8}$$



find change in slope for cf, $\omega = 2$, $\omega = 10$, $\omega = 20$, $\omega = 100$, $\omega = 1000$.

Find the slope of the line b/w the following cf's. 20 to 50, 50 to 100, 100 to 200 and high freq. asymptote.

find the slopes of the lines around following cf's.

$\omega = 2$, $\omega = 20$, $\omega = 50$, $\omega = 100$

for $$G(s)H(s) = \frac{s^3 (1+s/10)^2 (1+\frac{s}{200})^{10} (1+\frac{s}{50})^6}{(1+\frac{s}{2})^2 (1+\frac{s}{20})^5 (1+\frac{s}{100})^8 (1+\frac{s}{1000})^{20}}$$

change in slope = New slope − Earlier slope

$= 0 - (+60)$

$= -60$

change in slope is nothing but slopes of no. of finite poles & finite zeros at that corner freq.

(i).

$\omega = 2 \longrightarrow -40$

$\omega = 10 \longrightarrow +40$

$\omega = 20 \longrightarrow -100$

$\omega = 100 \longrightarrow 160$

$\omega = 50 \longrightarrow +120$

$\omega = 1000 \longrightarrow -400$

(include) (Exclude)
≥20 to < 50
3P
6Z
→
3Z ⇒ +60

≥100 to <1000
6P
10Z
4Z ⇒ +80

≥2 to <10
2P
3Z
1Z → +20

(Include) (Exclude)
≥50 to <100
7P
11Z
4Z ⇒ +80

100 to 200
15P
11Z
4P ⇒ −80

for
All this
type of questions
consider only below
given ck slope
of P & Z's
in that include
or exclude

High freq. Asymptote:
>1000
35P
21Z
14P ⇒ −280

Around 10
≤10 ≥10
(Exclude) (Include)
1P 3P
3Z 3Z
2Z 0
⇓ +40

Around 1000(I)
(E)
≤1000 ≥1000
10Z 10Z
6P 14P
4Z 4P
⇓ ⇓
+80 −80

(E) Around 2 (I)

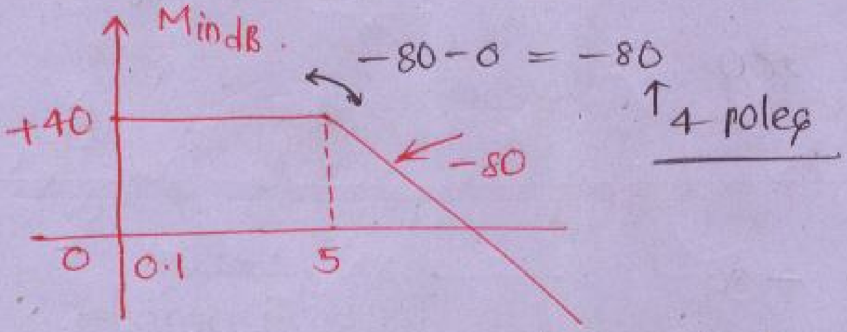
<2	>2
$3Z \rightarrow +60$	$3Z$
	$2P$
	$1Z \rightarrow +20$

(E) Around 20 (I)

<20	>20
$5Z$	$5Z$
$2P$	$7P$
$3Z \rightarrow 60$	$2P \rightarrow -40$

INVERSE PLOTS :

Q. find T/f for given magnitude plot.



Step 1: Observe the initial slope which gives no. of P & Z's at origin.

S2: find change in slope at each & every CF and write the no. of finite poles or finite zeros at that CF.

S3: find the k value.

Initial slope is '0'. So no poles & zeros at origin.

change in slope

$$\frac{k}{\left(1+\frac{s}{5}\right)^4}$$

$$40\Big|_{\omega=0.1} = \frac{k}{(1+s/5)^4}$$

$$40\Big|_{\omega=0.1} = 20\log k - 80\log\sqrt{1+\left(\frac{\overset{0.1}{\cancel{10}}}{5}\right)^2}$$ Neglect

$$= 20\log k - \cancel{80\log 1} \to 0$$

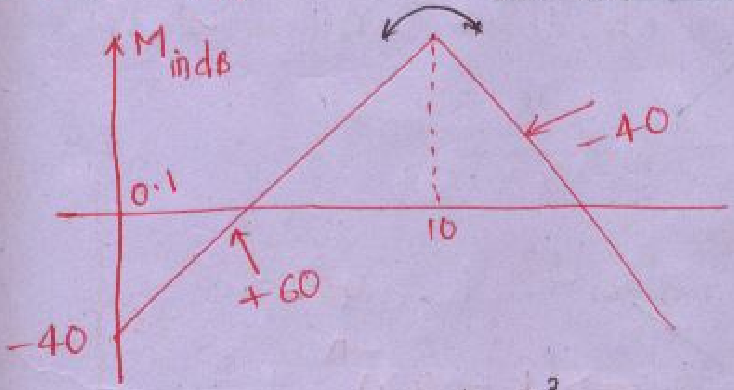
$$\Rightarrow k = 10^2 = 100$$

Don't write terms in dB if cf ≥ 0.1

$$TF = \frac{100}{(1+s/5)^4}.$$

Q.-find T/f — $-40-60 = -100$

↑ 5 poles.

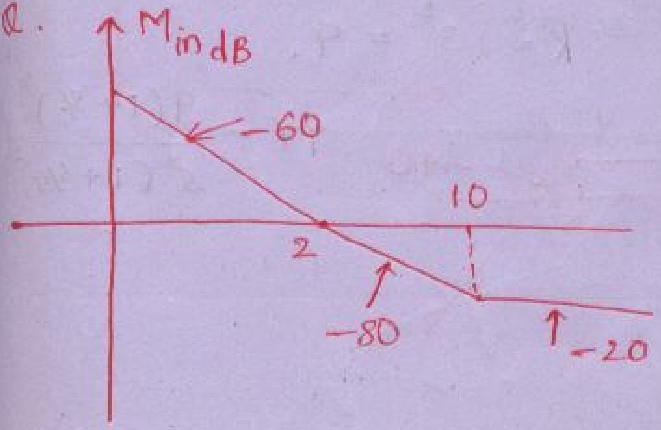


$$-40\Big|_{\omega=0.1} = \frac{k.s^3}{(1+s/10)^5}$$

$$\rightarrow -40 = 20\log k + 60\log \overset{0.1}{\cancel{\omega}}$$

$$\Rightarrow 20 = 20\log k \Rightarrow k = 10$$

Q.

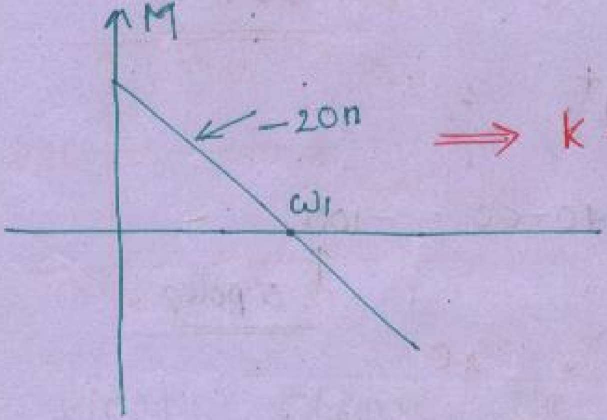


$$0\Big|_{\omega=2} = \frac{k\left(1+s/10\right)^3}{s^3\left(1+s/2\right)}$$

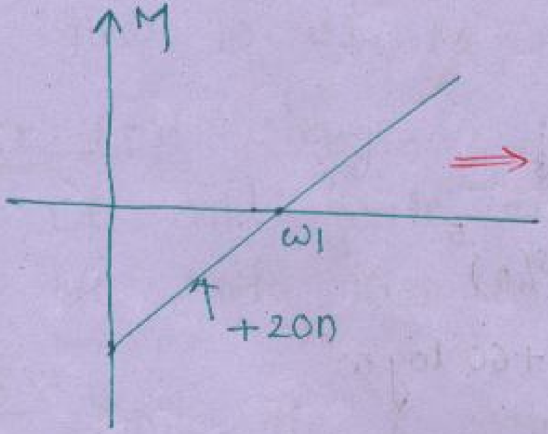
$$\Rightarrow 0 = 20\log k - 60\log 2$$

$$\Rightarrow 20\log k = 20\log 2^3$$

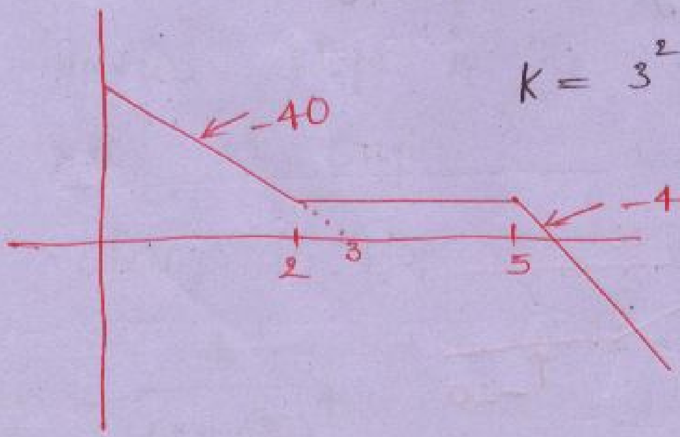
$$\Rightarrow k = 8$$



$$\Rightarrow k = (\omega_1)^n$$



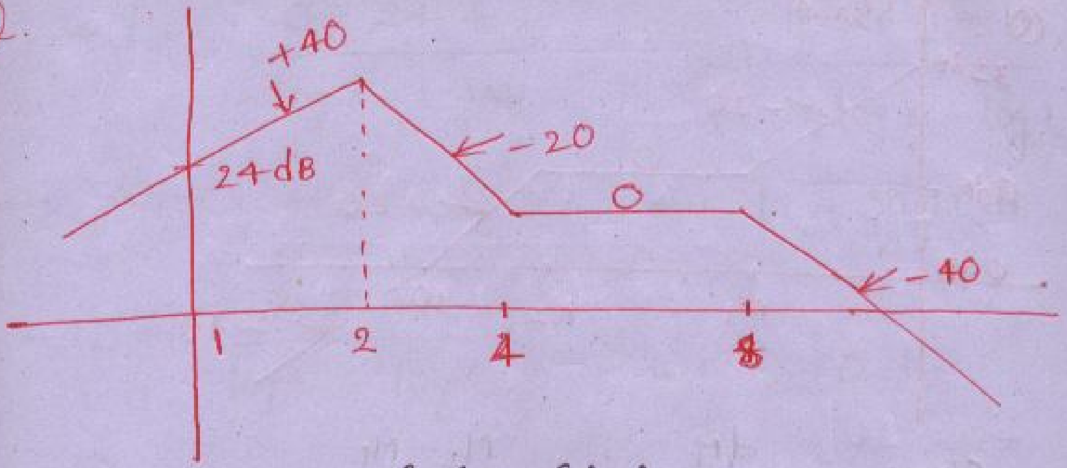
$$\Rightarrow k = (\omega_1)^{-n}$$



$$K = 3^2 = 9.$$

$$Tf = \frac{9(1+s/2)^2}{s^2(1+s/5)^2}$$

Q.



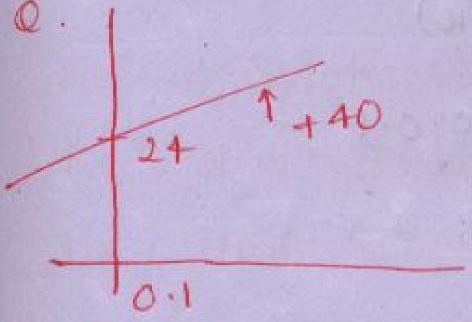
$$24\Big|_{\omega=1} = \frac{K.s^2(1+s/4)}{(1+s/2)^3\,(1+s/8)^2}$$

$$\Rightarrow \quad 24 = 20\log k + 40\,\cancel{\log 1}^{\to 0}$$

$$\Rightarrow \quad k = 10^{1.2} = 15.24$$

$$\simeq 16$$

Q.

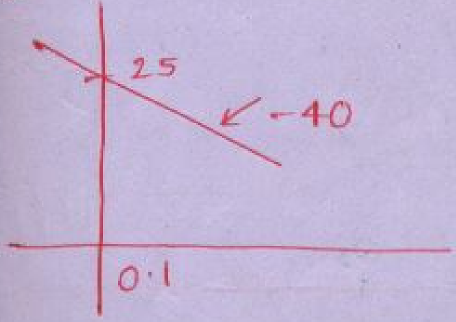


$$24\Big|_{\omega=0.1} = k.s^2$$

$$24 = 20\log k + 40\log 0.1$$

$$\Rightarrow \quad k = 10^{3.2}$$

Q.

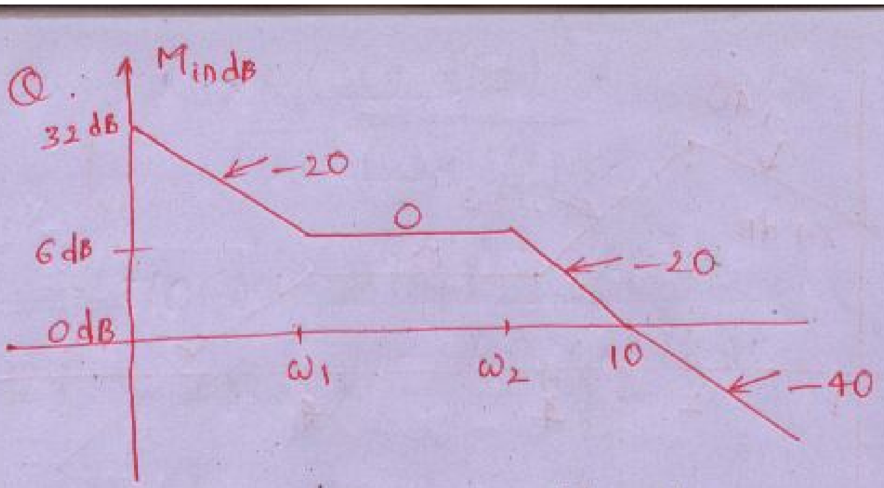


$$25\Big|_{\omega=0.1} = \frac{k}{s^2}$$

$$25 = 20\log k - 40\log 0.1$$

$$\Rightarrow \quad k =$$

Q.



$$\text{Slope} = \frac{dM}{d\log\omega} = \frac{M_2 - M_1}{\log\omega_2 - \log\omega_1}$$

$$-20 = \frac{6-32}{\log\omega_1 - \log 0.1} \quad ; \quad -20 = \frac{0-6}{\log 10 - \log\omega_2}$$

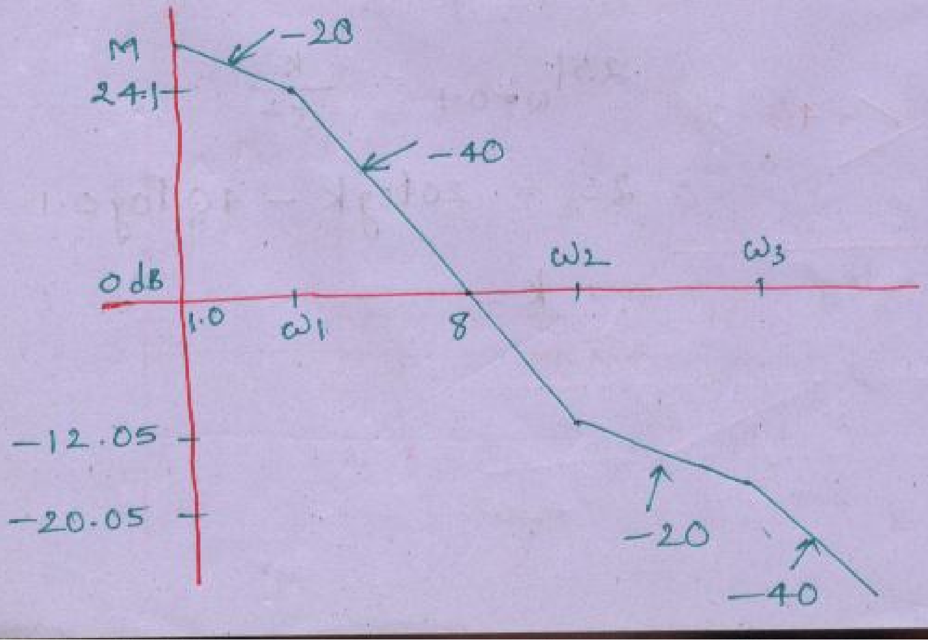
$$\Rightarrow \quad \omega_1 = 2 \text{ rad/sec} \quad \rightarrow \quad \omega_2 = 5 \text{ rad/sec}$$

$$TF = \frac{K(1+S/2)}{S(1+S/5)(1+S/10)}$$

$$32\Big|_{0.1} = 20\log k - 20\log 0.1$$

$$\Rightarrow \quad K = 10^{0.6} \simeq 4$$

Find magnitude M, $\omega_1, \omega_2, \omega_3$ & TF



$$-40 = \frac{0-24.1}{\log 8-\log\omega_1} \; ; \quad -20 = \frac{24.1-M}{\log 2-\log 1}$$

$$\Rightarrow \omega_1 = 2 \qquad \Rightarrow M = 30.12\,dB.$$

$$-40 = \frac{-12.05-0}{\log\omega_2-\log 8} \; ; \quad \omega_3 = 40.$$

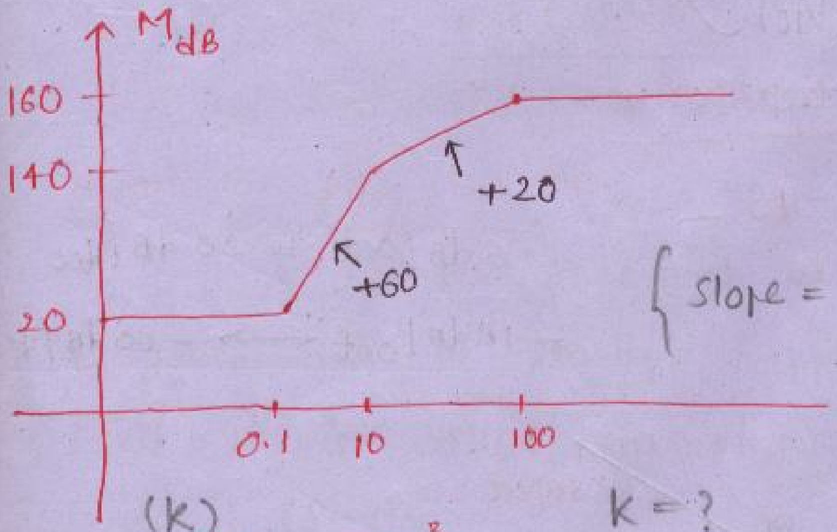
$$\Rightarrow \omega_2 = 16$$

$$T/f = \frac{k(1+s/16)}{s(1+s/2)(1+s/40)}$$

$$30.12\Big|_{\omega=1} = 20\log k - 20\log 1$$

$$\Longrightarrow k = 32.06$$

Q. The Asymptotic appr. bode ^{mag} plot of a minimum ph. system shown in fig. The T/F of the system is —?



$$\left\{ \text{Slope} = \frac{dM}{d\log\omega} \right\}$$

$$= \frac{140-20}{\log 10-\log 0.1}$$

k = ?

(a). $\frac{\overset{(K)}{\cancel{10}^{6}}(s+0.1)^3}{(s+10)^2(s+100)}$ (b) 10^8 " (c). 10^7 "

(d). 10^9 "

$$2\,\text{dec} \longrightarrow 120$$

$$1\,\text{dec} \longrightarrow ?\ \ 60$$

$$T/f = \frac{k\left(1+s/0.1\right)^3}{\left(1+s/10\right)^2\left(1+s/100\right)}$$

$$\Rightarrow 20\Big|_{0.1} = 20\log k$$

$$\Rightarrow k = 10$$

$$160\Big|_{\omega=100} = 20\log k + 60\log\overset{1000}{\sqrt{\underset{\substack{\uparrow\\ \text{Neg.}}}{1}+\left(\frac{\overset{100}{\cancel{\omega}}}{\underset{\substack{\uparrow\\ 1000}}{0.1}}\right)^2}}$$

$$-40\log\overset{10}{\sqrt{\underset{\substack{\uparrow\\ \text{Neg.}}}{1}+\left(\frac{\overset{100}{\cancel{\omega}}}{\underset{\substack{\uparrow\\ 10}}{10}}\right)^2}}$$

$$\Rightarrow 160 = 20\log k + 180 - 40$$

$$\Rightarrow k = 10.$$

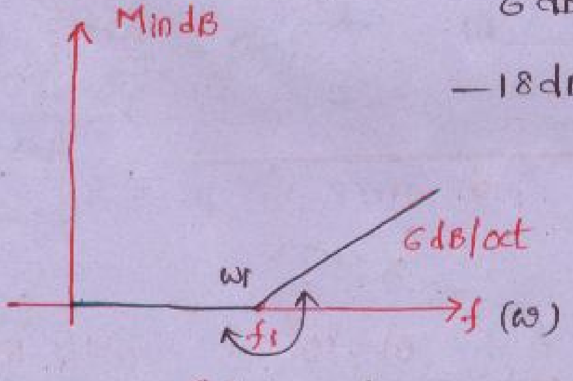
$$\frac{10\times10^2\times100\times10^3}{\underset{(1/10)^3}{(0.1)^3}} \Rightarrow k = 10^8.$$

* MON. 22/12/08 *

find
The T/f to _

$$6\,\text{dB/oct} = 20\,\text{dB/dec}$$

$$-18\,\text{dB/oct} \longleftrightarrow -60\,\text{dB/dec}$$



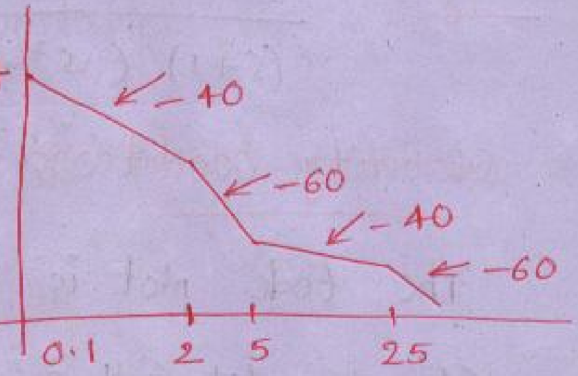
$$\underset{\swarrow 1}{K}\left(1+s/\omega_1\right) = 1+\frac{j\omega}{\omega_1}$$

$$= \left(1+\frac{j\omega}{\omega_1}\right)$$

$$= \left(1+\frac{j2\pi f}{2\pi f_1}\right) = \left(1+\frac{jf}{f_1}\right)$$

Q. The asymptote appr. of log-mag vs freq of a min. ph. system shown in fig. Its T/F — ?

$$\frac{K(1+S/5)}{S^2(1+S/2)(1+S/25)}$$



$$54\Big|_{\omega=0.1} = 20\log k - 40\log 0.1$$

$$\Rightarrow k = 10^{0.7} = \frac{5\times 2\times 25}{5}$$

$$= 50.$$

$$\therefore \quad T/F = \frac{50(s+5)}{s^2(s+2)(s+25)}$$

Minimum ph. System:

A system in which all the finite poles & zeros lies in the left half of s-plane then it is called min. ph. system.

eg: $\frac{(s+1)}{(s+2)(s+3)}$

ALPS:

A system in which zeros lies in right of s-plane & poles lies in left half s-plane which are symmetrical about ima. axis then it is called All pass system.

Eg: $$\frac{(s-1)(s^2-2s+2)}{(s+1)(s^2+2s+2)}$$

stability conditions:

The bode plot is drawn for o/l T/F.

$CE = 1 + GH = 0$

$\longrightarrow GH = -1 + j0$

The above eq. gives two conditions:

Angle condi: $\angle GH = \underline{|-1+j0}$

$= -180^\circ \longrightarrow \omega_{pc}$

Mag. condi:

$|G(j\omega).H(j\omega)| = 1$

$M_{in\ dB} = 0\,dB \longrightarrow \omega_{gc}$

$$GM = \frac{1}{|G(j\omega)H(j\omega)|}\bigg|_{\omega=\omega_{pc}} = -20\log|G(j\omega)H(j\omega)|_{\omega=\omega_{pc}}$$

$$PM = 180^\circ + \angle G(j\omega)H(j\omega)\Big|_{\omega=\omega_{gc}}$$

$\omega_{pc} > \omega_{gc} \longrightarrow$ stable

$\Longrightarrow$ GM > 1 (L) } & PM → +ve
+ve (in dB)

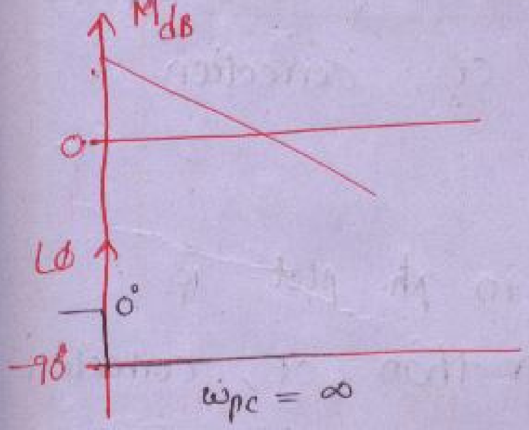
$\omega_{pc} = \omega_{gc} \longrightarrow$ M.S.

→ GM = 1 (L) } & PM = 0°
= 0 (dB)

$\omega_{pc} < \omega_{gc} \longrightarrow$ unstable.

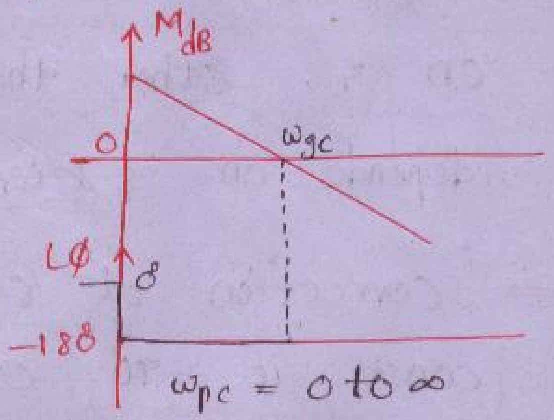
→ GM < 1 (L) } & PM → −ve.
−ve (dB)

Q. find system stability ?



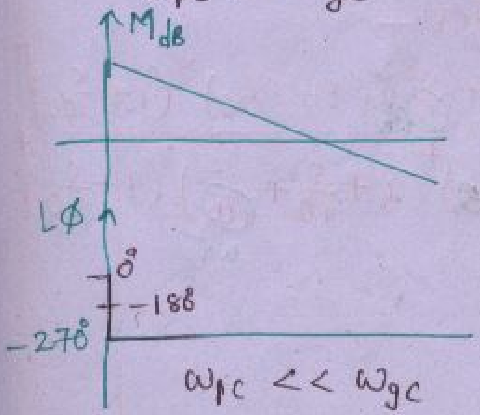
stable

$\omega_{pc} >> \omega_{gc}$



$\omega_{pc} = \omega_{gc}$

m.s.



$\omega_{pc} << \omega_{gc}$

unstable

* whenever plot maintains less -ve angle than -180° at all the freq-s then the $\omega_{pc} = \infty$.

* whenever system gives -180° at all the freq-s then the value of ω_{pc} may be any value b/w 0 to ∞. In this case the value of ω_{pc} decided by ω_{gc}.

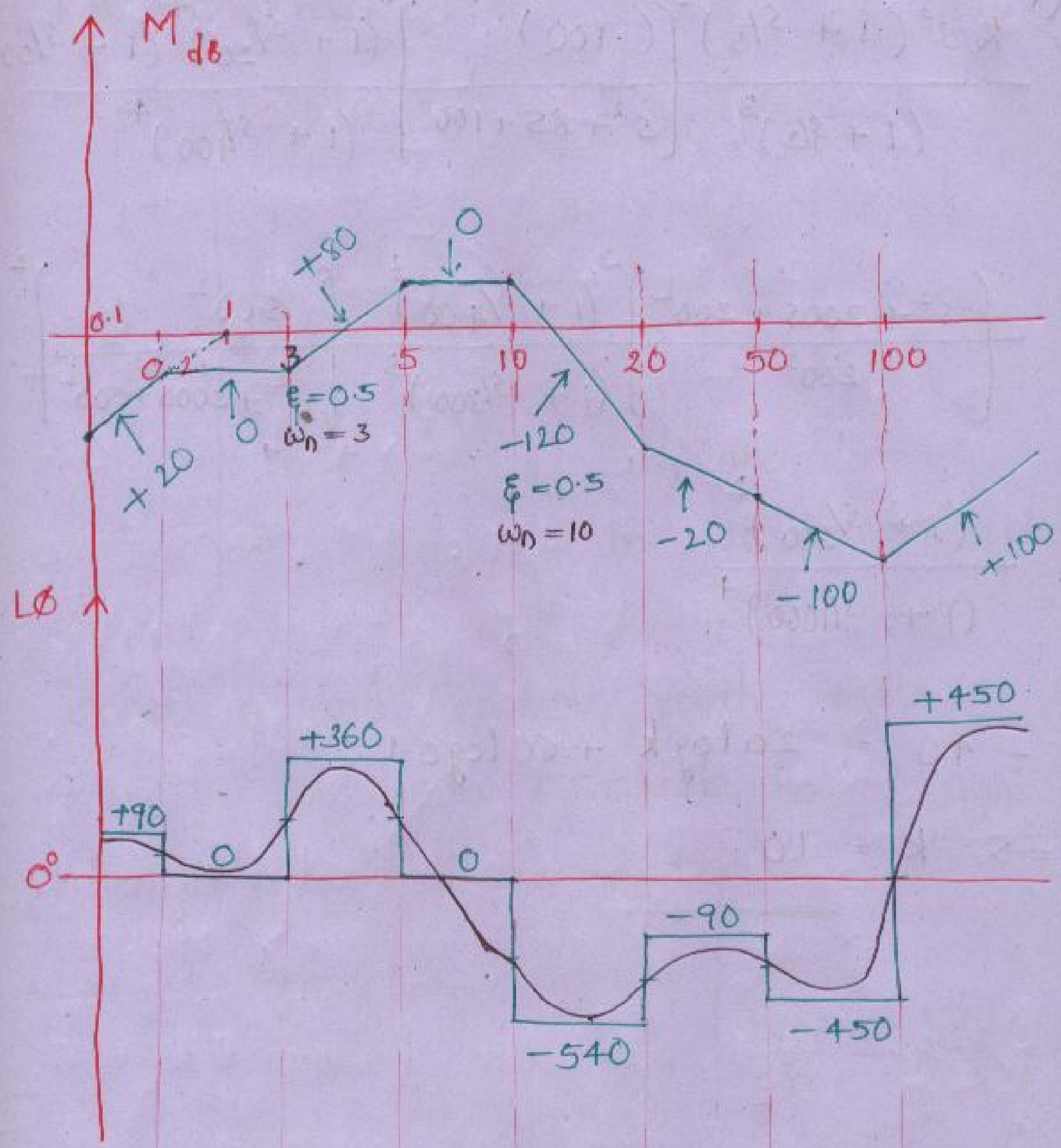
* whenever plot maintains the more -ve than -180° then $\omega_{pc} = 0$

→ correction at CF in mag. plot depends on ξ, other than CF, correction depends on ξ & ω_n.

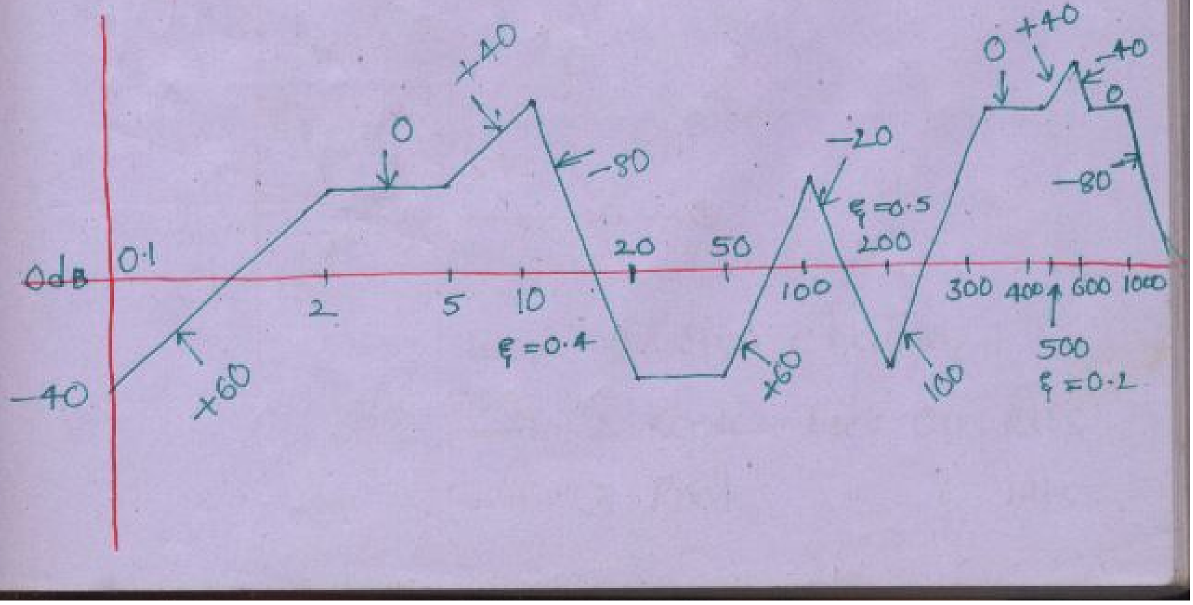
→ correction at CF in ph. plot is const. ie -90°. other than CF, correction depends on ξ & ω_n.

Q. Draw BODE PLOT for –

$$G(s)H(s) = \frac{s\left(1+s/3+\frac{s^2}{\underset{\omega_n^2}{9}}\right)^2\left(1+s/20\right)^5\left(1+\frac{s}{100}\right)^{10}}{1+\left(\frac{s}{0.2}\right)\left(1+s/5\right)^4\left(1+\frac{s}{10}+\frac{s^2}{\underset{\omega_n^2}{100}}\right)^3\left(1+\frac{s}{50}\right)^4}$$



Q. find the T/f to the given mag. plot.



TF:

$$\frac{k s^3 (1+s/5)^2}{(1+s/2)^3} \left[\frac{(100)}{s^2+8s+100}\right]^3 \frac{(1+s/20)^4 (1+s/50)^3}{(1+s/100)^4}$$

$$\left[\frac{s^2+200s+200^2}{200^2}\right]^3 \frac{(1+s/400)^2}{(1+s/300)^5} \left[\frac{500^2}{s^2+200s+500^2}\right]^2$$

$$\frac{(1+s/600)^2}{(1+s/1000)^4}$$

$$-40 = 20\log k + 60\log 0.1$$

$$\Rightarrow \quad \underline{\underline{k = 10.}}$$

R.H. CRITERIA:

Q. $s^4 + 2s^3 + 2s^2 + 4s + 8 = 0$

$$\begin{array}{c|ccc} s^4 & 1 & 2 & 8 \\ s^3 & \overset{1}{\cancel{2}} & \overset{2}{\cancel{4}} & \\ s^2 & \overset{\epsilon}{\cancel{0}} & 8 & \\ s^1 & \frac{2\epsilon-8}{\epsilon} & & \\ s^0 & 8 & & \end{array}$$

If any one coe. is zero in 1st column, replace zero by smallest +ve const. ϵ and continue routh table. finally sub. $\epsilon = 0$ & check no. of sign changes.

$$\begin{array}{c|l} s^4 & 1 \\ s^3 & 1 \\ s^2 & \overset{0}{\epsilon} \to 0 \\ s^1 & \frac{2\epsilon-8}{\epsilon} \to 2 - \frac{8}{\epsilon} = -\infty \quad (\epsilon \to 0) \\ s^0 & 8 \end{array}$$

$\Longrightarrow$ 2 sign changes.

Q. $s^5 + s^4 + 2s^3 + 2s^2 + 3s + 15 = 0$.

$$\begin{array}{c|ccc} s^5 & 1 & 2 & 3 \\ s^4 & 1 & 2 & 15 \\ s^3 & \overset{\epsilon}{\cancel{0}} & -12 & \\ s^2 & \boxed{\frac{2\epsilon+12}{\epsilon}} \to K & 15 & \\ s^1 & \frac{-12K-15\epsilon}{K} \longrightarrow -ve & & \\ s^0 & 15 & & \end{array}$$

$\Longrightarrow$ 2 sign changes.

$\rightarrow$ 2 Roots lies on RHS

$\rightarrow$ 3 Roots " " LHS.

Q $s^5 + s^4 + 3s^3 + 3s^2 + 2s + 2 = 0$

s^5	1	3	2
s^4	$1 s^4$	$3 s^2$	$2 s^0$
s^3	0 4	0 6	0 0 → AE
s^2	3/2	2	
s^1	2/3		
s^0	2		

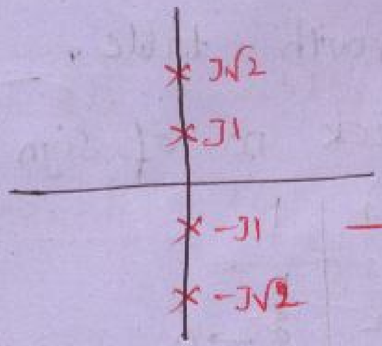
$s^4 + 3s^2 + 2 = 0$

$\xrightarrow{d} 4s^3 + 6s = 0.$

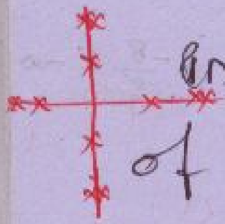
AE: $s^4 + 3s^2 + 2 = 0$

$\Rightarrow (s^2+2)(s^2+1) = 0$

$\rightarrow s = \pm j1, \pm j\sqrt{2}.$



→ Non-repeated roots on Ima. axis ⟹ M.S.



In Routh tabular form, the row of zeros occurs only when poles are located symmetrical about the origin.

In Routh, The AE consists only even powers b'coz roots of AE must be symmetrical about origin.

+
+
+
0 0 0 ⟹ Marginally stable.
+
+
+

* whenever only once one row of zeros occurs and all coe.s in 1st column are +ve then the system must be marginal stable. b'coz the poles must be on ima axis which are non repeated.

Q. $s^6 + 3s^5 + 4s^4 + 6s^3 + 5s^2 + 3s + 2 = 0.$

$$\begin{array}{c|cccc}
s^6 & 1 & 4 & 5 & 2 \\
s^5 & \cancel{3}^{1} & \cancel{6}^{2} & \cancel{3}^{1} & \\
s^4 & \cancel{\frac{2}{3}}\ 2s^4 & 4s^2 & 2s^0 & \\
s^3 & \cancel{0}^{8} & \cancel{0}^{8} & \cancel{0} & \\
s^2 & 2s^2 & 2s^0 & & \\
s^1 & \cancel{0}^{4} & \cancel{0} & & \\
s^0 & 2 & & &
\end{array}$$

AE$_1$: $2s^4 + 4s^2 + 2 = 0$

$\Longrightarrow (s^2+1)^2 = 0$

$\Longrightarrow s =$

AE$_2$: $2s^2 + 2 = 0$

$s = \pm j1.$

* whenever many times rows of zeros occurs & all coe.s are +ve, the system must be unstable. b'coz the poles on ima. axis which are repeated.

Q. Identify the location of poles for

(1).

s^7 | −

s^6 | + → AE$_1$ → 6th order ie 6 poles are symmetrical about origin.

s^5 | ~~0 0 0~~

s^4 | +

s^3 | ~~0 0 0~~

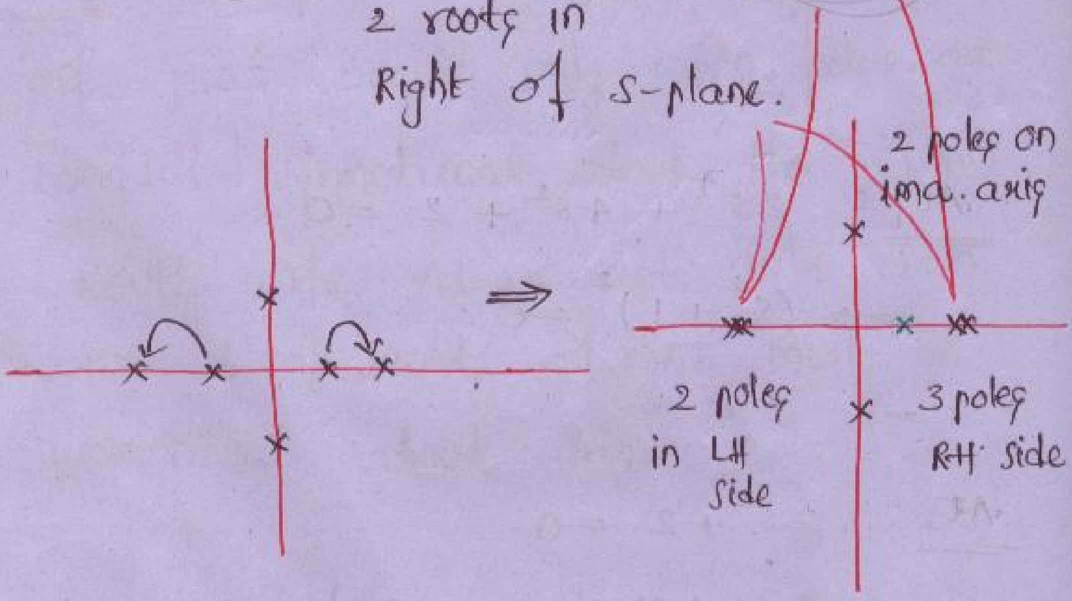
s^2 | +

s^1 | −

s^0 | +

2 sign changes ↓ 2 roots in Right of s-plane.

2 times row of zeros → 2 poles repeated



(2).

s^7	+
s^6	+
s^5	~~0 0 0~~
s^4	+
s^3	+
s^2	+
s^1	+
s^0	−

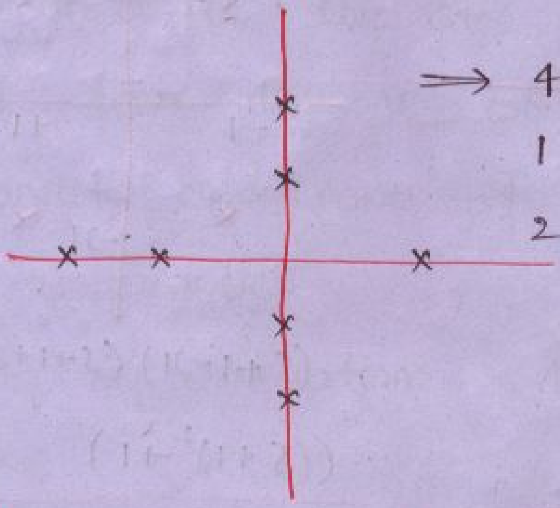
s^6 + → AE : → 6 th order.

6 poles symmetrical about origin.

One time row of zeros → so no repeated poles.

1 sign changes.

→ one pole on right side



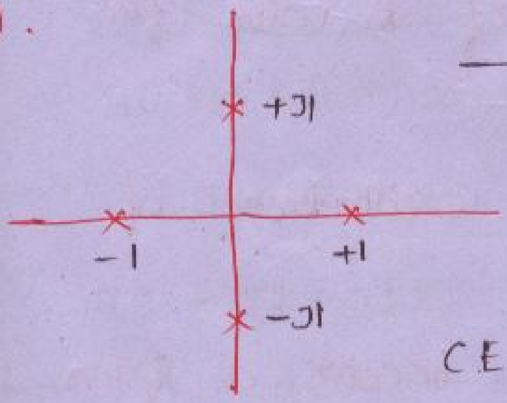
⟹ 4 pole → on ima. axis

1 pole → RH

2 poles → LH.

Q. Identify the routh tabular form for –

(1).



→ 1 time row of zeros.

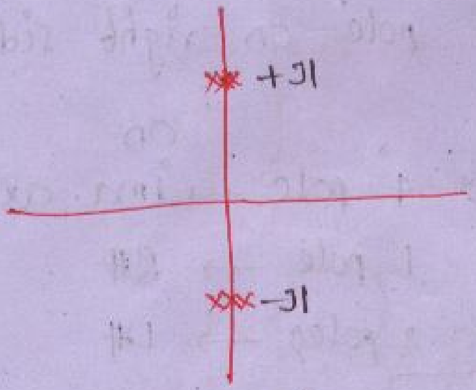
→ 1 sign changes.

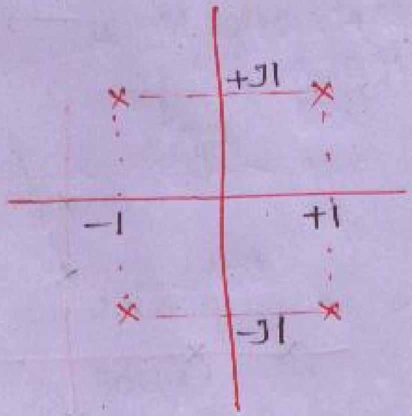
CE : $(s^2-1)(s^2+1)=0$

$\rightarrow s^4-1=0.$

$s^4 - 1 = 0$

$$\begin{array}{c|ccc}
s^4 & 1s^4 & 0s^2 & -1s^0 \\
s^3 & \cancel{0^4} & \cancel{0^0} & \cancel{0} \\
s^2 & \overset{\epsilon}{\cancel{0}} \rightarrow & -1 & \\
s^1 & 4/\epsilon & & \\
s^0 & -1 & &
\end{array}$$

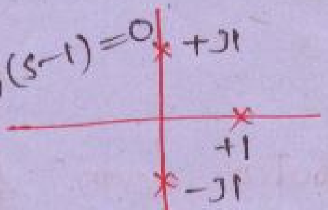




CE: $(s^2+1)^3 = 0$

$(s+1-j1)(s+1+j1)$

CE: $(s^2+1)(s-1) = 0$



$((s+1)^2+1)$

$(s^2+2s+2)(s^2-2s+2)=0$

Find the range of k value for system stability.

find the freq. of oscillations, if system is m.s.

find k value to become a system m.s.

(1). $s^3 + 5s^2 + 8s + k = 0.$

$(0 < k < 40)$

$8 \times 5 > k \times 1 \Rightarrow k < 40. \longrightarrow$ Stable.

$8 \times 5 = k \times 1 \Rightarrow k_{max} = 40.$

for m.s, not consider s^0 coe. b'coz if s^0 coe. $= 0$ then row of zeros occurs. for this case if form the AE, which consists odd power of s terms, which is undesirable.

$$\begin{array}{c|cc} s^3 & 1 & 8 \\ s^2 & 5 & k \\ s^1 & \frac{40-k}{5} & \\ s^0 & k & \end{array}$$

$\frac{40-k}{5} = 0 \rightarrow$ M.S.

$\frac{40-k}{5} > 0$, $k > 0 \; \} \rightarrow$ Ⓢ

freq. of oscillations : AE : $5s^2 + \overset{k}{\textcircled{40}} = 0$

$\Rightarrow s = \pm j\sqrt{8}.$

Q. $2s^3 + 9s^2 + 10s + (k+5) = 0.$

$90 > (k+5)2$ & $k+5 > 0$

$\Rightarrow -5 < k < 40 \longrightarrow$ stable.

$90 = (k+5)2$

$\Rightarrow k = 40 \longrightarrow$ M.S.

AE: $9s^2 + (\overset{40}{\cancel{k}}+5) = 0$

$\Rightarrow s = \pm j5 \longrightarrow$ freq of oscillations.

Q $GH = \dfrac{k}{s(s+2)(s+4)(s+6)}$

CE: $s(s+2)(s+4)(s+6) + k = 0$

$$\Rightarrow s\left[s^3 + 12s^2 + \overset{(8+12+24)}{44}s + \overset{48}{\cancel{32}}\right] + k = 0$$

$$\Rightarrow s^4 + 12s^3 + 44s^2 + \overset{48}{\cancel{32}}s + k = 0.$$

$$\begin{array}{c|ccc}
s^4 & 1 & 44 & k \\
s^3 & \overset{1}{\cancel{12}} & \overset{4}{\cancel{48}} & \\
s^2 & 40 & K & \\
s^1 & \dfrac{160-k}{40} > 0 & & \\
s^0 & K > 0 & &
\end{array}$$

AE: $40s^2 + 160 = 0$

$\rightarrow s = \pm j2$ (FOO)

$k_{mar} = 160$

$\rightarrow = 0 \rightarrow$ M.S

$\rightarrow$ Stable

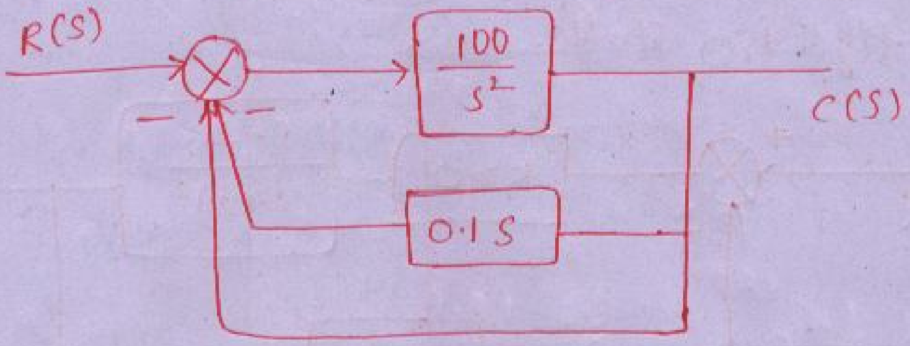
$= 0 < k < 160$

Q. Determine k & b, oscillates at a freq. of 2 rad/sec

$G(s) = \dfrac{k(s+1)}{s^3 + bs^2 + 3s + 1}$ & $H(s) = 1$.

The system oscillates at 2 rad/sec means system is M.S.

Q. find system stability — ?



$$T/f = \frac{C(S)}{R(S)} = \frac{100}{s^2+10s+100}$$

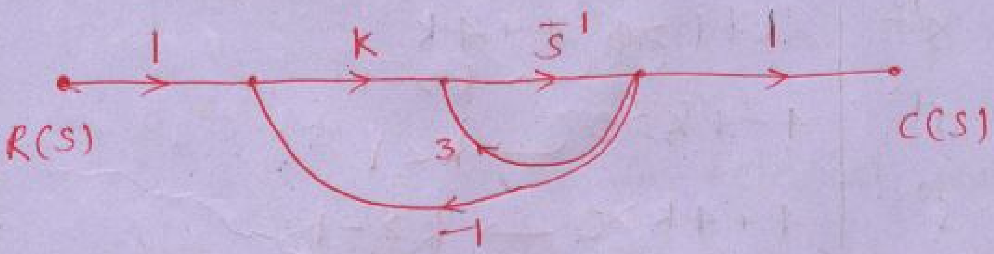
$s^2+10s+100=0.$

$a, b, c > 0 \implies$ stable.

s^2	1	100
s^1	10	
s^0	100	

+ve

Q. The system shown in fig. remain stable when (a). $k<-1$ (b). $k<1$ & $k>-1$ (c). $1<k<3$ [✓ (d).] $k>3$.



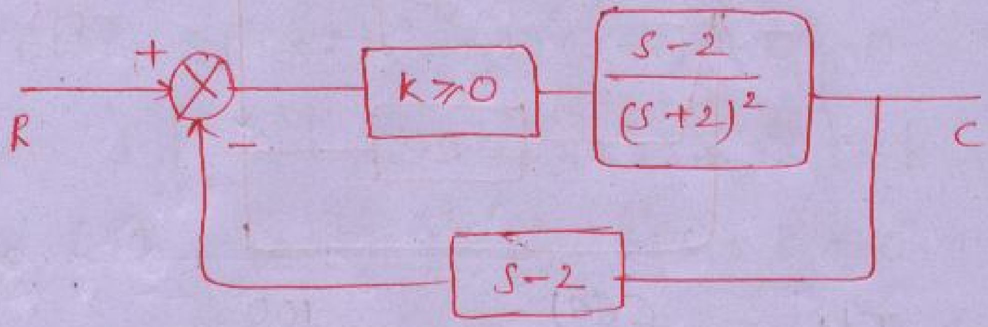
$$\frac{C}{R} = \frac{k/s}{1-3/s+\frac{k}{s}}$$

$$= \frac{k}{s-3+k}$$

s^1	1
s^0	$k-3>0$

$\implies k>3.$

Q. The F16 control system shown in fig.. is stable



(a). $\forall k \geq 0$ (b). only if $k \geq 0$

(c). only if $0 \leq k < 1$ (d). only if $0 \leq k \leq 1$.

$$\frac{C}{R} = \frac{k(s-2)}{(s+2)^2 + k(s-2)^2}$$

CE: $s^2(1+k) + s(4-4k) + 4+4k = 0$

$$\begin{array}{c|ll} s^2 & k+1 > 0 & 4+4k \\ & \hookrightarrow k > -1 & \\ s^1 & 4-4k > 0 \hookrightarrow k < 1 & \\ s^0 & 4+4k > 0 \longrightarrow k > -1 & \end{array}$$

Ans: $0 \leq k < 1$

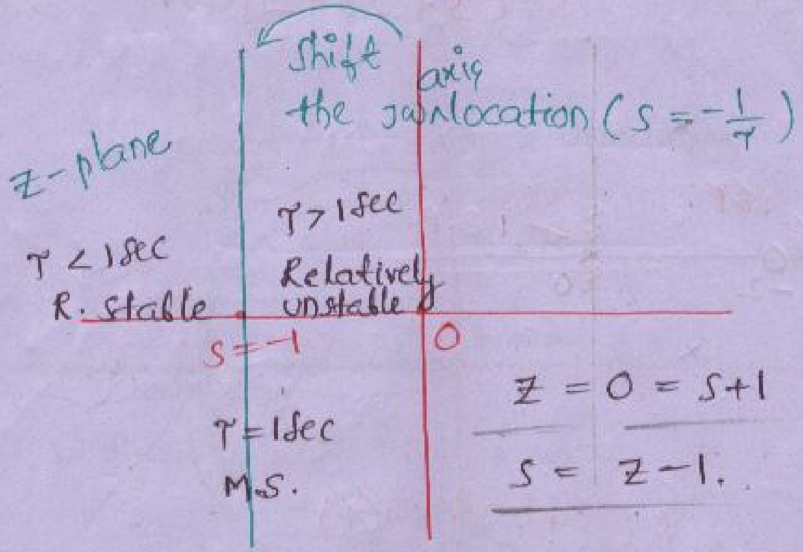
Relative stability is applicable only for c/L system.

Q. A system has $G(s) = \frac{2}{s(s+1)(s+2)}$

$H(s) = 1$. with RH criteria Determine

RS about the point or line $s = -1$.

CE: $s^3 + 3s^2 + 2s + 2 = 0 \rightarrow$ stable.



$z = 0 = s+1$

$s = z-1.$

$\rightarrow (z-1)^3 + 3(z-1)^2 + 2(z-1) + 2 = 0$

$\Rightarrow z^3 - z + 2 = 0 \rightarrow$ Relatively unstable.

z^3	1	-1
z^2	$\overset{\epsilon}{\cancel{0}} \rightarrow$	2
z^1	$\frac{-\epsilon-2}{\epsilon} \rightarrow -\infty$	
z^0	2	

2 roots b/w 0 & -1.

1 root at left of $s = -1$

∴ Relatively unstable.

Q. Check whether the τ is greater or lesser or equal to 1 sec, for $s^3 + 7s^2 + 25s + 39 = 0$.

CE: $s^3 + 7s^2 + 25s + 39 = 0$.

$s = z +$ axis shift location $[s = -\frac{1}{\tau}]$

$s = -1.$

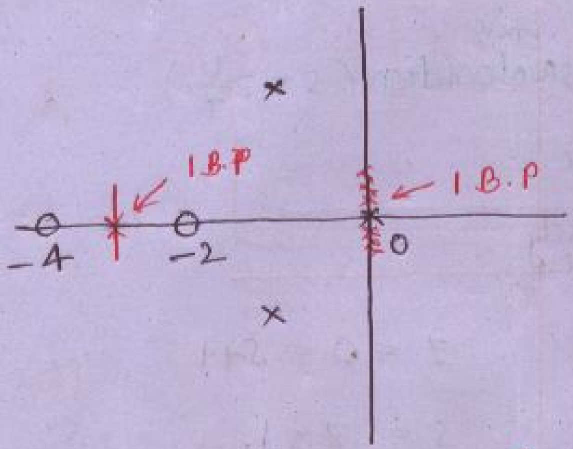
$s = z - 1$

$\Rightarrow z^3 + 4z^2 + 14z + 20 = 0.$

$\rightarrow$ Relatively stable ∴ $\tau < 1sec.$

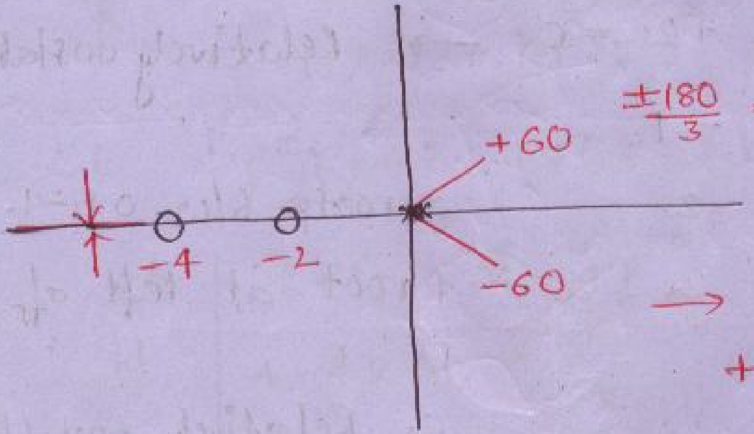
Q. Determine no. of break points.

$$GH(s) = \frac{K(s+2)(s+4)}{s^2(s^2+2s+2)}$$



$$GH(s) = \frac{K(s+2)(s+4)}{s^3}$$

$$\frac{\pm 180}{3} = \pm 60$$

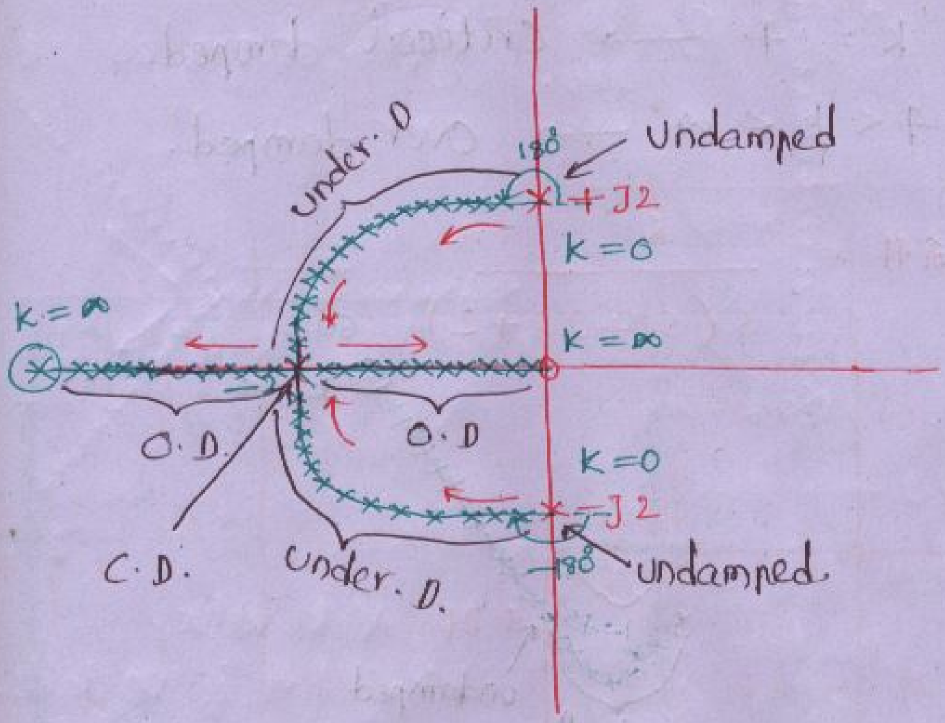


→ 2 B. points

+1 Break in

Q. $\frac{ks}{s^2+4} \longrightarrow$ Break point $= -2$.

(~~+2~~, -2)



Angle of departure $\phi_d : 180° - \phi$

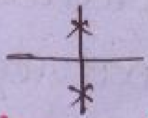
$\phi = 90° - 90° = 0.$

$\therefore \phi_d = 180°.$

$k > 0 \longrightarrow$ stable.

$\Rightarrow \quad 0 < k < \infty \longrightarrow$ stable.

$k = 0 \longrightarrow$ M.S.

when $k = 0 \longrightarrow$ UNDAMPED 

$$\left| \frac{ks}{s^2+4} \right|_{s=-2} = 1 \longrightarrow$$ {for finding, k value at break point.}

By using magnitude condition.

$\Rightarrow \quad k = 4$

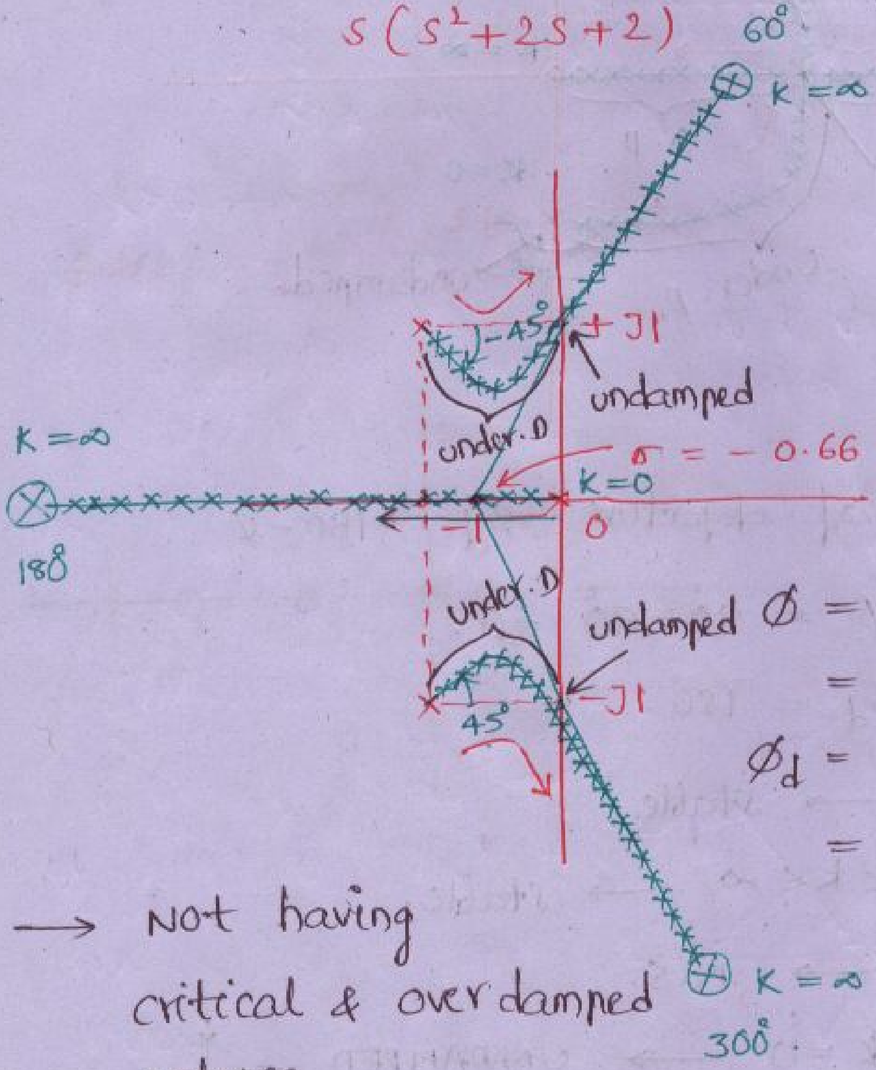
∴ $k = 0 \longrightarrow$ undamped.

$0 < k < 4 \longrightarrow$ under damped.

$k = 4 \longrightarrow$ critical damped

$4 < k < \infty \longrightarrow$ over damped

Q. $$GH = \frac{k}{s(s^2+2s+2)}$$



$\phi = 135 + 90$

$= 225$

$\phi_d = 180 - 225$

$= -45°.$

→ Not having critical & over damped natures.

CE : $s^3 + 2s^2 + 2s + k = 0$

$k_{max} = 4$

$0 < k < 4 \rightarrow$ under damped (stable)

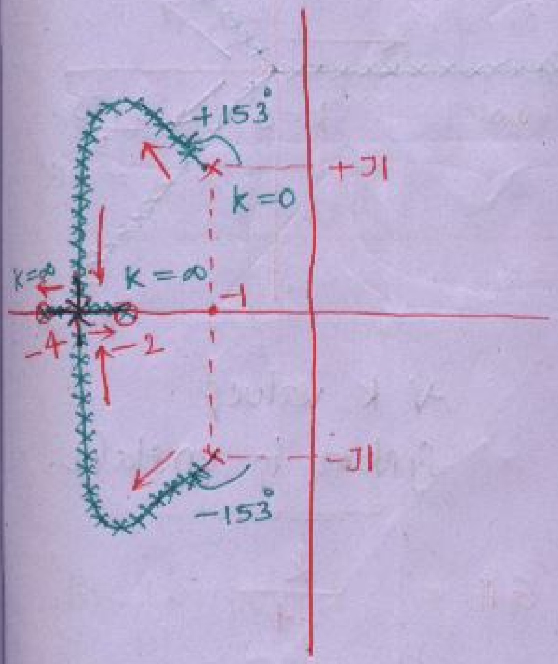
$k = 4 \rightarrow$ undamped (M.S)

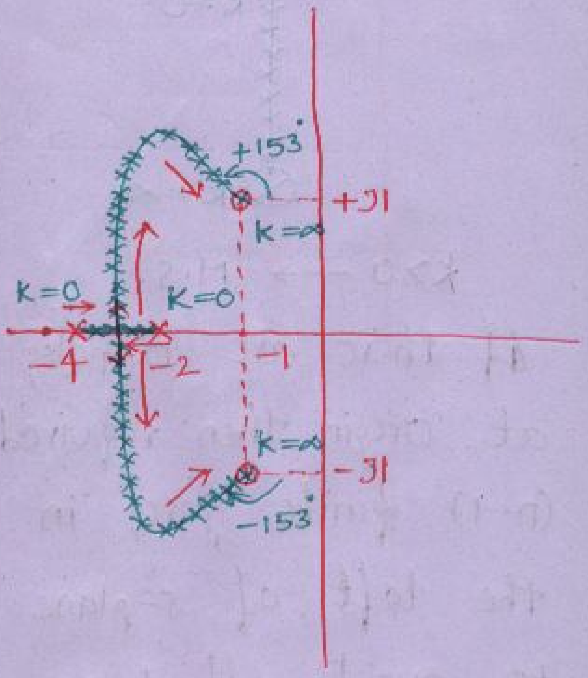
Q.

$$GH = \frac{k(s+2)(s+4)}{(s^2+2s+2)}$$

Q.

$$GH = \frac{k(s^2+2s+2)}{(s+2)(s+4)}$$





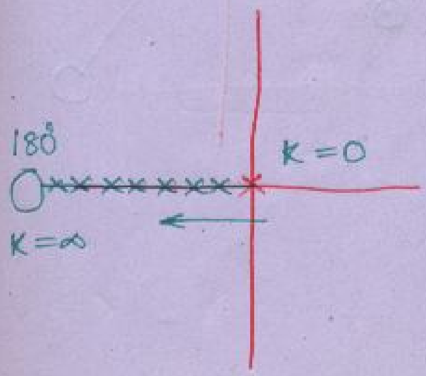
Q. $\frac{k}{s}$, $\frac{k}{s^2}$

NOTE: whenever T/F consists pole at origin then RL diagram is nothing but angle of Asymptote line.

$GH = \frac{k}{s}$

No. of Asymptotes (N) = 1

(θ) = 180°

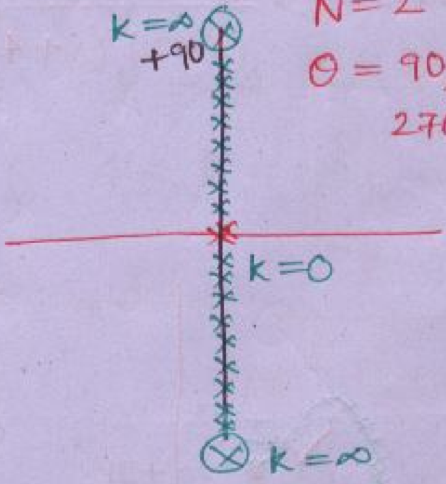


k > 0 ⟶ stable.

$$GH = \frac{k}{s^2}$$

N = 2

$\theta = 90, 270°.$

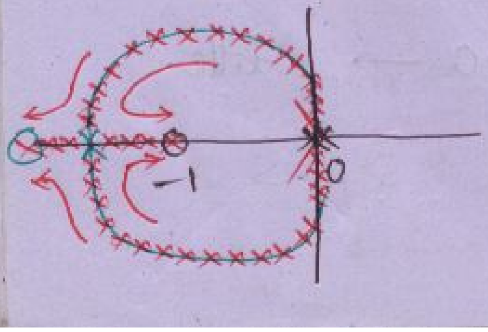


$k > 0 \longrightarrow$ M.S.

If there are n poles at origin then required (n−1) finite zeros in the left of s-plane to avoid affect on stability.

So by adding one finite zero, the above system becomes stable.

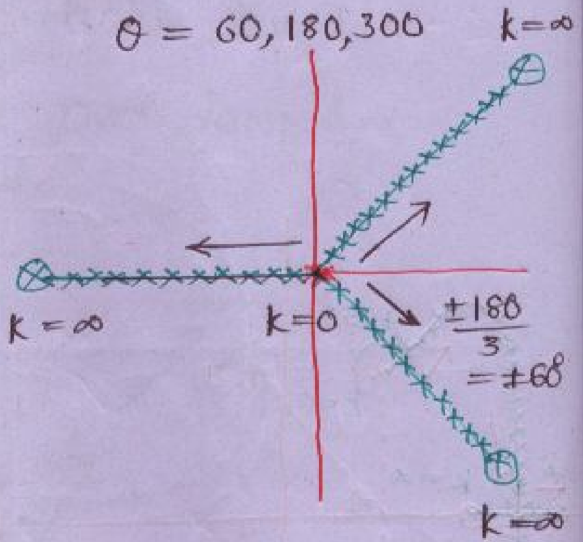
eg: $GH = \dfrac{k(s+1)}{s^2}$



$$GH = \frac{k}{s^3}$$

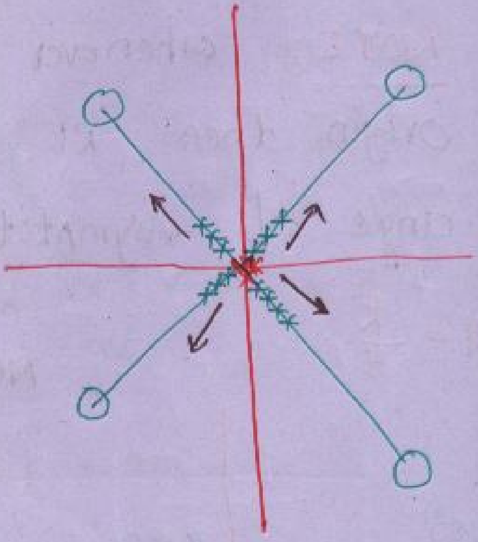
N = 3.

$\theta = 60, 180, 300$



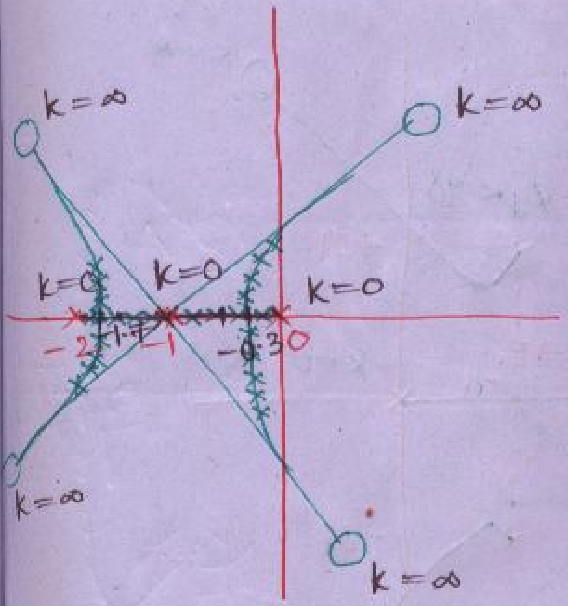
∀ k values system is unstable.

$$GH = \frac{k}{s^4}$$



Q.

$$GH = \frac{k}{s(s+1)^2(s+2)}$$



Break points :

$s(s+1)^2(s+2) = 0$

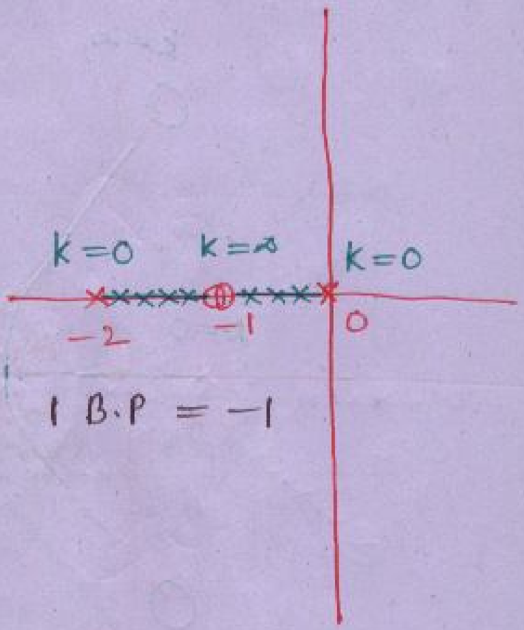
$\Rightarrow 4s^3 + 12s^2 + 10s + 2 = 0$

3 B.P'S.

$\rightarrow -0.3, -1, -1.7$

Q.

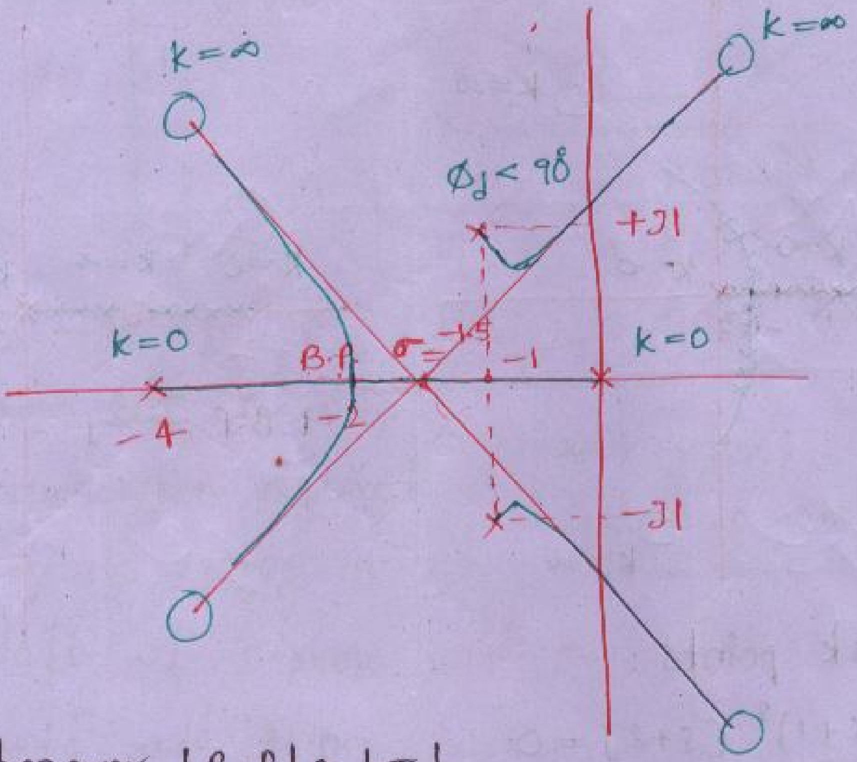
$$GH = \frac{k(s+1)^2}{s(s+2)}$$



1 B.P = −1

Q. $GH = \dfrac{K}{s(s+k_1)(s^2+2s+2)}$

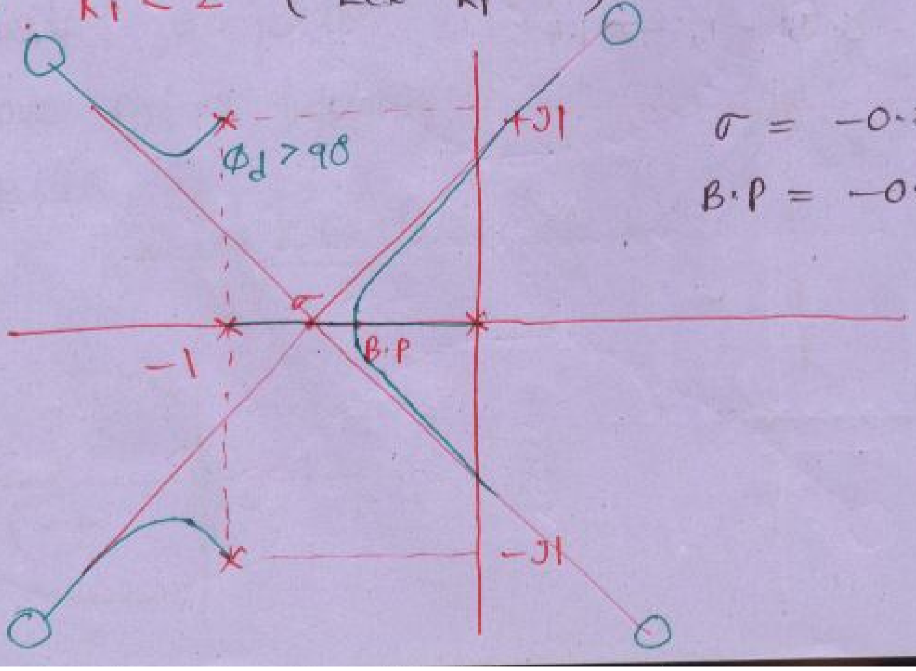
(i). $k_1 > 2$ (Let $k_1 = 4$) ; B.P = −2.



whenever |B.P| > |σ|

then $\phi_d < \mp 90°$.

(ii). $k_1 < 2$ (Let $k_1 = 1$).



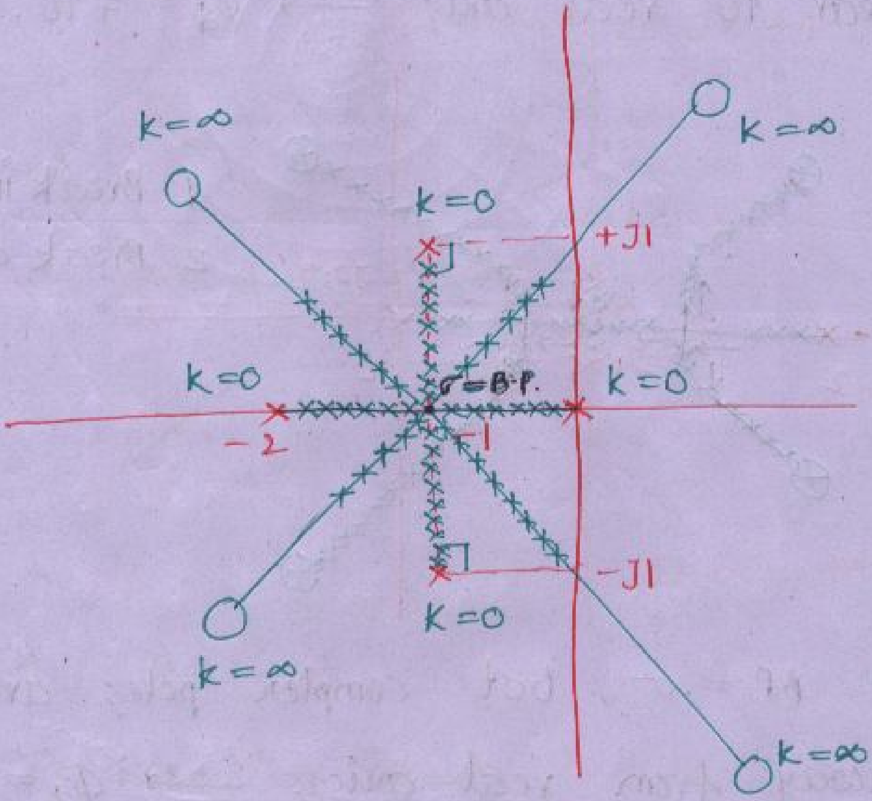
σ = −0.75

B.P = −0.5

whenever $|B.P| < |\sigma|$ then $\phi_d > \mp 90$.

(iii). $k_1 = 2$.

$$GH = \frac{k}{s(s+2)(s^2+2s+2)}$$



whenever $B.P = \sigma$ then $\phi_d = \mp 90$.

In above system, the no. of poles meet at B.P = 4.

The k value at B.P is 1.

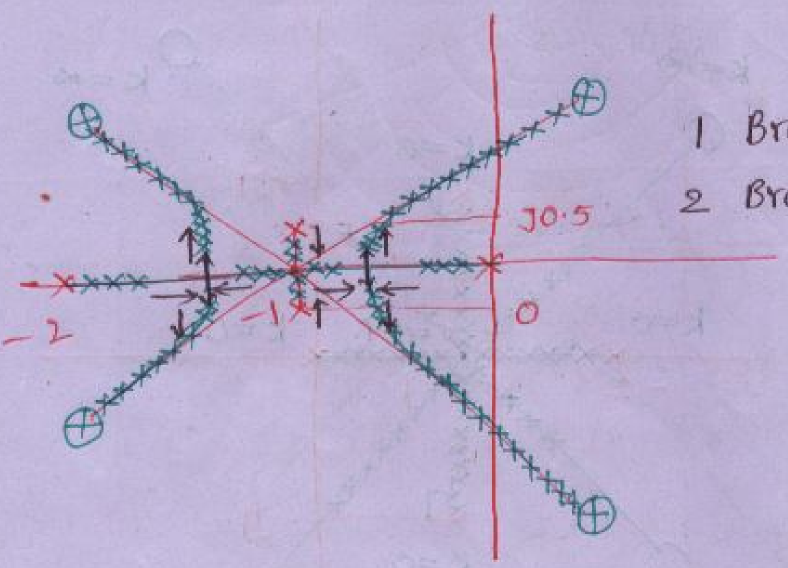
$$\left| \frac{k}{s(s+2)(s^2+2s+2)} \right|_{s=-1} = 1$$

$\rightarrow$ $k = 1$.

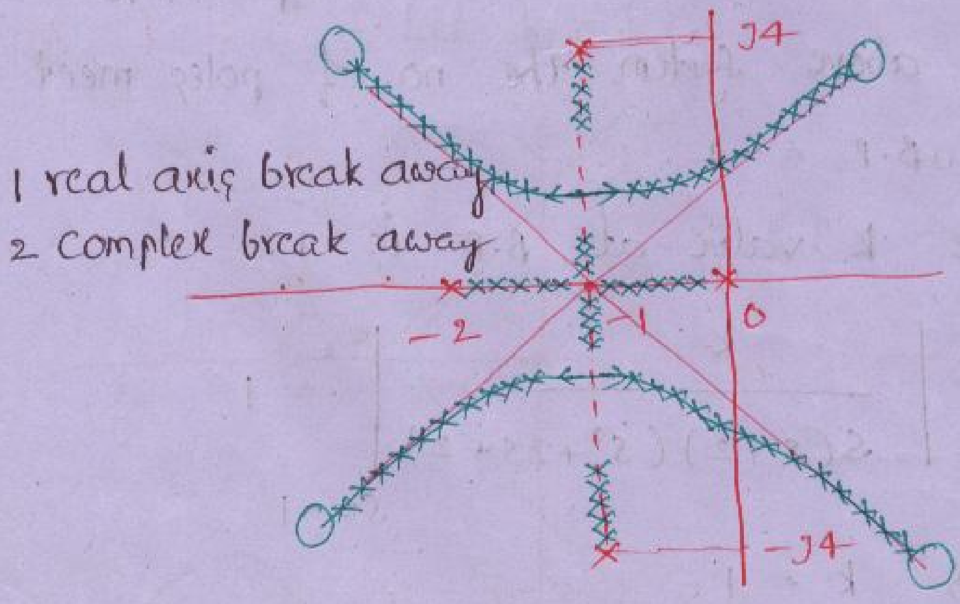
The C/L TF at B.P is $\frac{1}{(s+1)^4}$

$$\left\{ \frac{1}{s(s+2)(s^2+2s+2)+1} \right\}$$

** B.P = σ, but complex poles very near to real axis → $\phi_d = \mp 90$.

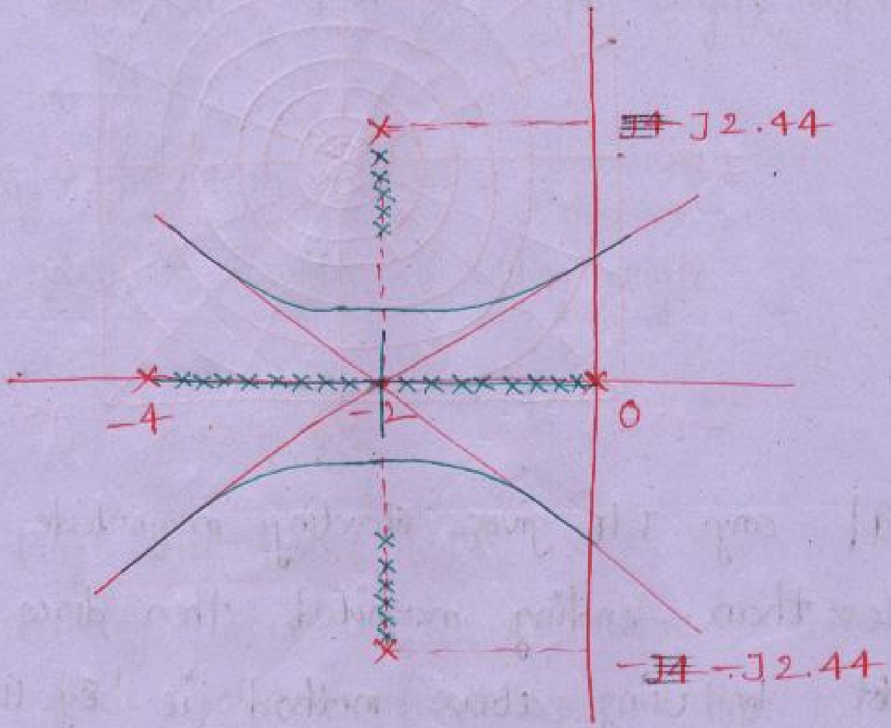


** BP = σ, but complex poles are away from real axis. → $\phi_d = \mp 90$



whenever centroid = real part of a complex pole

Q. Draw RL for $GH = \frac{k}{s(s+4)(s^2+4s+20)}$



$$s^4 + 8s^3 + 36s^2 + 80s$$

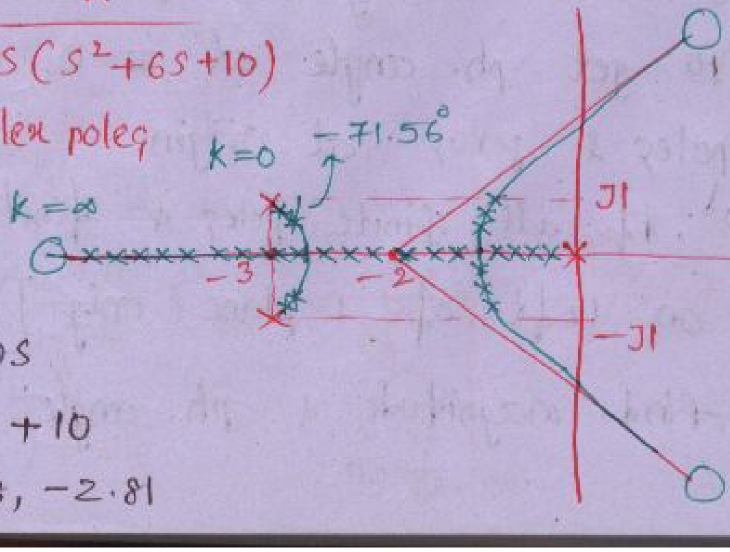
$$\xrightarrow{d} 4s^3 + 24s^2 + 80 = 0$$

$$— \quad s = -2 \pm j2.44, -2$$

$$\sigma = B.P = -2.$$

Q. $GH = \frac{K}{s(s^2+6s+10)}$

BP ≠ σ & complex poles near to real axis.



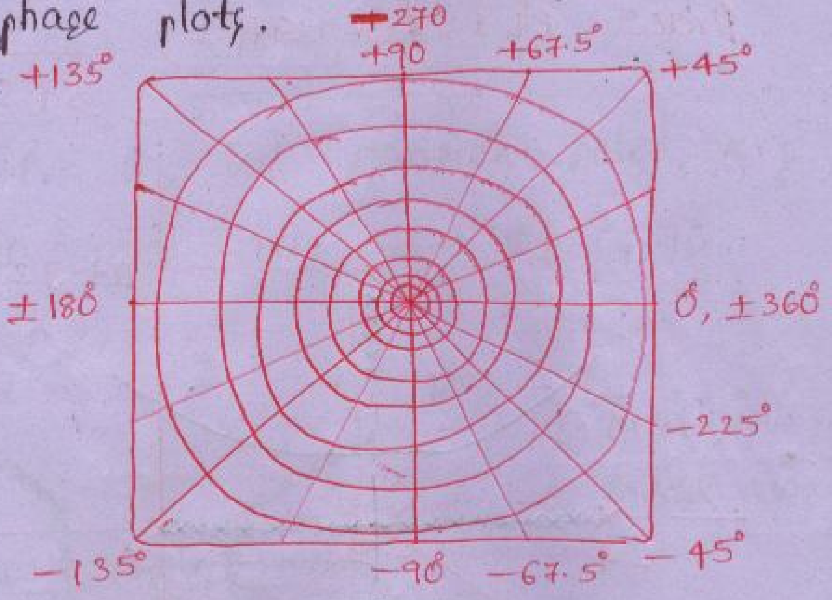
$$s^3 + 6s^2 + 10s$$

$$\xrightarrow{d} 3s^2 + 12s + 10$$

$$\longrightarrow s = -1.18, -2.81$$

POLAR PLOTS:

→ polar plots are nothing but Magnitude Vs phase plots.



If any T/F gives starting magnitude is less than ending magnitude then draw polar plot by using above method ie Eg (i) & (ii) $\left\{GH = \frac{1}{1+sT}\right\}$ in old notes.

PROCEDURE TO DRAW POLAR PLOT:-

* find magnitude & ph. angle at $\omega = 0$.

To get magnitude at $\omega = 0$, simply substitute $s = 0$ in the given T/F.

To get ph. angle at $\omega = 0$, consider no. of poles & zeros at origin.

[valid, If all finite poles & finite zeros lies on left of s-plane only].

* find magnitude & ph. angle at $\omega = \infty$.

To find magnitude at $\omega = \infty$, Simply substitute $s = \infty$ in the given T/F.

To get ph. angle at $\omega = \infty$, consider the algebraic sum of ph. angles of all poles & zeros.

* Ending direction : [ED]

= starting angle − ending angle

= +ve ⟶ cw

= −ve ⟶ ccw

* starting direction : [SD]

starting direction is considered when all finite poles & finite zeros lies in the 1st quadrant only.

If finite pole is near to ima. axis, the plot push towards cw dirc.

If finite zero is near to ima. axis then plot push towards ccw dirc.

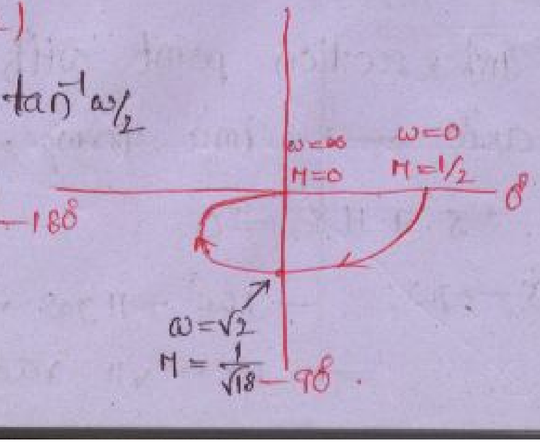
Q. $GH = \frac{1}{(s+1)(s+2)}$

$-90^\circ = -\tan^{-1}\omega - \tan^{-1}\omega/_2$

$90 = \tan^{-1}\left(\frac{\omega + \omega/_2}{1 - \omega^2/_2}\right)$

$\Rightarrow \infty = \frac{\omega + \omega/_2}{\left(1 - \frac{\omega^2}{2}\right)} \rightarrow = 0$

$\Longrightarrow \omega = \sqrt{2}$.



$$M\Big|_{\omega=\sqrt{2}} = \frac{1}{\sqrt{(1+\omega^2)(4+\omega^2)}}$$

$$= \frac{1}{\sqrt{18}}$$

Intersection point $= \left(0, -\frac{j}{\sqrt{18}}\right)$

$$\frac{K}{(1+sT_1)(1+sT_2)} \; ; \quad M = \frac{K\sqrt{T_1T_2}}{T_1+T_2}$$

$$= \frac{0.5\sqrt{1\times 1/2}}{1+1/2}$$

$$= \frac{1}{\sqrt{18}}.$$

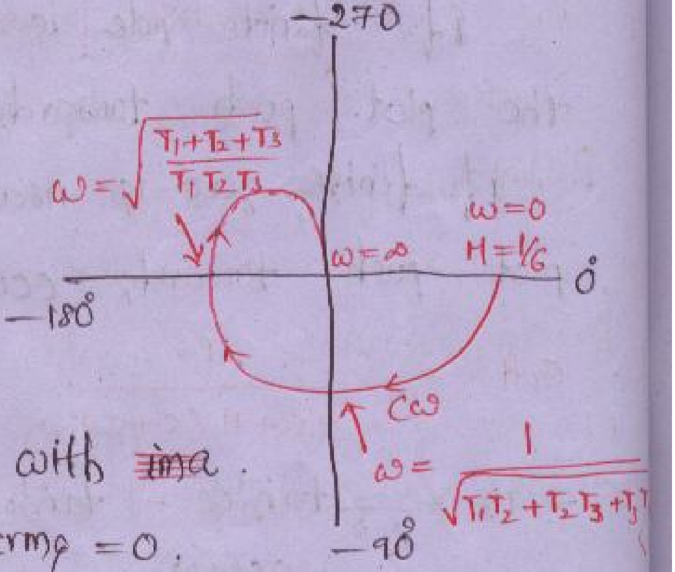
Q. $GH = \frac{1}{(s+1)(s+2)(s+3)}$

At $\omega = 0$, $1/6 \angle 0^\circ$.

At $\omega = \infty$, $0 \angle -270^\circ$

ED : $\longrightarrow$ CW

SD : $\longrightarrow$ CW



Expand terms:

$s^3 + 6s^2 + 11s + 6$

Intersection point with ~~ima~~ real axis $\longrightarrow$ ima. terms $= 0$.

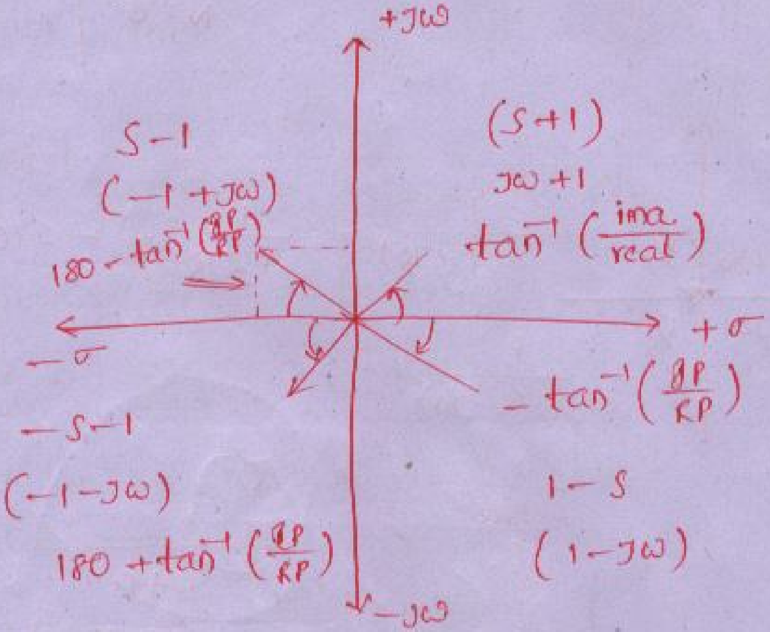
$s^3 + 11s = 0$

$s \to j\omega$; $\quad -j\omega^3 + 11j\omega = 0$

$\Rightarrow \omega = \sqrt{11}$ rad/sec.

$$M = \frac{1}{\sqrt{(1+\omega^2)(4+\omega^2)(9+\omega^2)}}\Bigg|_{\omega=\sqrt{11}}$$

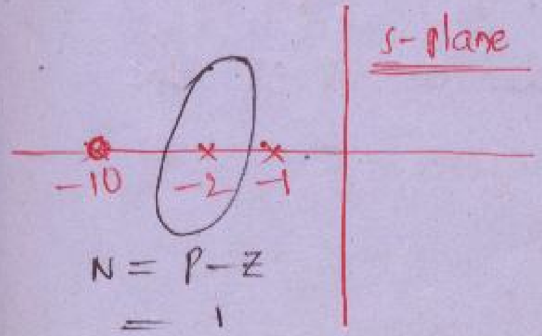
$$= \frac{1}{60}.$$



NYQUIST PLOTS:

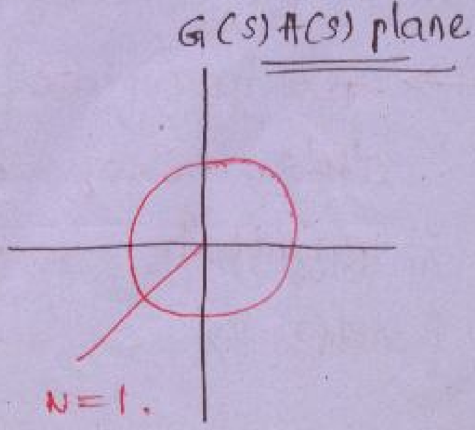
- * To draw complete freq. response of o/L T/F.
- * To findout c/L system stability.
- * To findout no. of c/L poles in the right of s-plane.
- * To find range of k-value for system stability.
- * To find gain margin, ph. margin, ω_{gc}, ω_{pc}.

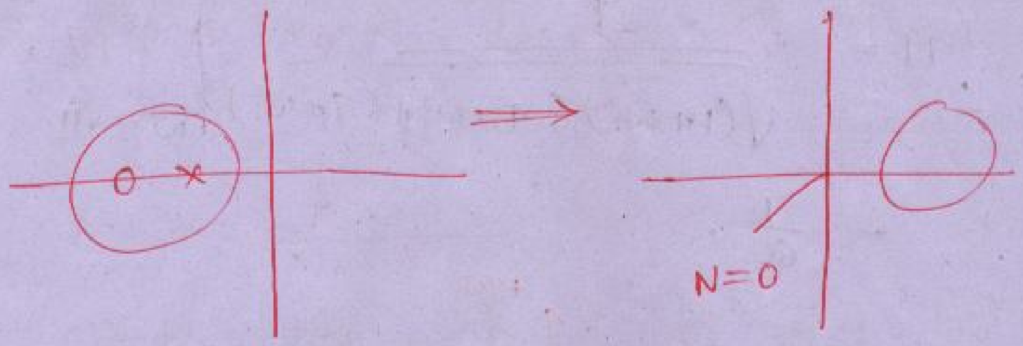
$$GH = \frac{(s+10)}{(s+1)(s+2)}$$

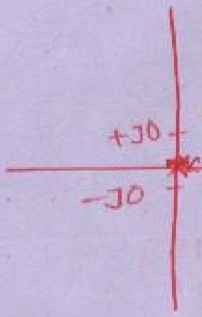


$N = P - Z$

$= 1$







$$\frac{1}{s^3} = \frac{1}{3(\angle +j0)} = \frac{1}{+270} = -270^\circ.$$

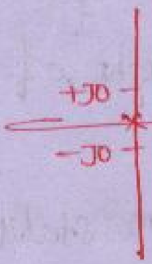
$$\frac{1}{s^3} = \frac{1}{3(\angle -j0)}$$

$$= \frac{1}{-270^\circ} = +270^\circ$$

$$\overline{540}$$

$$=(180 \times 3)$$

→ no. of infinite radius half o'le's = no. of poles at origin.



$$\frac{1}{s} = \frac{1}{\angle +j0} = \frac{1}{+90} = -90^\circ$$

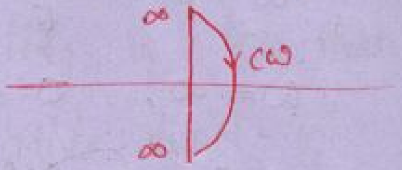
$$\frac{1}{s} = \frac{1}{\angle -j0} = \frac{1}{-90} = +90$$

$$\overline{180}$$

$$=(180 \times 1)$$

→ for zeros we dont get infinite radius o'le's. b'coz at ω = 0- & 0+ the magnitude becomes zero.

→ The infinite radius half o'le dirc. always in the cw b'coz the infinite radius half o'le dirc. completely depends on Nyquist contour dirc.



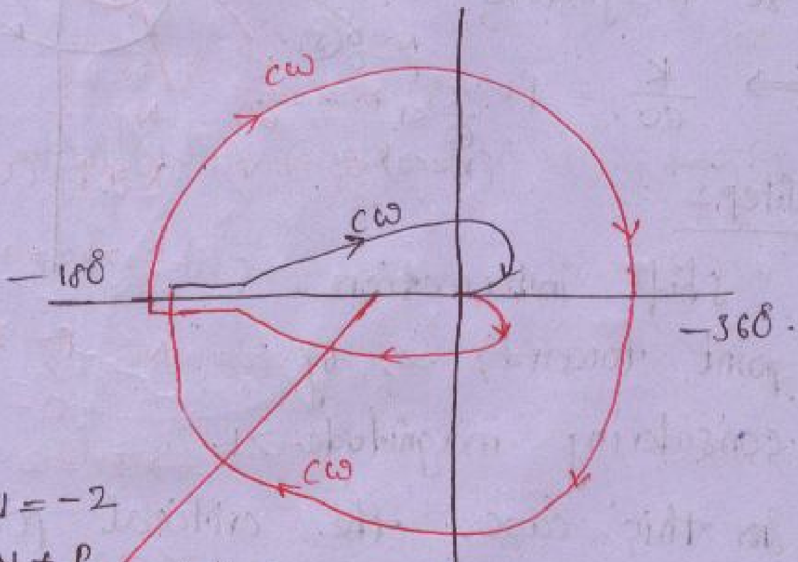
Q. $GH = \dfrac{10}{s^2(s+1)(s+2)} \xrightarrow{} OL\ (P=0).$

$\omega = 0 \Rightarrow \infty \angle -180^\circ$

$\omega = \infty \Rightarrow 0 \angle -360$

ED → cw

SD → cw.



$N = -2$

$N \neq P$ (unstable)

The no. of cl poles given in RH s-plane is given by principle of arguments.

$N = P - Z$

$\Rightarrow -2 = 0 - Z \Rightarrow Z = 2$ [cl poles in RH s plane].

Q. find range of k-value for system stability. $GH = \frac{k}{(s+1)(s+2)(s+3)}$

$\omega = 0, \quad \frac{k}{6} \angle 0$

$\omega = \infty, \quad 0 \angle -270$

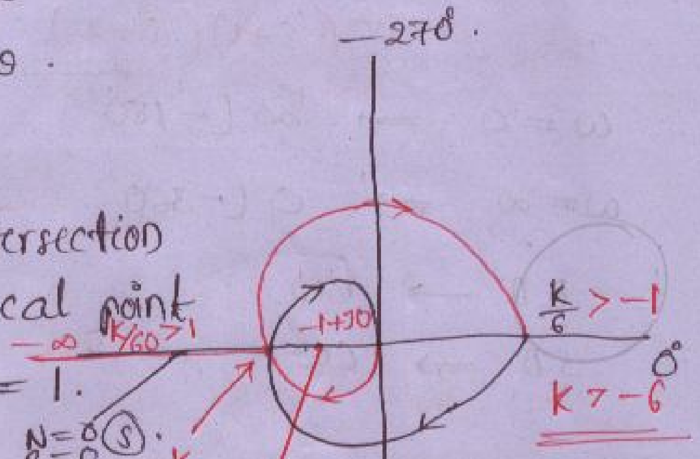
ED → CW

SD → CW.

Step 1 :

Assume intersection point = critical point ie magnitude = 1.

$\Rightarrow \frac{k}{60} = 1.$ $\frac{k}{60} < 1$ $(k < 60)$



$M = \frac{k}{60}$ $\omega = \sqrt{11}$

N = −2

N ≠ P (unstable)

$\frac{k}{60} > 1 \Rightarrow k > 60$ (U.S).

N = P − Z

−2 = 0 − Z

⇒ Z = 2 CL poles on RH s-plane.

Step 2 :

Shift intersection point towards ∞, by considering magnitude >1. In this case, the critical point lies in side the loop.

for this case find no. of encirclements and get one condi. for stability.

Step 3 :

Shift intersection point towards origin by considering mag. < 1, in this case critical point is outside the loop, for this case find no. of encirclements and get the condition for stability

If the condi. for stability is less than certain value then the other limit is given by intersection point with 0° by considering magnitude of intersection point > −1.

$\Longrightarrow \quad -6 < K < 60 \longrightarrow$ stable.

Q. $GH(s) = \dfrac{K(s+2)}{(s+1)(s-1)}$.

$\longrightarrow P = 1$

$\phi = -\tan^{-1}(\omega) - (180 - \tan^{-1}\omega) + \tan^{-1}\frac{\omega}{2}$.

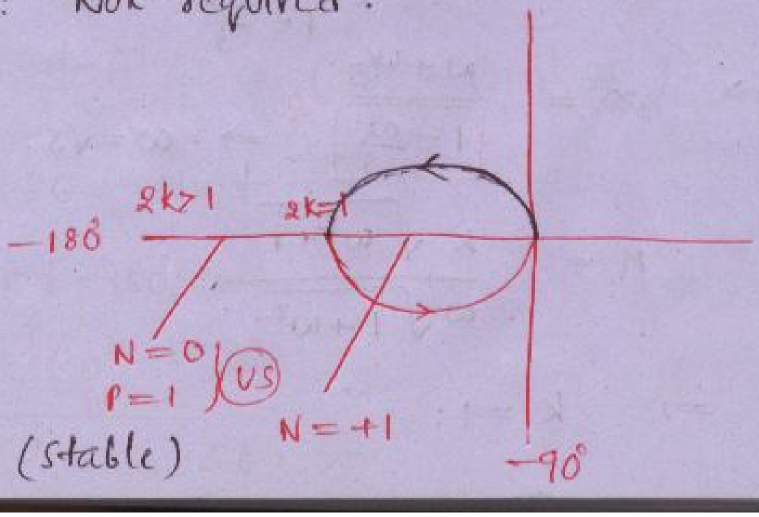
$= -180 + \tan^{-1}\frac{\omega}{2}$.

$\omega = 0; \quad 2K \angle -180^\circ$

$\omega = \infty; \quad 0 \angle -90^\circ$

ED : ACW

SD : Not required.



N = P

2k > 1

⟶ K > ½ (stable)

$N = P - Z$

$0 = 1 - Z$

$\Rightarrow Z = 1$ (CL RH pole)

Q. $GH = \dfrac{K(s+3)}{\textcircled{s}(s-1)} \longrightarrow \left.\begin{matrix} P=1 \\ N=1 \end{matrix}\right\} \textcircled{S}$

(1). half ole.

$K > 1 \rightarrow \textcircled{S}$.

$\phi = -90 - (180 - \tan^{-1}\omega) + \tan^{-1}\left(\frac{\omega}{3}\right)$

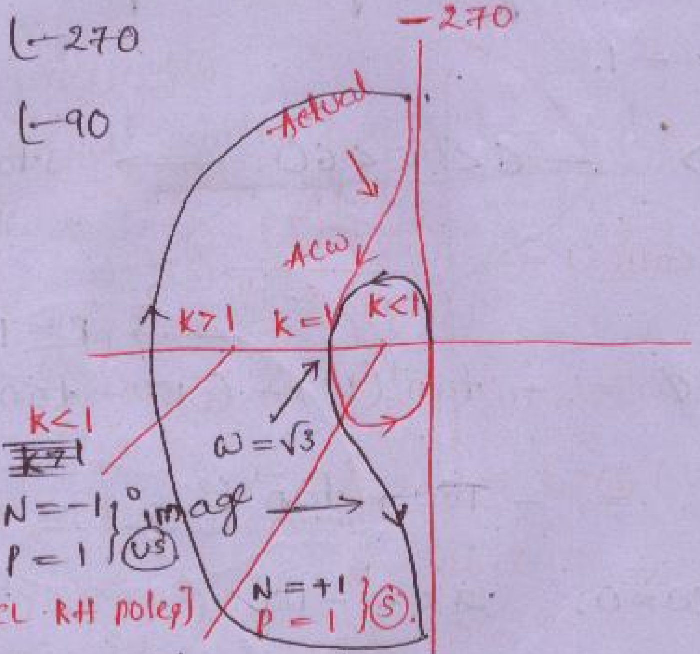
$= -270 + \tan^{-1}\omega + \tan^{-1}\omega/3$.

$\omega = 0; \quad \infty \angle -270$

$\omega = \infty; \quad 0 \angle -90$

ED: ACW

SD: X



$N = P - Z$

$\Rightarrow -1 = 1 - Z$

$\Rightarrow Z = 2$ [2 CL RH poles]

$-180 = -270 + \tan^{-1}\omega + \tan^{-1}\frac{\omega}{3}$ -90

$90 = \tan^{-1}\omega + \tan^{-1}\frac{\omega}{3}$

$\Rightarrow 90 = \tan^{-1}\left[\dfrac{\omega + \omega/3}{1 - \frac{\omega^2}{3}}\right]$

$\Rightarrow \infty = \dfrac{\omega + \omega/3}{\cancel{\boxed{1 - \frac{\omega^2}{3}}}\xrightarrow{}0} \Rightarrow \omega = \sqrt{3}$.

$M = \dfrac{K\sqrt{\omega^2+9}}{\omega\sqrt{1+\omega^2}}$

$\Rightarrow K = 1$;

Q. $GH = \frac{K(s+2)}{(s-2)} \longrightarrow \left.\begin{matrix} P=1 \\ N=1 \end{matrix}\right\} ⓢ$ $K>1$ ⓢ.

$$\phi = -\left(180 - \tan^{-1}\frac{\omega}{2}\right) + \tan^{-1}\frac{\omega}{2}.$$

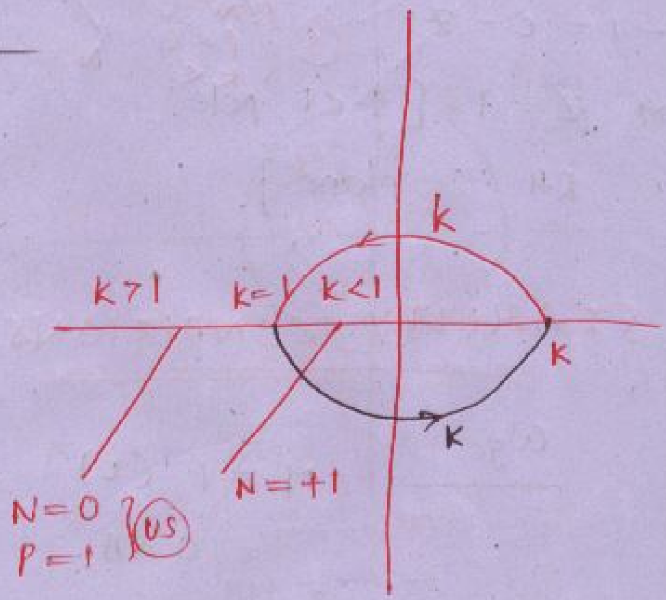
$$= -180 + 2.\tan^{-1}\frac{\omega}{2}.$$

$\omega = 0;\ K\angle{-180}$

$\omega = \infty;\ K\angle 0$

ED: Acw

SD: ×



$N = P - Z$

$0 = 1 - Z$

$\Rightarrow Z = 1$ [1 cl pole in RH s-plane]

Q. $GH = \frac{K(s-2)}{(s+2)}$

$$\phi = -\tan^{-1}\frac{\omega}{2} + \left(180 - \tan^{-1}\frac{\omega}{2}\right)$$

$$= 180 - 2.\tan^{-1}\frac{\omega}{2}.$$

$\omega = 0;\ K\angle{-180^\circ}$

$\omega = \infty;\ K\angle 0^\circ$

ED: cw SD: ×.

$$k\frac{(s-2)}{(s+2)}$$

$\longrightarrow P = 0.$

$N = -1$

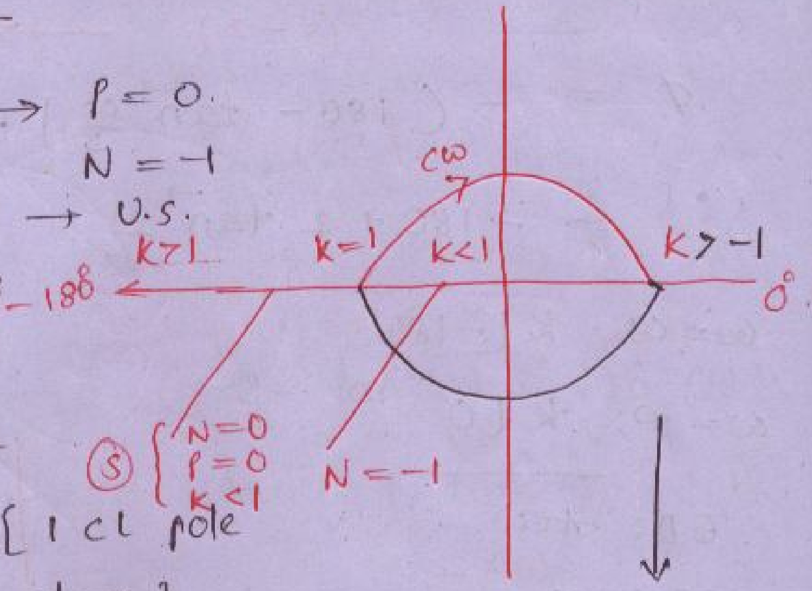
$\Rightarrow N \neq P \rightarrow$ U.S.

$K > 1 \rightarrow$ U.S

$N = P - Z$

$-1 = 0 - Z$

$\Rightarrow Z = 1$ [1 CL pole RH s-plane].



$-1 < K < 1$

STABILITY CONDITIONS:

* 29/12/08 *

$$\omega_{gc} \longrightarrow M = 1 \ (L) = 0 \text{ dB}.$$

$$\omega_{pc} \longrightarrow -180^\circ.$$

$$GM = \frac{1}{|M|}\Big|_{\omega=\omega_{pc}} \quad (L)$$

$$= -20\log|GH(j\omega)|\Big|_{\omega=\omega_{pc}}.$$

$$PM = 180 + \angle GH\Big|_{\omega=\omega_{gc}}$$

$$\omega_{pc} > \omega_{gc} \longrightarrow Ⓢ \left.\begin{matrix} GM>1 \\ +ve\ dB \end{matrix}\right\} PM\ +ve$$

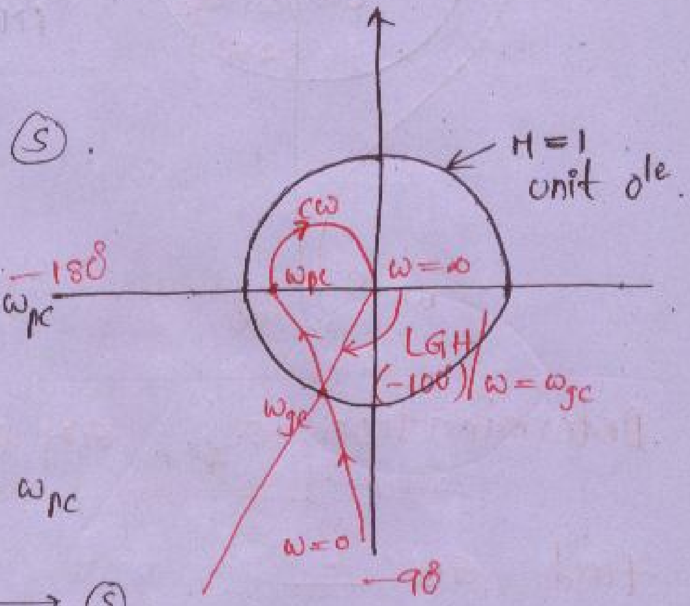
$\omega_{pc} = \omega_{gc} \xrightarrow{\quad} M.S$ $\left.\begin{matrix} GM = 1 \\ 0\,dB \end{matrix}\right\}$ $PM = 0$.

$\omega_{pc} < \omega_{gc} \xrightarrow{\quad} U.S$ $\left.\begin{matrix} GM < 1 \ (L) \\ -ve \text{ in } dB \end{matrix}\right\}$ PM -ve

Q. Identify stability.

$\omega_{pc} > \omega_{gc} \xrightarrow{\quad}$ Ⓢ.

$GM = \frac{1}{|M|/_{\omega=\omega_{pc}}}$



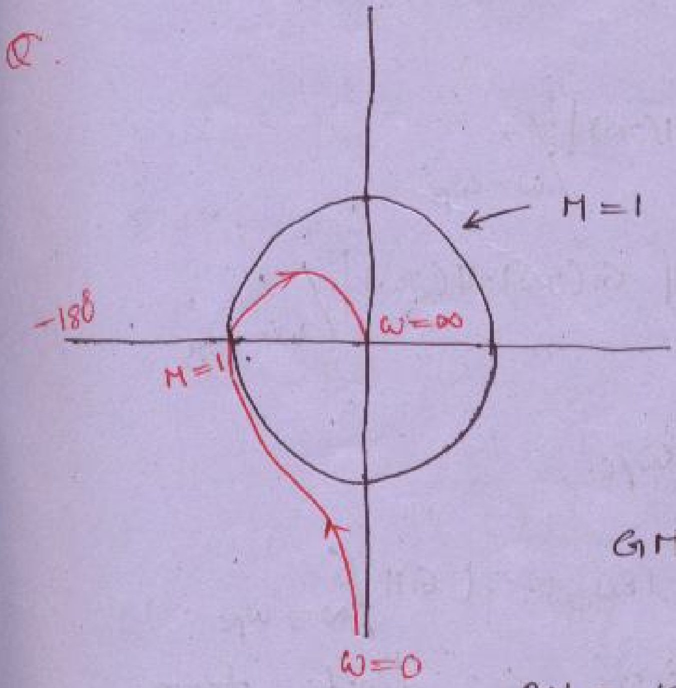
$= \frac{1}{<1/_{\omega=\omega_{pc}}}$

$\Longrightarrow$ $GM > 1 \longrightarrow$ Ⓢ.

$PM = 180 - 100$

$= +80$

Q.

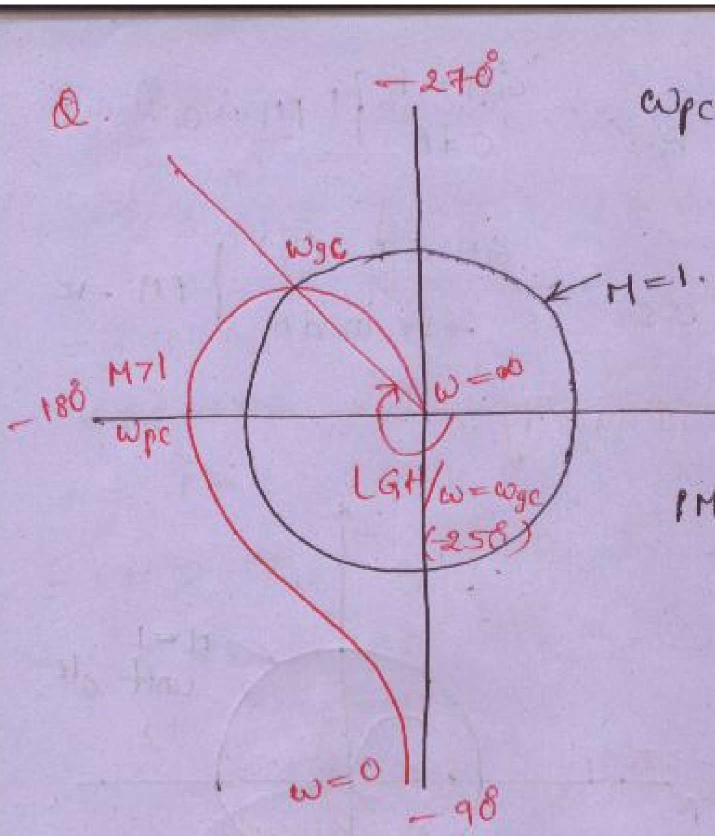


$\omega_{pc} = \omega_{gc} \xrightarrow{\quad} M.S.$

$GM = \frac{1}{1/_{\omega=\omega_{pc}}} = 1$

$PM = 180 - 180 = 0°$.

Q.



$\omega_{pc} < \omega_{gc} \longrightarrow$ U.S.

$$GM = \frac{1}{M/_{\omega=\omega_{pc}}}$$

$$= \frac{1}{>1} = <1 \text{ (U.S)}$$

$$PM = 180 - 250°$$

$$= +(-70) \text{ (U.S)}.$$

Determination of GM & PM:

find GM ___

$$GH = \frac{1}{s(s+1)(s+2)}$$

$$GM = \frac{1}{|G(j\omega)H(j\omega)|/_{\omega=\omega_{pc}}}.$$

$$= -20\log|G(j\omega)H(j\omega)|/_{\omega=\omega_{pc}}$$

Step 1 : find ω_{pc},

Method 1 : $-180 = \angle GH/_{\omega=\omega_{pc}}$.

if T/F consists ≤ 2 finite terms.

(or) T/F consists exponential terms.

Method 2 :

Expand the terms and make odd power s - terms = 0. [if T/F consists ≥ 3 finite terms].

$$-180 = \angle GH \Big/_{\omega=\omega_{pc}}$$

$$\Rightarrow -180 = -90 - \tan^{-1}\omega - \tan^{-1}\omega/2$$

$$\Rightarrow 90 = \tan^{-1}\omega + \tan^{-1}\frac{\omega}{2}$$

$$\Rightarrow 90 = \tan^{-1}\left[\frac{\omega+\omega/2}{\boxed{1-\omega^2/2} \rightarrow 0}\right]$$

∞

$$\Rightarrow \omega_{pc} = \sqrt{2}.$$

$$M = \frac{1}{\omega\sqrt{(1+\omega^2)(4+\omega^2)}}\Big/_{\omega_{pc}=\sqrt{2}}$$

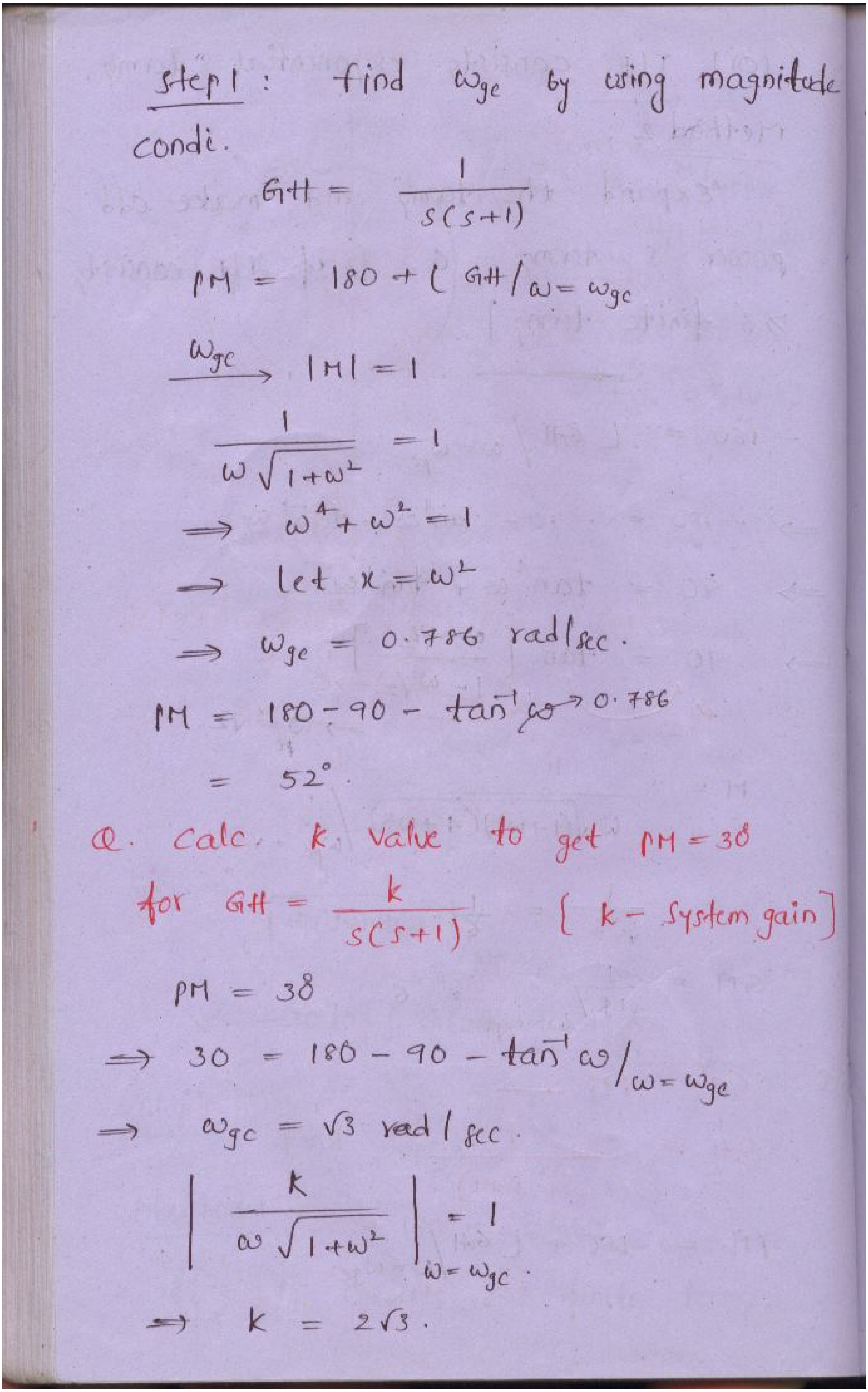
$$= \frac{1}{\sqrt{36}} = \frac{1}{6}.$$

$$GM = \frac{1}{M}\Big/_{\omega=\omega_{pc}} = 6.$$

Q. Calc. PM ___

$$GH = \frac{1}{s(s+1)}.$$

$$PM = 180 + \angle GH \Big/_{\omega=\omega_{gc}}$$



Step 1 : find ω_{gc} by using magnitude condi.

$$GH = \frac{1}{s(s+1)}$$

$$PM = 180 + \angle GH \Big|_{\omega=\omega_{gc}}$$

$$\omega_{gc} \longrightarrow |M| = 1$$

$$\frac{1}{\omega\sqrt{1+\omega^2}} = 1$$

$$\Rightarrow \omega^4 + \omega^2 = 1$$

$$\Rightarrow \text{let } x = \omega^2$$

$$\Rightarrow \omega_{gc} = 0.786 \text{ rad/sec.}$$

$$PM = 180 - 90 - \tan^{-1}\omega \rightarrow 0.786$$

$$= 52^\circ.$$

Q. Calc. k value to get PM = 30

for $GH = \dfrac{k}{s(s+1)}$ [k − System gain]

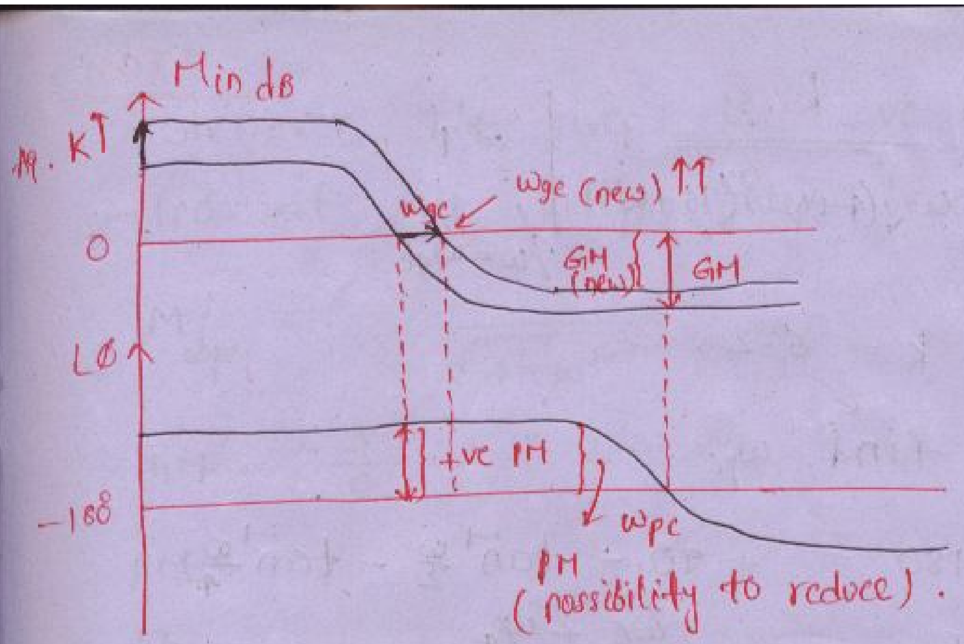
$$PM = 30$$

$$\Rightarrow 30 = 180 - 90 - \tan^{-1}\omega \Big|_{\omega=\omega_{gc}}$$

$$\Rightarrow \omega_{gc} = \sqrt{3} \text{ rad/sec.}$$

$$\left| \frac{k}{\omega\sqrt{1+\omega^2}} \right|_{\omega=\omega_{gc}} = 1$$

$$\Rightarrow k = 2\sqrt{3}.$$



As Gain k increases;

→ ω_{gc} ↑↑

→ No change in ω_{pc}

→ GM ↓↓

→ PM ↓↓.

Q. find k value to get PM = 60.

find k value to get GM = 20 dB.

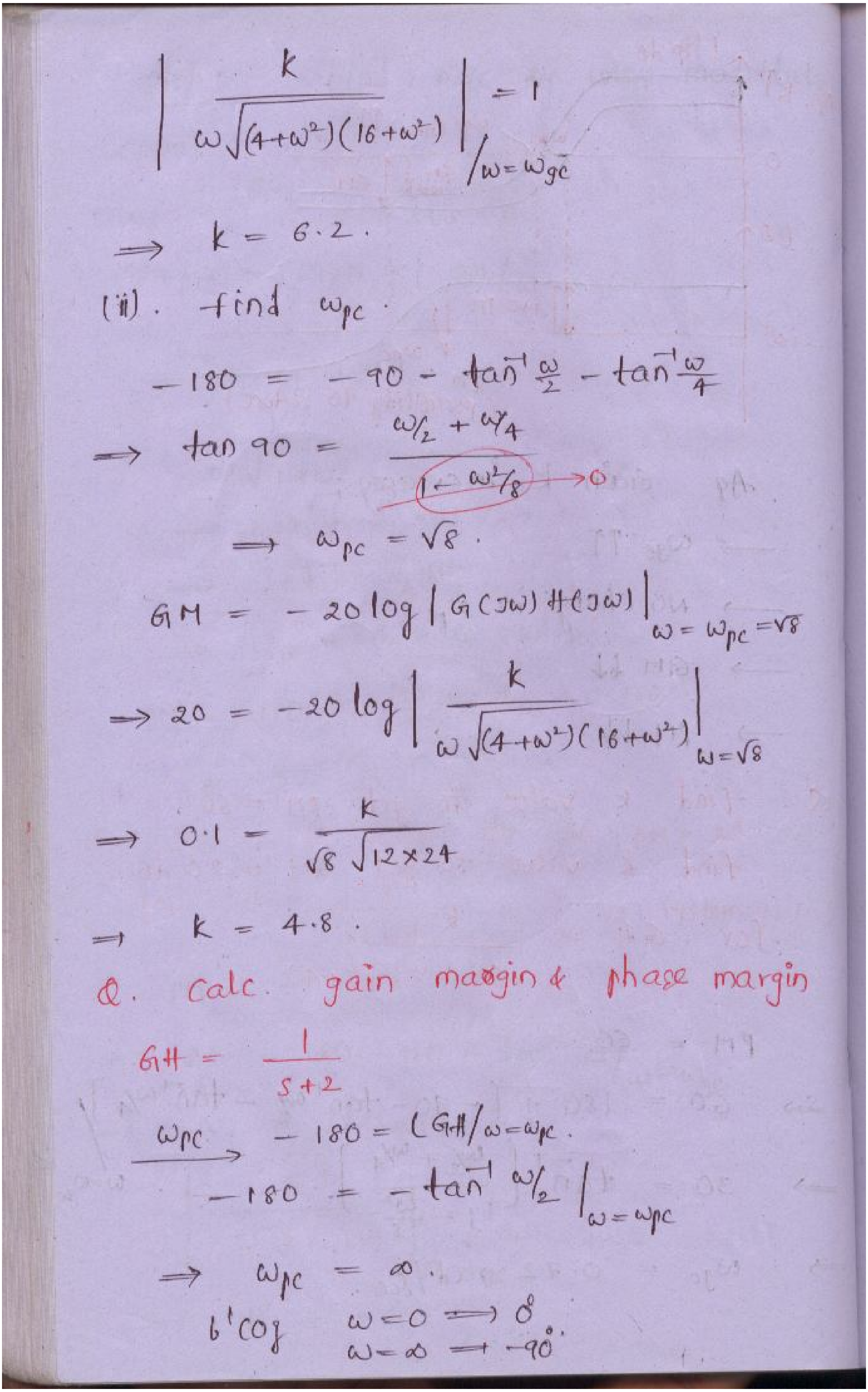
for . $GH = \frac{k}{s(s+2)(s+4)}$

PM = 60.

$$\Rightarrow 60 = 180 + \left[-90 - \tan^{-1}\omega/2 - \tan^{-1}\omega/4\right]\Big|_{\omega=\omega_{gc}}$$

$$\Rightarrow 30 = \tan^{-1}\left(\frac{\omega/2 + \omega/4}{1 - \frac{\omega^2}{8}}\right)$$

$$\Rightarrow \omega_{gc} = 0.72 \text{ rad/sec.}$$



$$\left|\frac{k}{\omega\sqrt{(4+\omega^2)(16+\omega^2)}}\right|_{/\omega=\omega_{gc}} = 1$$

$\Rightarrow$ $k = 6.2.$

(ii). find ω_{pc}.

$$-180 = -90 - \tan^{-1}\frac{\omega}{2} - \tan^{-1}\frac{\omega}{4}$$

$$\Rightarrow \tan 90 = \frac{\omega/2 + \omega/4}{\cancel{1-\omega^2/8} \rightarrow 0}$$

$$\Rightarrow \omega_{pc} = \sqrt{8}.$$

$$GM = -20\log\left|G(j\omega)H(j\omega)\right|_{\omega=\omega_{pc}=\sqrt{8}}$$

$$\Rightarrow 20 = -20\log\left|\frac{k}{\omega\sqrt{(4+\omega^2)(16+\omega^2)}}\right|_{\omega=\sqrt{8}}$$

$$\Rightarrow 0.1 = \frac{k}{\sqrt{8}\sqrt{12\times24}}$$

$\Rightarrow$ $k = 4.8.$

Q. calc. gain margin & phase margin

$$GH = \frac{1}{s+2}$$

$$\omega_{pc} \xrightarrow{\quad} -180 = \angle GH/_{\omega=\omega_{pc}}.$$

$$-180 = -\tan^{-1}\omega/2\Big|_{\omega=\omega_{pc}}$$

$$\Rightarrow \omega_{pc} = \infty.$$

b'coz $\omega=0 \Rightarrow 0^{\circ}.$

$\omega=\infty \rightarrow -90^{\circ}$

NOTE :

whenever T/F gives less -ve than -180 at all freq-s then ω_{pc} becomes ∞.

$$M\Big|_{\omega_{pc}} = \frac{1}{\sqrt{4+\cancel{\omega^2}\to\infty}} = 0.$$

$$GM = \frac{1}{0} = \infty.$$

PM :

$$\xrightarrow{\omega_{gc}} \quad M = 1$$

$$\frac{1}{\sqrt{4+\omega^2}} = 1$$

$$\omega = 0 \longrightarrow 0.5$$

$$\omega = \infty \longrightarrow 0$$

$$\Longrightarrow \omega_{gc} = 0.$$

NOTE:

whenever T/F gives less magnitude than 1 at all freq-s then $\omega_{gc} = 0$.

$$PM = 180 - \tan^{-1}\cancel{\omega}/2 \to 0$$

$$= 180 \longrightarrow (\infty) \quad Ⓢ. \ \underline{\text{stable}}$$

Ans: (∞, ∞)

Q. $GH = \frac{1}{s}$.

GM :

$$\xrightarrow{\omega_{pc}} \quad -180 = \angle GH$$

$$-180 = -90$$

$$\Longrightarrow \omega_{pc} = \infty.$$

$M|_{\omega_{pc}} = \frac{1}{\omega} = 0.$

$GM = \frac{1}{0} = \infty.$

PM:

$\xrightarrow{\omega_{gc}} M = 1.$

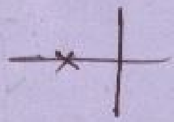
$\frac{1}{\omega} = 1$

$\longrightarrow \omega_{gc} = 1.$

$PM = 180 - 90$

OL T/F $\frac{1}{s}$

$= 90 \longrightarrow$ Ⓢ. (CL T/F $\frac{1}{s+1}$)



Q. $GH = \frac{1}{s^2}$

GM:

$\xrightarrow{\omega_{pc}} -180 = -180$

$\Rightarrow \omega_{pc} = 0$ to ∞.

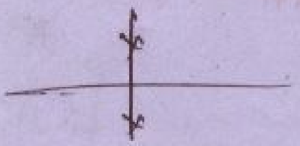
In this case ω_{pc} is decided by ω_{gc}.

$\omega_{gc} = \frac{1}{|\omega^2|} = 1$

$\Rightarrow \omega_{gc} = 1 = \omega_{pc}.$

CL T/F. $\frac{1}{s^2+1}$

$GM = \frac{1}{1} = 1$ (L)



$PM = 180 - 180$

$= 0$ (M.S).

Q. $GH = \frac{1}{S^3}$.

GM:

$\xrightarrow{\omega_{pc}} \quad -180° = -270°$

$\rightarrow \omega_{pc} = 0$

whenever System gives more −ve than −180 at all freq.s then $\omega_{pc} = 0$.

$M|_{\omega_{pc}} = \frac{1}{\omega^3} = \infty$

$\Longrightarrow GM = \frac{1}{\infty} = 0.$

PM:

$\xrightarrow{\omega_{gc}} \quad M = 1 \; ; \quad \frac{1}{\omega^3} = 1$

$\rightarrow \omega_{gc} = 1.$

$PM = 180 - 270 = -90.$

Q. In G(s)H(s) plane the Nyquist plot of loop T/F $G(s)H(s) = \frac{\pi \cdot e^{-0.25S}}{S}$ passes through −ve real axis at the point is ___.

(a). (−0.25, j0) (b). (−0.5, j0)

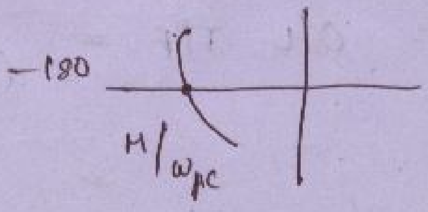
(c). (−1, j0) (d). (−2, j0).

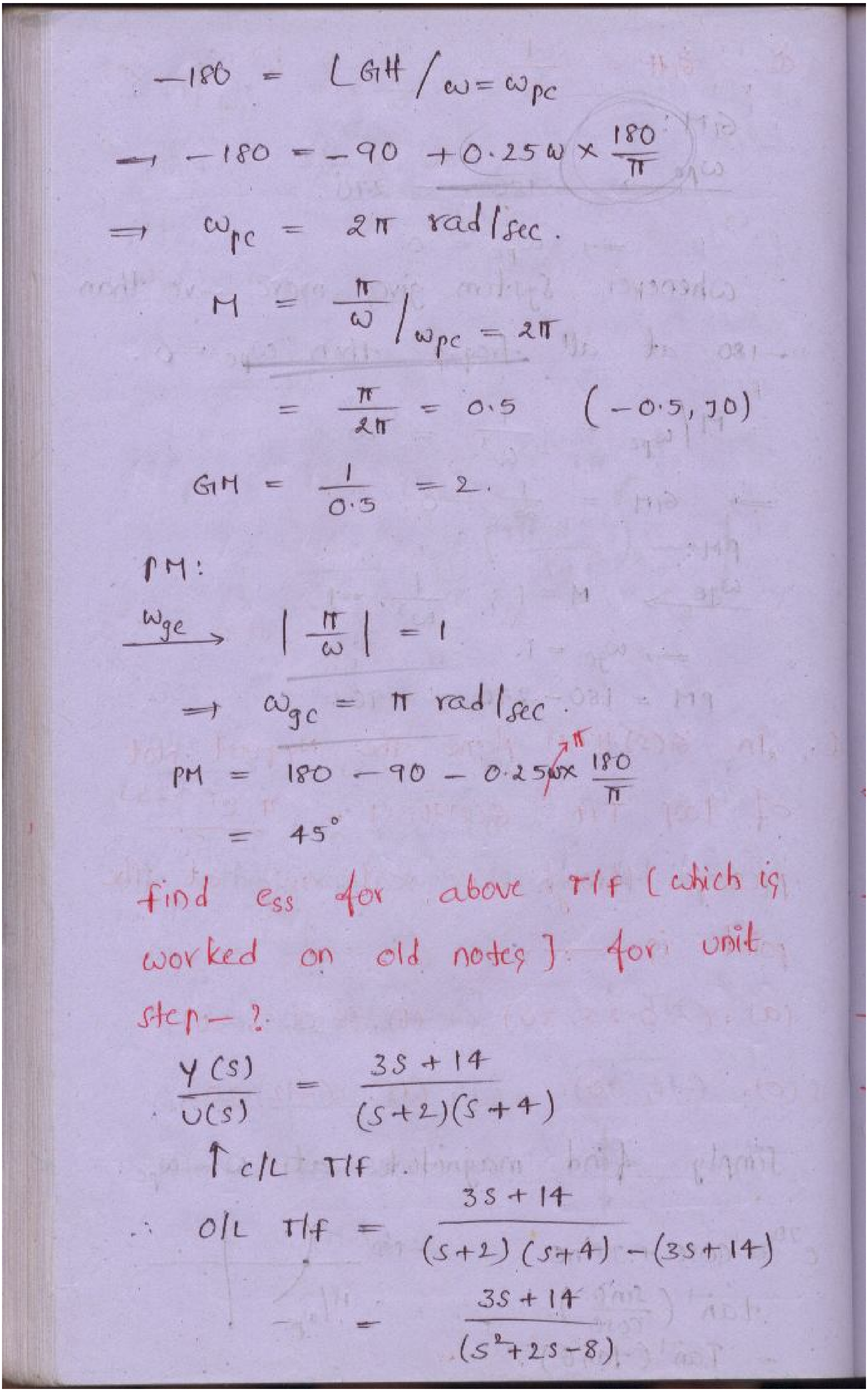
Simply find magnitude at $\omega = \omega_{pc}$.

$e^{j\theta} = \cos\theta + j\sin\theta$

$\tan^{-1}\left(\frac{\sin\theta}{\cos\theta}\right)$

$= \tan^{-1}(\tan\theta).$





$$-180 = \angle GH \Big/_{\omega=\omega_{pc}}$$

$$\rightarrow -180 = -90 + 0.25\omega \times \frac{180}{\pi}$$

$$\Rightarrow \omega_{pc} = 2\pi \text{ rad/sec.}$$

$$M = \frac{\pi}{\omega}\Big/_{\omega_{pc}=2\pi}$$

$$= \frac{\pi}{2\pi} = 0.5 \qquad (-0.5, j0)$$

$$GM = \frac{1}{0.5} = 2.$$

PM:

$$\xrightarrow{\omega_{gc}} \left|\frac{\pi}{\omega}\right| = 1$$

$$\Rightarrow \omega_{gc} = \pi \text{ rad/sec.}$$

$$PM = 180 - 90 - 0.25\overset{\pi}{\cancel{\omega}} \times \frac{180}{\pi}$$

$$= 45^\circ$$

find e_{ss} for above T/F (which is worked on old notes] for unit step – ?

$$\frac{Y(s)}{U(s)} = \frac{3s+14}{(s+2)(s+4)}$$

↑ c/L T/F.

$$\therefore \text{O/L T/F} = \frac{3s+14}{(s+2)(s+4)-(3s+14)}$$

$$= \frac{3s+14}{(s^2+2s-8)}$$

$$e_{ss} = \frac{A}{1+k_p}$$

$$= \frac{1}{1+\frac{14}{-8}} = -\frac{8}{6} = -1.33.$$

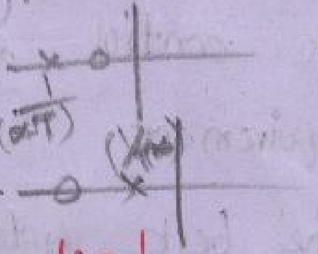
Compensator:-

A compensator is a n/w which add 1 finite pole, 1 finite zero, such that system performance is improved.

lag, lead } $\underline{\underline{T/f:}}$ $\frac{1+TS}{\boxed{1+\alpha TS}}$ pole

$\alpha < 1 \longrightarrow$ lead

$\alpha > 1 \longrightarrow$ lag.



* Lead - lag, Lag - lead } T/f : $\underbrace{\left[\frac{1+T_1 s}{1+\alpha T_1 s}\right]}_{}$ $\left[\frac{1+T_2 s}{1+\alpha T_2 s}\right]$

(lead) (lag)

→ $\omega_m = \frac{1}{T\sqrt{\alpha}}$ [freq. when max. ph. lead or ph. lag occurs].

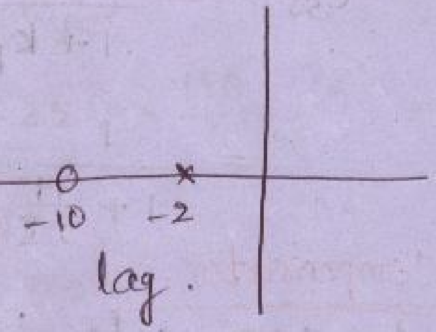
→ $\phi_m = \sin^{-1}\left[\frac{1-\alpha}{1+\alpha}\right] \longrightarrow$ lead. ($\alpha < 1$)

→ $\phi_m = \sin^{-1}\left\{\frac{\alpha-1}{\alpha+1}\right] \longrightarrow$ lag ($\alpha > 1$)

→ $M/_{\omega_m} = 10 \log\left(\frac{1}{\alpha}\right)$.

Q. $GH = \frac{s+10}{s+2}$

$$\frac{1+\tau s}{1+\alpha \tau s} = \frac{10(1+0.1s)}{2(1+0.5s)}$$



$\tau = 0.1$

$\alpha\tau = 0.5$

$\Rightarrow \alpha = 5.$

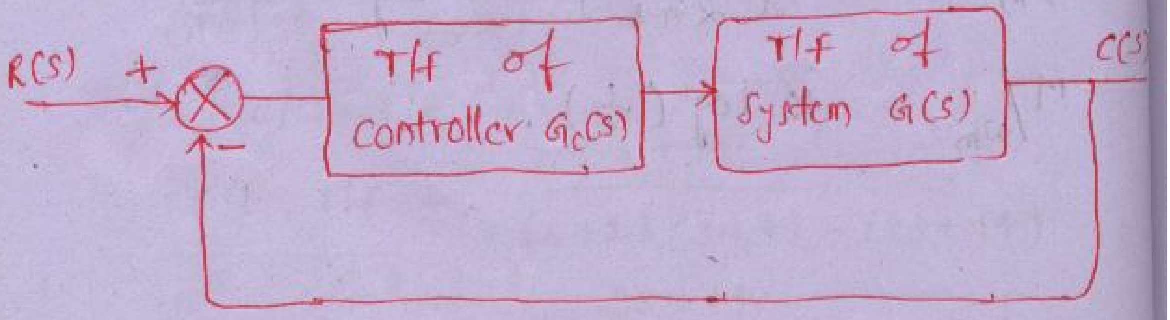
$$\omega_m = \frac{1}{0.1\sqrt{5}}$$

PID controllers :

A controller is a device which is used to control ss & tr. response as per requirement.

The best system demands smallest t_r, smallest t_s, smallest e_{ss} and smallest M_p, which is not possible without PID controllers.

Block diagram with controller shown in fig.



P- CONTROLLER:

To change tr. response as per requirement. T/F of P- controller is $= k_p$

Let consider the system $G(s) = \frac{1}{s(s+10)}$

$$\Longrightarrow \frac{C(s)}{R(s)} = \frac{1}{s^2+10s+1}$$

$\omega_n = 1$

$\xi = 5 \longrightarrow$ over damped.

with controller,

$$G(s) = \frac{k_p}{s(s+10)}$$

$$\frac{C(s)}{R(s)} = \frac{k_p}{s^2+10s+k_p}$$

If $k_p = 100$, $\omega_n = 10$

$\xi = 0.5 \longrightarrow$ under damped

$k_p = 25$, $\omega_n = 5$

$\xi = 1 \longrightarrow$ Critical damped.

→ The main dis. adv. in p- controller is as k_p value increases, ξ decreases hence % M_p increases.

As % M_p ↑↑, the system becomes less stable.

By using p-controller we get required nature by changing k_p value.

I - CONTROLLER:

purpose: To decrease ss error.

The T/f of Integral controller is $\frac{k_i}{s}$

I- controller add one ~~zero~~ pole at origin hence type is increased. As type increases the ss error decreases but system stability effected.

Eg: $G(s) = \frac{1}{s(s+10)}$ (without controller)

type = 1

CE: $s^2+10s+1 \longrightarrow$ Ⓢ.

with controller,

$$G(s) = \frac{k_i}{s^2(s+10)}$$

type = 2

CE: $s^3+10s^2+k_i=0 \longrightarrow$ (U.S).

D - CONTROLLER :

purpose : To improve the stability.

T/f of d- controller is $k_D . s$

D- controller add one zero at origin hence type is decreased. As type decreases stability improved but ss error increased.

Eg : $G(s) = \frac{1}{s^2(s+10)}$

type = 2

CE : $s^3 + 10s^2 + 1 = 0 \longrightarrow$ (U.S).

with controller.

$G(s) = \frac{k_D \cancel{s}}{s^{\cancel{2}}(s+10)}$

Type = 1

CE : $s^2 + 10s + k_D = 0$

PI - CONTROLLER :

purpose : To decrease ss error without effecting stability

T/f of PI - controller is $k_p + \frac{k_i}{s}$.

ie $\frac{sk_p + k_i}{s}$.

PI - controller add one pole at origin hence type is increased.

As Type increases e_{ss} decreases.

PI - controller add one finite zero in left of s-plane, which avoid effect on stability.

PD - CONTROLLER

PURPOSE :

To improve stability without effecting e_{ss}.

T/f of PD - controller is $k_p + k_D s$

PD - controller add only one finite zero in the left of s-plane hence System stability improved.

No change in type with PD - controller, hence no effect on ss error.

ξ value with PD - controller is

$$\xi_{PD} = \xi + \frac{\omega_n k_D}{2}$$

As ξ_{PD} ↑↑, % M_p ↓↓ → more stable.

PID - CONTROLLER:

Purpose:

To improve stability as well as to decrease e_{ss}.

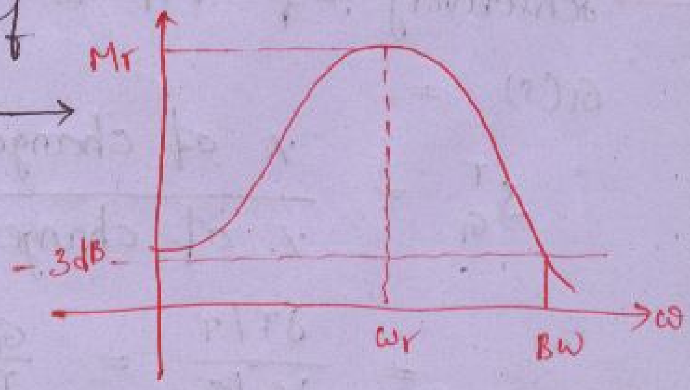
T/f of PID = $k_p + \frac{k_I}{s} + k_D s$

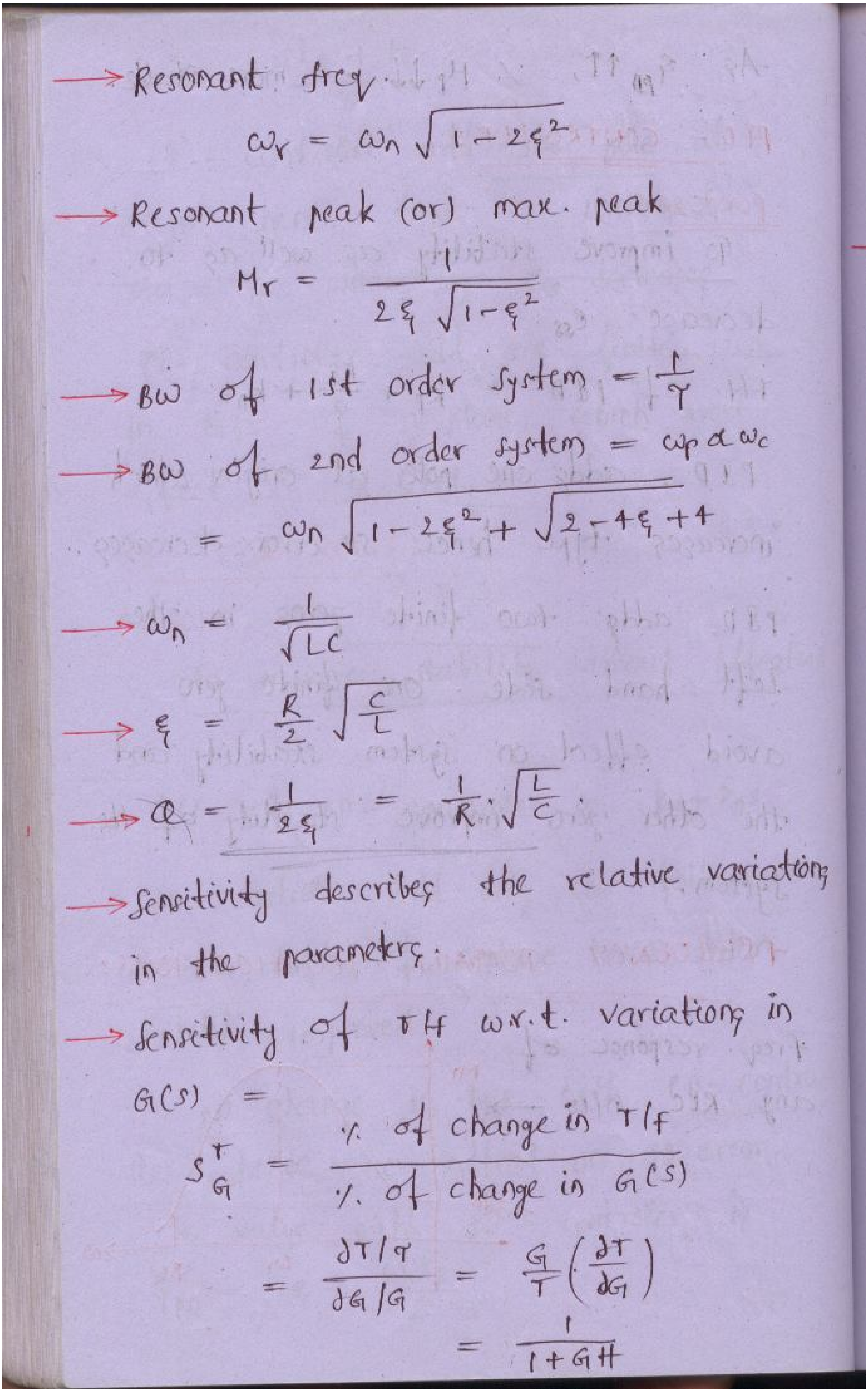
PID, adds one pole at origin which increases type hence ss error decreases.

PID, adds two finite zeros in the left hand side. One finite zero avoid effect on system stability and the other zero improve stability of the system.

FREQUENCY DOMAIN SPECIFICATIONS:

freq. response of any RLC n/w →





→ Resonant freq.

$$\omega_r = \omega_n \sqrt{1-2\xi^2}$$

→ Resonant peak (or) max. peak

$$M_r = \frac{1}{2\xi\sqrt{1-\xi^2}}$$

→ BW of 1st order system $= \frac{1}{\tau}$

→ BW of 2nd order system $= \omega_p \,\alpha\, \omega_c$

$$= \omega_n \sqrt{1-2\xi^2+\sqrt{2-4\xi+4}}$$

→ $\omega_n = \frac{1}{\sqrt{LC}}$

→ $\xi = \frac{R}{2}\sqrt{\frac{C}{L}}$

→ $Q = \frac{1}{2\xi} = \frac{1}{R} \cdot \sqrt{\frac{L}{C}}$

→ Sensitivity describes the relative variations in the parameters.

→ Sensitivity of T/F w.r.t. variations in G(s) =

$$S_G^T = \frac{\text{\% of change in T/F}}{\text{\% of change in G(s)}}$$

$$= \frac{\partial T/T}{\partial G/G} = \frac{G}{T}\left(\frac{\partial T}{\partial G}\right)$$

$$= \frac{1}{1+GH}$$

$$S_H^T = \frac{H}{T}\frac{\partial T}{\partial H} = \frac{-GH}{1+GH}.$$

o/l Sensitivity $= 1 = S_T^{o/l} > S_T^{c/l \text{ loop}}$

→ The o/l system is more sensitive compare to c/l system. In a c/l system f/b n/w is more sensitive compare to forward path.

FRI.
15/08/08

Subjects to be Targetted:

1. Signals & Systems — 20 M.
2. Digital & MP — 15 to 20 M
3. Mathematics — 20 M
4. EDC — 15 M
5. Networks — 15 to 20 M.
6. Control Systems — 10 to 15 M
7. Measurements — 10 to 15 M
8. Machines — 20 to 25 M
9. Power Systems — 20 to 25 M
10. Power Electronics — 10 to 15 M

Previous papers:

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